THE PETROV-GALERKIN FORMULATION APPLIED TO A CONVECTIVE DRYING MODEL

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Abstract. *The process of artificial drying of grains can be used to decrease the moisture content of agricultural products, facilitating the process of storage and conservation. This work presents the grain drying process simulation in a dryer cylinder with axial and radial flow of air with stationary bed was . Equations of bed moisture content, bed grain temperature, absolute humidity and air temperature of air were numerically solved. The problem is a pure convective problem and its was solved using the Finite Element Method (FEM), employing the Galerkin method formulation or the Streamline Upwind Petrov-Galerkin(SUPG). Both formulations shown satisfactory results using the same time steps. The application of the SUPG method has to have a large time step size of the classical formulation of the FEM.*

Keywords*: Drying, FEM, Petrov-Galerkin*

1. INTRODUCTION

França and Fortes (1994) present a model for one-dimensional and two-dimensional dryer using finite volume and finite element, based on equations of drying-layer. This model was used in the simulation of grain drying in stationary bed. Souza (1996) presents a model based on the model of França and Fortes (1994) to simulate an axis-symmetric dryer. This model describes the process of grain drying in a dryer of radial axial flow of air. Equations of Absolute Air Humidity and temperature of bed grains are solved using the finite element method and finite differences are employed to solve bed moisture content and bed temperature equations.

The model proposed by Souza (1996) uses the finite element method with the Galerkin formulation and time marching is done using an constant time step. In problems where the convection is dominant, the classical Galerkin formulation of finite elements method may present spurious oscillations which can be circumvented through appropriate choice of spatial and temporal mesh.

In this paper the equations are solved by (FEM) using the Petrov-Galerkin formulation, due to be purely convective model treaty, in order to compare with solutions obtained by Souza (1996).

The dimensionless Peclet number, shown in Eq.1, is used to evaluate the contribution of convective and diffusive terms in the partial differential equation discretization.

$$Pe = Uh/2k \tag{1}$$

When $Pe < 2$ indicates that the contribution of the diffusive term is predominant in the resolution, when Pe is greater than this limit means that the greater contribution of convective term (Zienkiewicz & Taylor, 2000). In this case the Galerkin formulation can lead to distorted solutions. The Petrov-Galerkin formulation suggests a new weighting function able to minimize or eliminate these problems.

The Petrov-Galerkin formulation in this paper is the *Streamline Upwind Petrov-Galerkin* (SUPG). This method introduces a consistent stability in the direction of the lines of current. One form of treatment of the Petrov-Galerkin formulation is presented by Sampaio (1991), and is used in this paper. Following the criteria of the formulation for convective-diffusive problems, it obtained a new weight function (w).

2. METHODOLOGY

2.1. Simulation of grain drying in a axisymmetric dryer – Numerical Model

This paper presents the model described by Souza (1996) to simulate the grain drying in a axisymmetric dryer. The solution of differential equations, which will be studied in this paper was performed using the Finite Element Method and Finite Differences.

The results presented in Souza (1996) in their work were compared with experimental data obtained in tests, performed by EMBRAPA, in an axis-symmetric dryer.

The partial differential equations to be solved together with their respective initial conditions and control are summarized below.

- Moisture content of the bed:

$$\frac{\partial M}{\partial t} = \frac{M_e - M}{3600\sqrt{A^2 + (1/900)Bt}} \qquad (2)$$

$0 \le z \le 3{,}60 \quad t > 0 \quad 0{,}40 \le r \le 1{,}0$

Initial Conditions:

$$M = M_0 \qquad 0 \le z \le 3{,}60 \quad t = 0 \qquad 0{,}40 \le r \le 1{,}0$$

where M is the moisture content of the bed;
M_0 is the equilibrium moisture content;
A e B values are specific to the maize desired range of pressure.

- Absolute humidity of air:

$$\frac{\partial W}{\partial t} + \left(\frac{\vec{v}_r}{\epsilon}\right)\frac{W}{r} + \frac{1}{\epsilon}\left[\vec{v}_r\frac{\partial W}{\partial r} + \vec{v}_z\frac{\partial W}{\partial z}\right] = \frac{\rho_p}{\epsilon\rho_a}\frac{\partial W}{\partial t} \qquad (3)$$

$0 \le z \le 3{,}60 \quad t > 0 \quad 0{,}40 \le r \le 1{,}0$

Boundary conditions 1:

$$W = W_{in} \qquad z=0 \quad t > 0 \qquad 0{,}40 \le r \le 1{,}0$$

Boundary conditions 2:

$$W = W_{in} \qquad r=0{,}40 \quad t > 0 \qquad 0 \le z \le 3{,}6$$

Initial conditions:

$$W = W_0 \qquad 0 \le z \le 3{,}60 \quad t = 0 \qquad 0{,}40 \le r \le 1{,}0$$

Where W is the absolute humidity of air;
ε is the bed porosity (volume of pores / total volume);
ρ_p is the product density;
ρ_a is the air density;
$\vec{v}$ is the air velocity vector.

- Air temperature:

$$\frac{\partial T}{\partial t} + \left(\frac{\vec{v}_r}{\epsilon r}\right)T + \frac{\vec{v}_r}{\epsilon} + \frac{\vec{v}_r}{\epsilon}\frac{\partial T}{\partial r} + \frac{\vec{v}_z}{\epsilon}\frac{\partial T}{\partial z} = \frac{ha(T-\theta)}{\epsilon\rho_a(c_a + Wc_v)} \qquad (4)$$

$0 < z \le 3{,}60 \quad t > 0 \quad 0{,}40 < r \le 1{,}0$

Boundary conditions 1:

$$T = T_{in} \qquad \begin{array}{l} z=0 \\ 0{,}40 \leq r \leq 1{,}0 \end{array} \qquad t > 0$$

Boundary conditions 2:

$$T = T_{in} \qquad \begin{array}{l} r=0{,}40 \\ 0 \leq z \leq 3{,}6 \end{array} \qquad t > 0$$

Initial conditions:

$$T = T_0 \qquad \begin{array}{l} 0 \leq z \leq 3{,}60 \\ 0{,}40 \leq r \leq 1{,}0 \end{array} \qquad t = 0$$

where
T is the air temperature;
h is the convective heat transfer coefficient (W/m² K);
c is the specific heat (J/kg K);

- Temperature of the bed grain:

$$\frac{\partial \theta}{\partial t} = \frac{h_a(T-\theta)}{\rho_p(c_p + c_w M)} + \frac{h_{fg} + c_v(T-\theta)}{(c_p + c_w M)} \frac{\partial M}{\partial t} \tag{5}$$

$0 \leq z \leq 3{,}60 \quad t > 0 \quad 0{,}40 \leq r \leq 1{,}0$

Initial conditions:

$$\theta=\theta_0 \qquad \begin{array}{l} 0 \leq z \leq 3{,}60 \\ 0{,}40 \leq r \leq 1{,}0 \end{array} \qquad t = 0$$

where θ is the temperature of the bed grain;
h_{fg} is the latent heat of vaporization of water (J.kg)

2.2. Discretization of Equations

The problem addressed in this paper presents the equation of Absolute Humidity of Air and Air Temperature. These equations are purely convective and are used to obtain the additional weighting function following the methodology proposed by Sampaio (1991),

Absolute humidity of air:

$$\frac{\partial W}{\partial t} + \frac{1}{\epsilon}\left[\vec{v}_r \frac{\partial W}{\partial r} + \vec{v}_z \frac{\partial W}{\partial z}\right] = -\frac{\rho_p}{\epsilon \rho_a} \frac{\partial W}{\partial t} \tag{6}$$

Equation (3) is discretized in time by a central scheme:

$$A + \left(\frac{\rho_p}{\epsilon \rho_a} \frac{\partial W}{\partial t}\right)^{(n+1/2)} = 0 \tag{7}$$

where

$$A = \frac{w^{n+1} - w^n}{\Delta t} + \frac{(\vec{v}_r + \vec{v}_z)}{\epsilon}.\nabla W^{1/2}$$
$$\nabla W^{n+1/2} = \frac{1}{2}(W^{n+1} + W^n)$$

Discretization of the Absolute Humidity of Air in the space is done using finite elements

$$W^{\hat{n}+1} = N_j W_j^{n+1}$$

where W_j^{n+1} is the nodal value at the time **n+1** e $\mathbf{N_j}$ is the shape function associated.
A can be discretized as **Â** accordingly (eq. 8),

$$\hat{A} = \frac{W^{\hat{n}+1} - \hat{W^n}}{\Delta t} + \frac{(\vec{v}_r + \vec{v}_z)}{\epsilon}.\nabla W^{n\hat{+}1/2} \tag{8}$$

The square of the residue to approximation of A by Â is given by:

$$R = \int_\Omega [\hat{A} - A]^2 d\Omega \tag{9}$$

Replacing (7) in (9) get

$$R = \int_\Omega \left[\hat{A} + \left(\frac{\rho_p}{\epsilon \rho_a}\frac{\partial W}{\partial t}\right)^{(n+1/2)}\right]^2 d\Omega \tag{10}$$

The square of the residue should be minimized with respect to free parameters W_j^{n+1}

$$\frac{\partial R}{w_j^{n+1}} = \int_\Omega 2\left[\hat{A} + \left(\frac{\rho_p}{\epsilon \rho_a}\frac{\partial W}{\partial t}\right)^{(n+1/2)}\right]\frac{\partial \hat{A}}{\partial w_j^{n+1}} d\Omega = 0 \tag{11}$$

The equation above is equivalent to the Petrov-Galerkin method with the weight functions $N_i + w_i = w_a$.

$$\int_\Omega (w_a)\left[\hat{A} + \left(\frac{\rho_p}{\epsilon \rho_a}\frac{\partial W}{\partial t}\right)^{(n+1/2)}\right] d\Omega = 0 \tag{12}$$

$$w_i = \frac{\Delta t}{2}\left(\frac{\vec{v}_r + \vec{v}_z}{\epsilon}\right)\nabla N_i \tag{13}$$

Discretizing the equation of Absolute Humidity of Air, using the formulation of Sampaio (1991) and rewriting the Eq. (12), get:

$$\int_\Omega (w_a)\left[\left(\frac{W^{\hat{n}+1} - \hat{W^n}}{\Delta t} + \frac{(\vec{v}_r + \vec{v}_z)}{\epsilon}.\nabla W^{n\hat{+}1/2}\right) + \left(\frac{\rho_p}{\epsilon \rho_a}\frac{\partial W}{\partial t}\right)^{(n+1/2)}\right] d\Omega = 0 \tag{14}$$

$$\int_\Omega (w_a)\left(\frac{W^{\hat{n}+1} - \hat{W^n}}{\Delta t} + \frac{(\vec{v}_r + \vec{v}_z)}{\epsilon}.\nabla W^{n\hat{+}1/2}\right) d\Omega + \int_\Omega (w_a)\left(\frac{\rho_p}{\epsilon \rho_a}\frac{\partial W}{\partial t}\right)^{(n+1/2)} d\Omega = 0 \tag{15}$$

Considering the limit as steady state, where the values of $\hat{W}$ at time **n+1** tend to the values of $\hat{W}$ at time **n**, the Sampaio formulation for equation above is

$$\int_{\Omega}(w_a)\left(\frac{(\vec{v}_r+\vec{v}_z)}{\epsilon}.\nabla W^{\hat{n+1}}\right)d\Omega+\int_{\Omega}(w_a)\left(\frac{\rho_p}{\epsilon\rho_a}\frac{\partial W}{\partial t}\right)^{(n+1)}d\Omega=0 \quad (16)$$

In a similar the air temperature equation can be manipulated resulting in eq 17.

$$\int_{\Omega}(w_a)\left(\frac{(\vec{v}_r+\vec{v}_z)}{\epsilon}.\nabla T^{\hat{n+1}}\right)d\Omega+\int_{\Omega}(w_a)\left(\frac{ha(T-\theta)}{\epsilon\rho_a(c_a+Wc_v)}\right)^{(n+1)}d\Omega=0 \quad (17)$$

3. RESULTS AND DISCUSSION

Using a constant time step size of 40 seconds as proposed by Souza (1996), the results for the formulation of Galerkin and SUPG formulation are very close.

Increasing indiscriminately the time step size, large distortions are observed with the entry of the initial conditions, when using the direct method of initialization. A technique that to solve the problem is the use of a smoothing function (Vieira and JR., 2000), to promote a reduction in the time step size in the first times.

The smoothing function presented in Vieira and Jr (2000) has the form $\eta(\phi) = 1 - e^{\phi}$ where $\phi = t/\tau$ and τ is an appropriate smoothing factor. The initial step in time was 4 seconds of time to step up to 100 seconds.

The domain simulated considered an axisymmetric dryer with the following dimensions: external diameter of 2 m, height 3.90 m, 0.80 m diameter of the central duct. The experimental measurements were performed at 4, 8 and 12 hours of drying in the dryer at three distinct elevations of 0.36 m, 1.80 m and 3.60 m (Fig.1). A total of nine points in three positions along the radial coordinate at each time.

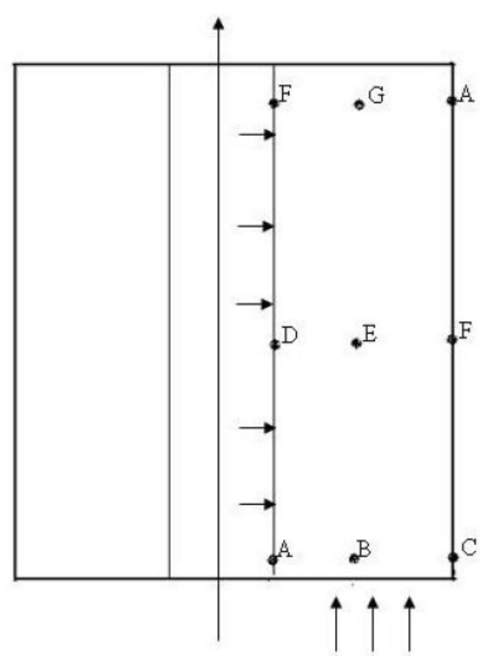

Figure 1. Representation of the dryer with the collection points of experimental data

In Fig.2 and Fig.3 solutions are presented along the radius of the dryer, comparing experimental data and numerical results from the Galerkin and SUPG formulation. In fig.2 the SUPG results are closer the experimental data than the Galerkin results. The results are closer as the dryer radius increases. The SUPG results using the smallest time step are closer to the experimental data. The influence of the time step can be observed for a radius value greater than 0,65 m where the results present a dispersion.

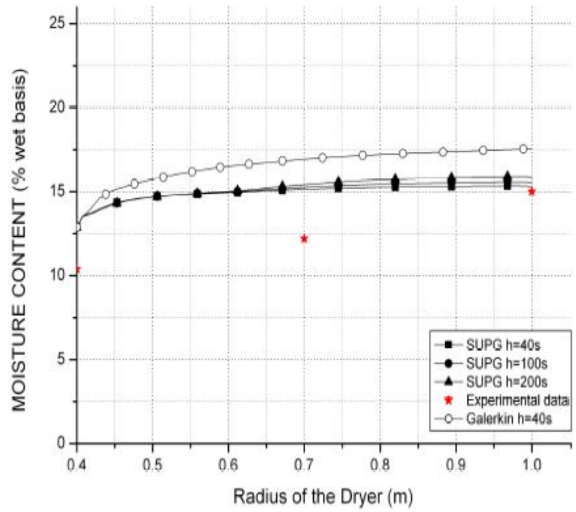

Figure 2. Moisture content profile of the bed after 12 hours of drying to a height of 0.36 m, with the air temperature of 42°C

In fig.3 the Galerkin results are closer to the experimental data than the SUPG results. The results are closer as the dryer radius decreases. The SUPG results using a time step of 100s are closer to the experimental data. The influence of the time step can be observed for a radius value greater than 0,75 m where the results present a dispersion.

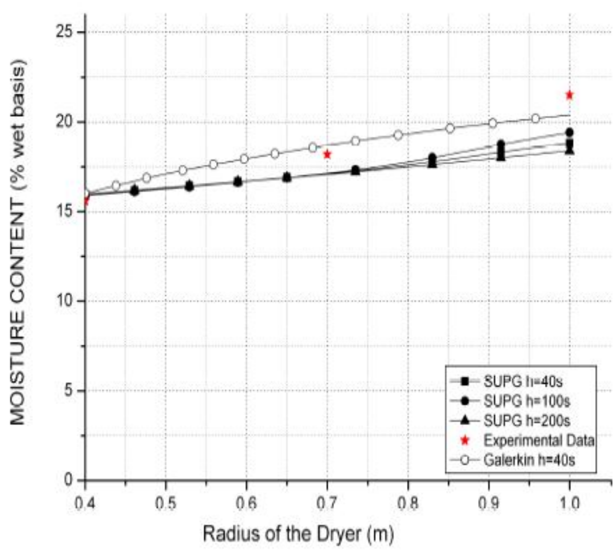

Figure 3. Moisture content profile of the bed after 8 hours of drying to a height of 1.80 m, with the air temperature of 42°C

The Fig.4 shows the moisture content of bed in time for a flow of air 9500 m³/s the air temperature of 42ºC into the dryer. The SUPG method presents the best solution after 12 hrs of drying. Unfortunately it was not possible to present experimental data curve of Middle of Moisture Content over time (Alves, 2001). It is clear that the SUPG method consistent with the reality of an Middle of Moisture Content lower, considering the experimental data at the point of collection along the time.

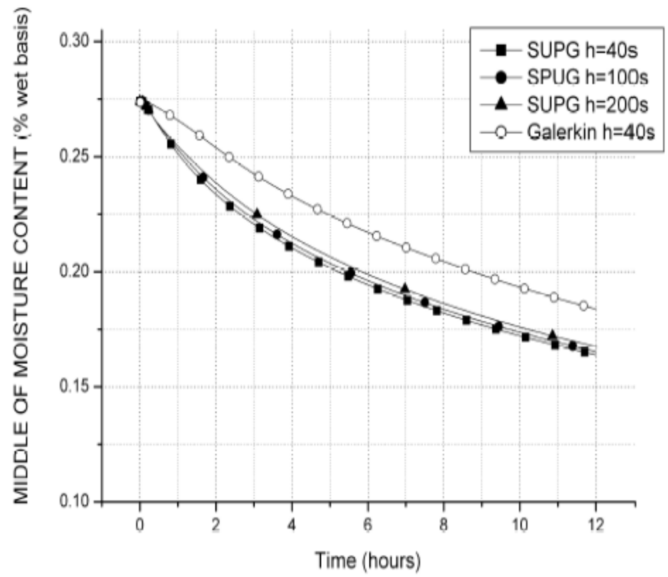

Figure 4. Moisture content profile of the bed with time (Air Flow=9500 m^3/s)

4. CONCLUSIONS

The results suggest the economy of time and computational effort in adopting the SUPG method applied to the drying problem proposed by Souza (1996). The results are confirmed by experimental data presented. The refinement of the time step is appropriate and results in acceptable solutions.

In the model presented by Souza (1996) the maximum time step size of 40 seconds was obtained using the formulation of Galerkin. The SPUG method proposed using larger time steps size, up to 200 seconds, lead to solutions closer to those obtained by Souza Souza (1996).

A proposed future is the adoption of an adaptive time step, where the choice of the time step size is done automatically, through the local error, since the formulation of Petrov-Galerkin allows time steps higher than those allowed by the classical formulation of Galerkin.

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