SOME ASPECTS AND APPLICATIONS OF STATE OBSERVERS METHODOLOGY FOR CRACK DETECTION, LOCALIZATION AND EVALUATION IN CONTINUOUS SYSTEMS

Edson Luiz Valverde Castilho Filho, edslufil@aluno.feis.unesp.br
Vinícius Fernandes, viniciusfer@aluno.feis.unesp.br
Gilberto Pechoto de Melo, gilberto@dem.feis.unesp.br
Universidade Estadual Paulista "Júlio de Mesquita Filho" – Faculdade de Engenharia de Ilha Solteira, Av. Brasil Centro, nº56, Ilha Solteira – SP, Brasil.

Abstract. *New techniques of fault detection and localization at mechanical systems that are dynamically loaded have been developed to attend the industry demand caused by the technology progress. Even the tools for theoretical analysis of dynamic systems being sophisticated, there are great difficulties at the prediction of the dynamic behavior of some structural components and at the fault diagnosis, caused by the inaccuracy of the theoretical model, or caused by the difficulty on measuring some state variables. The methodology of state observers is perfectly inserted on this reality, because its capability of estimate the state variables of a system based on the measurement of the output and control variables. The methodology becomes more attractive because it makes possible the reconstruction of the states where the measurement is hard or just impossible, detecting failures at points that are not available to be measured and monitored trough the reconstruction of its states. Because of the magnitude of its effects, the crack nucleation or propagation demands essential care at mechanical systems. Knowing that this kind of fault can appear with the deterioration caused by vibrations and dynamical conditions, it becomes an excellent object for studying the use of the State Observers methodology to detect, locate and evaluate cracks conditions. For the suggested system, a coupled cantilever beam, were used a Finite Element Method, which showed itself the best one to do this kind of analysis, with beam elements at an elastic foundation, obeying a crack model. It was simulated with conditions of impulsive impact and harmonic excitation, and analyzed the results supplied by the State Observers through RMS differences between the two function curves. A complete observation system with a Global Observer of the process and Robust Observers, dedicated to accompany the stiffness variation of each element, was used, locating the fault and evaluating the percentage of penetration of the crack in the beam.*

Keywords: *State Observer, Crack, Continuous Systems.*

1. PURPOSE

The purpose of this work is the study of a new mathematical model to discretize cracks at continuous mechanical systems, applying all the available properties at computational simulations using the methodology of State Observers to detect, localize and evaluate the crack conditions.

2. THEORETICAL FUNDAMENTALS

Basically a state observer estimates the state variables using as base the measurements of the output and control variables. This technique consists in a method capable of reconstruct the states of a system where the measurement is compromised or impossible, being able to detect faults at these points without the knowledge of its measurements (Melo, 1995). That can also be monitored with the reconstruction of its states. In 1966, Luenberger demonstrated in his work that if a system is linear, its state array can approximately be reconstructed trough the project of an observer and, in 1971, the same author introduces the concept of many types of observers, as example, the Identity Observer (Marano, 2002) which uses a Linear Transformation of the data acquired from the output of the system to compare to the observers results. Despites the definition be relatively old, the proposed observers still being utilized, being theme of a great number of researches.

In 1990 was established the stiffness matrix of the cracked element and also studied the motion equation of a cracked cantilever bean "Qian *et al*, (1990)", using the equations provided by the fracture mechanic, turning possible modeling a cracked system. In 1995 was created a model of a beam using the Finite Element Method which can apply the Qian's model of the crack, turning possible to study a new form to detect this kind of problem. This model simplifies the best model that can be achieved, that should consider the crack propagation and growth, the dynamical characteristics of the cracked element and the influence of the crack presence at the tension field near the element.

Today the study of crack detection in mechanical systems lies over the developing of new observers and in the research and construction of crack models, making, then, more accurate the predictions that the simulations can bring.

2.1. State Obsevers

Since 1964, the observers have been performing part of numerous projects of control systems, where a small part has been shown in an explicit way. The simplicity of its project and resolution make the state observer an attractive component of the project, because it can reconstruct the non measured states of the system.

A state observer for an original dynamic system with the state {x(t)}, {y(t)}, as the output and the input being {u(t)}, is an auxiliary dynamic system. In other words, it is a copy of the original system that has the same input of this system and has the capability of estimate the unknown system states from states that are known. Figure 1 shows this definition, considering [L] as the State Observer Matrix.

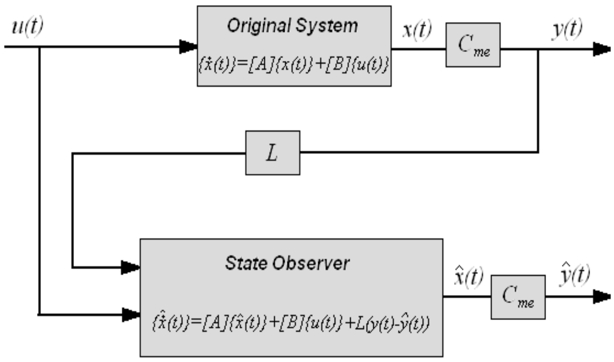

Figure 1. The State Observer definition

The construction of an observer is just possible if the original system is able to be observed or at least detectable. Differing from the system $\dot{x}(t)$, that is physical, the system $\dot{\hat{x}}(t)$ is something abstract and generated by a computer program. There are a great number of kinds of state observers, but the identity observer had been chosen to the realization of the research, because it has good convergence and easy implementation.

2.1.1. Identity Observer

That is considered, for the description of the Identity Observe, the linear and time invariant system shown by equation (1):

$$\begin{aligned}\{\dot{x}(t)\} &= [A]\{x(t)\} + [B]\{u(t)\} \\ \{y(t)\} &= [C_{me}]\{x(t)\} + [D]\{u(t)\}\end{aligned} \tag{1}$$

Where $[A] \in R^{nxn}$, $[B] \in R^{nxp}$, $[C_{me}] \in R^{kxn}$, $[D] \in R^{kxp}$, n the order of the system, p the number of inputs {u(t)} and k the number of outputs {y(t)}.Taking the system as completely observable (Melo, 1995).

An observer for a system like this is:

$$\{\dot{\bar{x}}(t)\} = [A]\{\bar{x}(t)\} + [B]\{u(t)\} + [L](\{\{y(t)\} - \bar{y}(t)\}) \tag{2}$$

And

$$\{\bar{y}(t)\} = [C_{me}]\{\bar{x}(t)\} \tag{3}$$

Where, [L] is the state observer matrix.

The estimation error for the state is:

$$\{e(t)\} = \{\bar{x}(t)\} - \{x(t)\} \tag{4}$$

And the estimation error at the output (residue):

$$\{\varepsilon(t)\} = \{\bar{y}(t)\} - \{y(t)\} \tag{5}$$

Now, substituting the equations (1), (2) and (3) in (4) and (5), what is got is:

$$\{\dot{e}(t)\} = ([A] - [L][C_{me}])\{e(t)\} + [L][D]\{u(t)\} \tag{6}$$

And

$$\{\varepsilon\ (t)\} = [C_{me}]\{e(t)\} - [D]\{u(t)\} \tag{7}$$

Where, the expression $\{\dot{e}(t)\} = d\{e(t)\}\,/\,dt$ represents the evolution of the error from the observer.

2.1.2. Robust and Global Observers

Two kinds of observers are used at the detection and localization of faults at dynamic systems. The global observer is the responsible for detect a possible fault at the system, while the robust observer is capable of locating the parameter that failed (Marano, 2002).

The global observer is nothing more than a copy of the original system. So, is possible to make a comparison of the collected parameters with the ones that have constructed by the global observer. If any difference appears between the behaviors of the curves can be concluded that the real system is failing.

From this information, the new focus is the search of the faulty parameter, constructing robust observers for every parameter that is able to fail. These observers are constructed with a gradual alteration on its dynamic matrix at the respective parameters for which they are robust. If the behavior of the robust observer of a determinate parameter gets close of the real behavior of the system, then that can be concluded that this parameter is failing (Marano, 2002).

This methodology can be employed for any mechanical system that is intended to control. The main idea is to construct a monitoring bank of robust observers to each parameter of the system, because they can constantly send the RMS differences between the obtained signals and the graphic generated by the observer, to a logic decision unit, that will judge if there is a fault at a parameter (Lemos, 2004).

The figure below represents the functioning of a logic decision unit based on the information given by the observers.

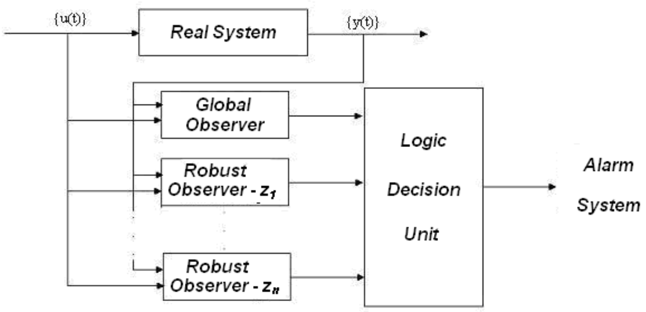

Figure 2. Principle of a system monitoring with Robust Observers

2.2. Cracked Beams

Knowing the risks that one crack can produces inside a mechanical system, its occurrence and identification is indispensable for the structure health analysis. The crack position and its dimensions can be detected by the disturbance of the natural frequency and mode shapes of the system. When a beam is dynamically loaded, and it has a crack inside, this crack will open and close alternately, depending on the vibration direction, causing a variation of the physical parameters of the system, as an example, a stiffness variance.

The presence of a crack at a beam, according to Saint- Venant's Principle, causes a neighborhood perturbation at the tension distribution. This perturbation is especially relevant when the crack is opened and determines a local reduction of the stiffness, so if the crack is closed, can be considered that there is no disturbances at the system.

When this kind of system is discretized by Finite Elements, it is necessary to take an essential care with the construction of its mass, stiffness and dumping matrices.

Making the assumption that with a crack there is no mass losses, can be concluded that the mass matrices won't suffer any effect of the crack, because even the cracked element still have its mass matrix [*M*] unaltered:

$$[M] = \frac{mL}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & 54 & -3l^2 & 4l^2 \end{bmatrix} \tag{8}$$

Where m is the element mass and L is the total length of the beam.

The dumping matrix is very hard to be obtained with theoretical procedures, so it is considered structural and obtained with the assumption that the beam is a single degree system. So, using some beam parameters could be found the logarithm decrement (ξ) of the beam displacement behavior, and using the natural frequency of the system an equation for the equivalent dumping can be found:

$$\mathrm{C_{eq}} = 4\pi.\mathrm{m.f_n}.\xi \tag{9}$$

Where C_{eq} is the equivalent dump, m is the beam mass and f_n is the natural frequency of the beam.

Placing the result at a 4x4 identity matrix can be obtained the dumping matrix for the element:

$$[\mathrm{C}] = C_{eq}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

2.2.1. Stiffness Matrix of a Cracked Element

The biggest problem to determine how a cracked system can be described is the stiffness matrix. All the approximations that can be done come from a complex theory developed to be used at numerical methods. According to Saint- Venant's Principle, the tension field is only affected at the adjacent region of the crack. So, the stiffness matrices of the elements, with the exception of the cracked element, can be considered unaltered under a certain limitation of the element size and it fits in the theory of Euler Bernoulli with hermitian function:

$$[\mathrm{K}] = \begin{bmatrix} \frac{Ebh^3}{l^3} & \frac{Ebh^3}{2l^2} & -\frac{Ebh^3}{l^3} & \frac{Ebh^3}{2l^2} \\ \frac{Ebh^3}{2l^2} & \frac{Ebh^3}{3l} & -\frac{Ebh^3}{2l^2} & \frac{Ebh^3}{6l} \\ -\frac{Ebh^3}{l^3} & -\frac{Ebh^3}{2l^2} & \frac{Ebh^3}{l^3} & -\frac{Ebh^3}{2l^2} \\ \frac{Ebh^3}{2l^2} & \frac{Ebh^3}{6l} & -\frac{Ebh^3}{2l^2} & \frac{Ebh^3}{3l} \end{bmatrix} \tag{11}$$

Where E is the beam material elasticity modulus, b is the width and h is the beam hight.

Because of the discontinuity of the deformation at the cracked element, it's very hard to find an appropriated function to express, approximately, the potential elastic energy. The calculus of the additional tension energy has been deeply studied through the fracture mechanics.

Then, the expression of the cracked element matrix (k_{crack}) is an explicit function of a lot of parameters, such as flexibility coefficients and cracks dimensions. However, the matrix can be writing as a relation of evaluation coefficients, where the coefficient for the condition for open crack (α) is tabled. This coefficient is a function of the crack depth and the relation between height and length of the cracked element. It has direct influence at the stiffness matrix of the cracked element:

$$k_{Crack} = \alpha_1 \begin{bmatrix} k_{11}\alpha_2 & k_{12}\alpha_2 & k_{13}\alpha_2 & k_{14}\alpha_2 \\ k_{12}\alpha_2 & k_{22}\alpha_3 & k_{23}\alpha_2 & k_{24}\alpha_4 \\ k_{13}\alpha_2 & k_{23}\alpha_2 & k_{33}\alpha_2 & k_{34}\alpha_2 \\ k_{14}\alpha_2 & k_{24}\alpha_4 & k_{34}\alpha_2 & k_{44}\alpha_3 \end{bmatrix} \quad (12)$$

Still, for each robust observer designed, there is a change in matrix stiffness in the position of the broken part, and reduced their values according to the proportional constant. Therefore, each new dynamic matrix built, Eq. (13), should be recalculated the quadrant related to the stiffness matrix of the same (in Quadrant 3).

$$[A]_{2nx2n} = \begin{bmatrix} [0]_{nxn} & \vdots & [I]_{nxn} \\ \cdots & \cdots & \cdots \\ -\left([M]^{-1}[K]\right)_{nxn} & \vdots & -\left([M]^{-1}[C]\right)_{nxn} \end{bmatrix} \quad (13)$$

Where [A] is the Dynamic Matrix, [0] is a Zero Matrix, [I] is an Identity Matrix, [M] is the Mass Matrix, [C] is the Dumping Matrix and [K] is the Stiffness Matrix.

2.2.2. Motion Equation of a Cracked Element

The dynamic response of the beam in the intervals of time that the crack is closed can be considered, for simplicity, such as a beam without crack. This is because the crack interfaces interact with each other completely. Under the action of the force of excitement, the opening and closing of the crack will alternate against time.

The equation of motion of the cracked beam discretized by N finite elements and subjected to a vector of external excitation f(t) can be written as Eq. (14), where M is the mass matrix, C is the matrix of damping, (Ku -- $\gamma\Delta K$) is the matrix of stiffness and ΔK = Ku – Kc. As definition $\gamma = 1$ when the crack is open and $\gamma = 0$ when the crack is closed.

$$M\ddot{u}(t) + C\dot{u}(t) + (K_u - \gamma\Delta K)u(t) = f(t) \quad (14)$$

Was considered $\gamma = 1$, because as long as the crack remains closed ($\gamma = 0$) the stiffness matrix is composed only by the portions where the crack is not considered, so at that moment there is no failure.

3. METHODOLOGY

For the suggested system, a coupled cantilever beam, were used a Finite Element Method with beam elements at an elastic foundation "Choy *et al*, (1995)" obeying the Crack model described before. The beam is discretized in five elements and at one of them placed the crack**.** The cracked element was modeled with the open crack parameters during all the simulation because were found difficulties to implement the dynamic crack.

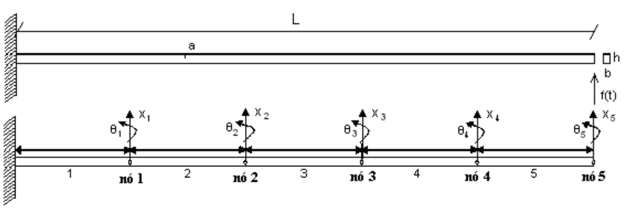

Figure 3. Cracked Beam scheme with the Crack placed at the second element

For this system were simulated conditions of impulsive impact and harmonic excitation, and analyzed the results supplied by the State Observers through RMS differences between the two function curves A complete observation system with a Global Observer of the process and Robust Observers, dedicated to accompany the stiffness variation of each element, was used, locating the fault and evaluating the percentage of penetration of the crack in the beam.

The simulations were computationally developed at MATLAB® software in two steps: First was constructed a stiffness matrix creation algorithm through the study of the beam parameters, and after, with these results, was developed a routine that has the detecting, finding and evaluating faults as function, with graphical plotting and numerical analysis.

4. RESULTS

The results were obtained considering previously that the crack position is already known and the study object is only the evaluation of the crack condition. However the fault localization isn't too much different, because the same process is done, but now, using all percentage of penetration for all the stiffness parameters of each element. In this case the global observer detected the fault and the robust observers are placed just at one element, the cracked one.

For this simulation a mild steel cantilever beam was considered having the dimensions of length L=0.6m, height h=12.5x10^{-3}m, width b=17.2x x10^{-3}m, Elasticity modulus E=2.07x10^{11}N/m, density ρ=7850 kg/m³ and Element length of l=0.12m, being the initial condition for all the simulations a null displacement for all the elements. The time interval chosen was from 0 to 0.4 seconds for impulsive excitation and harmonic excitation, being both of them shared into 1024 points to plot.

Considering that each element has its own stiffness matrix Eq. (11), mass matrix Eq. (8) and dumping matrix Eq. (10), it`s necessary respect the coupling effects and the crack alterations at them. So the matrices are changed from its usual configuration.

As said before, the only alteration did to adjust the model to be more close to the real situation is the stiffness matrix for the system alteration, because there is no mass significant loss and the dumping is calculated from the Structural Dumping. The alteration is done by proportional parameters that depend on the nature of the crack, being searched and included at Robust Observers.

For the realized computational simulations could be verified the detection and localization of the fault comparing the global system without fault and the global observer for the system calibration. The RMS difference of 10E-13 shows a curve coincidence (This value has been taken as a pattern through all the simulations). It means that, if the real system stay practically equal to the global observer (with no fault), there is no fault at the system.

Once the faults are put into the system, they are detected by a divergence between the curves already mentioned, and, trough the action of the robust observers they are found and evaluated. Exemplifying, on the second line and second column of the Tab. 1, can be verified a detection of 5% of fault, so, the crack reach 5% of the total height of the element. At the sequence, were inserted faults varying from 5% to 5% until reaching 20% of height of the element, accompanied by the robust observers which identified them. The simulations were done by an impact force, described by a unitary impulse function (or not null initial velocity), and by an harmonic force, of periodic actuation and described by a function F(t)=A.seno(φ + ω(t)), with amplitude of 10N and a frequency of 100Hz.

Table 1. Result of the RMS differences for an Impact Force Simulation

	Without Fault	5% of Fault	10% of Fault	15% of Fault	20% of Fault
Global Obsv.	9.0647e-12	1.6981e-03	1.5585e-02	5.4116e-02	9.2969e-02
Obsv. 5%	2.0342e-03	1.1934e-11	1.5669e-02	5.3738e-02	9.3116e-02
Obsv. 10%	6.1713e-03	9.2233e-03	1.0736e-11	3.4128e-02	7.5412e-02
Obsv. 15%	2.5082e-02	2.7802e-02	2.0286e-02	8.8463e-13	3.7926e-02
Obsv. 20%	5.1826e-02	5.4075e-02	4.3481e-02	2.3017e-02	7.6434e-12

Table 2. Result of the RMS differences for an Harmonic Force Simulation

	Without Fault	5% of Fault	10% of Fault	15% of Fault	20% of Fault
Global Obsv.	1.2029e-13	2.9342 e-02	8.7346e-03	4.6538e-03	2.3652e-03
Obsv. 5%	2.6341e-02	3.7426e-12	9.6035e-02	7.6985e-02	9.3336e-02
Obsv. 10%	3.8246e-02	3.1402e-03	9.7826e-12	5.1245e-03	8.3652e-03
Obsv. 15%	9.7542e-03	4.4948e-03	1.9896 e-03	1.2645e-11	1.3652e-03
Obsv. 20%	3.0129 e-02	5.9874 e-02	5.1765e-03	2.4578e-02	4.6548 e-11

As observed, there was a coincidence between the curves of the simulated system and the State Observers, projected to detect the respective size percentage of the crack, represented by the hatched elements, because the RMS difference values between the curves tends to zero, what show the efficiency of the State Observers bank that was used.

The graphical solution of the problem clarifies more didactically how the functioning of the State Observers is. At the figures below, can be observed a System excited harmonically with a crack size of 30% of the beam height.

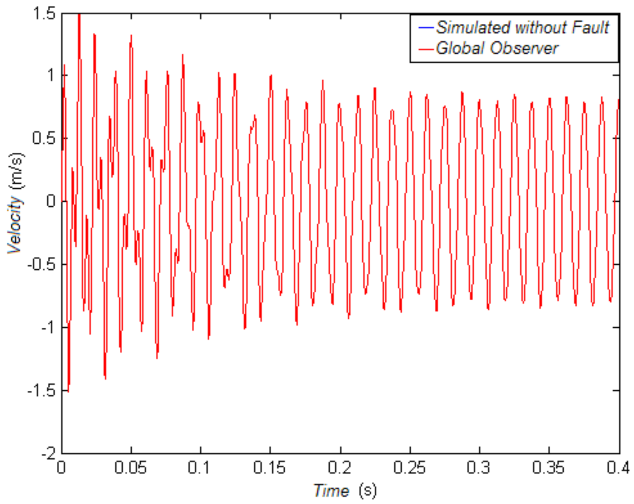

Figure 4. Global Observer overlaps the Simulated System without fault at the Simulation.

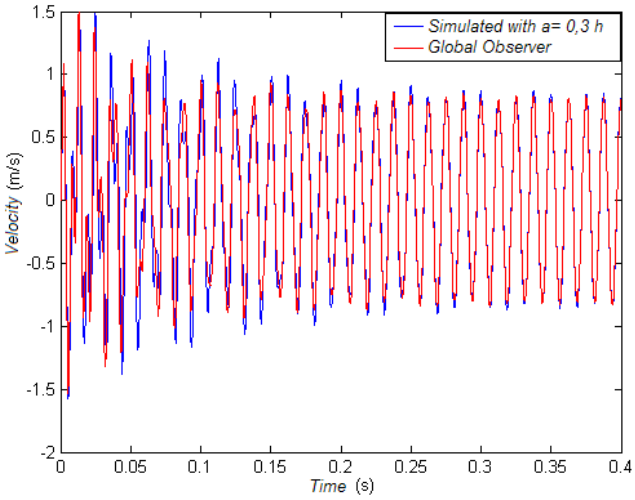

Figure 5. Disagreement between Global Observer and the Simulated System, with crack size of 30% of height.

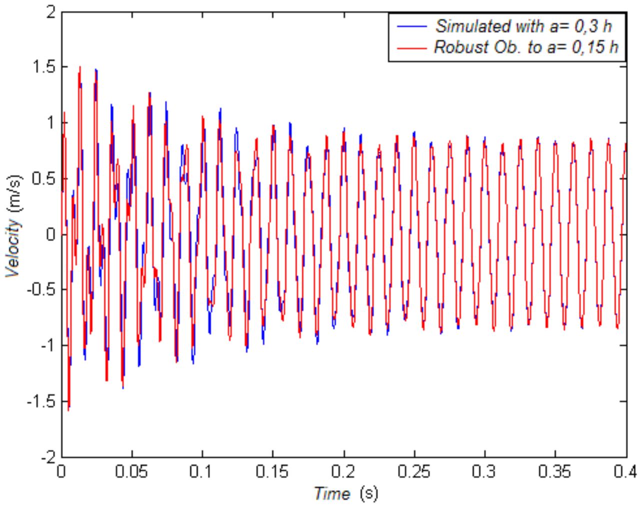

Figure 6. Disagreement between Robust Observer, to crack size of 15%, and the Simulated System, with crack size of 30% of height.

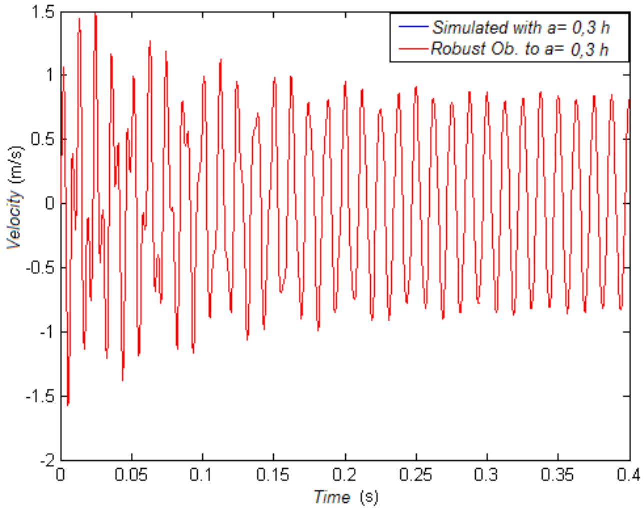

Figure 7. Overlapping of Robust Observer, to crack size of 30%, and the Simulated System, with crack size of 30% of height.

First the simulated system was compared to the global observer. The system wasn't working in perfect conditions, then there was detected a divergence between the Observer and the Simulated System curves. Knowing about a irregularity of the system, the Robust Observers keep on looking after the faulty parameter, being then, founded by a superposition of the function curves.

In both simulations, the kinematical magnitude used to do the analysis was the velocity. It is common to use this variable of the process because the measurement equipment is an accelerometer which gives, as result of the measurement, the acceleration of the vibration and with the numerical derivation of the curve, given by the own equipment, there is velocity as response.

As seen, the values of the RMS difference between the measured system and the State Observer, at the faulty parameters, doesn't show a great difference if compared to the cases that there isn't fault. The problem created with this misfortune is that, at some cases, the Observers may not detect the fault correctly because the RMS difference may not be as bigger as the value calculated to be, as understood from the system, the Logic Decision Unit may not trigger the Alarm System. It will probably happen to more complex systems, with more excitations forces or systems that require a finest mesh.

5. CONCLUSIONS

Trough this work there was noticed that in the study realized to faulty continuous system, the localization of the faults is obtained with a large number of structural variables measurements. The state observer technique uses fewer measurements with the reconstruction of the other states, what leads to cost and time reduction.

The computational analysis for the developed method have shown good results for the simulated system with a crack, what shows that the mathematic used to model the system have generated results that are applicable at real systems. There could be observed also, that only the robust state observer designated to a specific crack percentage can detect the irregularity presented, showing that the method not only detect and localizes the fault, but also can avail the problem magnitude.

In the future, the experimental analysis will show if the magnitude of the differences offers a difficulty to implement the method to real structures with measurements provided from an accelerometer, that will be included at the Finite Element model as part of the structure, coupled to the own structure.

6. ACKNOWLEDGEMENTS

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