EFFECT OF HARMONIC COMPONENTS IN AN ONLY OUTPUT BASED MODAL ANALYSIS

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Abstract. *This paper discusses the effect of the presence of harmonic components in the process of identification of modal parameters. The concept of probability density function (pdf) is used as tool to separate the operational modes (harmonic) from the structural ones. Experimental tests for an unbalance beam structure with an electric motor coupled in an extremity are presented and the modal parameters are estimated for different conditions. The results of the analyzed case showed that the approach could be a good indicator of harmonic components in model response and its application could improve the identification process, leading to the correct structural parameter of the model. They also evidenced its potentiality for real applications.*

Keywords: *Operation Modal Analysis, modal parameters, harmonic components*

1. INTRODUCTION

Structural modeling actually is becoming more and more necessary, mainly, with the increasing demands of high performance and velocity of machineries and equipments. These demands become even more important in study and the understanding of the dynamic behavior of a model and the correct knowledge of their dynamic parameters is of extreme importance. These parameters are crucial for the evaluation of the operation conditions, as well as for performance and safety indexes.

Experimental tests are used to study the dynamic behavior of structures through the identification of their modal parameters. Usually, this is done through the experimental modal tests, which are based fundamentally on the input-output relationship of the model. In this case, it demands the measurement of the input and the output of the model which sometimes it is not easy or practicable. Alternatively, the modal analysis techniques based only in the response, called Operational Modal Analysis (OMA), unlike the classical modal analysis, it allow one to obtain the modal parameters of the model without measuring the input forces acting in the model. In this case, the modal parameters of the model can be obtained by using the only output data, which makes it an interesting option to investigate the dynamic behavior of the model, since the parameters can be estimate by using the own operating conditions of the model as excitation.

The study of the structural behavior of the model in this case, is much more attractive when compared with experimental tests performed in laboratories, since the identification of modal parameters of the model are based in the operation conditions .

However, an important aspect to be considered in the operational modal analysis is the type of the input forces actuating in the system, that are mainly random forces and the eventual presence of harmonic components, mainly due to rotating systems, added to the excitation forces will affect the identification process. The presences of harmonic components in the excitation could mask the identification of the modal parameters and it will affect the robustness of the identification process, providing unsatisfactory results. In this case, the study of the harmonic components is crucial for the correct identification of the structural parameters of the model.

This paper discusses the effect of the presence of harmonic components in the process of identification of modal parameters. The concept of probability density function (*pdf*) is used as tool to separate the operational modes (harmonic) from the structural ones. Experimental tests for an unbalance beam structure with an electric motor coupled in an extremity are presented and the modal parameters are estimated for different conditions of operation. The obtained results showed the potential application of the *pdf* in the identification of harmonic components of structural systems.

2. THEORY AND BASIC CONCEPTS

The use of the concept of probability density function (*pdf*) to deal with harmonic components in the context of operational modal analysis is very attractive. The approach is based in the statistic concept that takes into account the random character of the analyzed data, therefore, ones can analyze the responses of the model and based in their behavior, it is possible to study the effect of eventual harmonics components into operation modal analysis of the model.

Taking a distribution function of a random variable, x, with a cumulative distribution function $F(x)$, one can directly related this function with the probability density function, Eq. (1).

$$f(x)=\frac{dF(x)}{dx} \tag{1}$$

For the case of two stochastic variables x and y, where $g(y)$ *is* a probability density function of the random variable y and $f(x)$ the probability density function of the random variable x one can define a relation of these two random variables through change of variable. Assuming that variable x is a white Gaussian noise, that could represent the input of a modal model and y represent the output (response signal) of the model, it is possible to define the variable y in terms of x, if for each value of x correspond an unique value of y and vice-versa, by the relation $y=h(x)$. By using the concept of discrete and continuous random variables, as discussed in Spiegel (1977), it is possible to relate the probability density function of y, given by $g(y)$ with $f(y)$, Eq. (2).

$$g(y)dy=f(x)dx \tag{2}$$

or,

$$g(y)\frac{dy}{dx}=f(x) \tag{3}$$

From the x and y relationship, it gives

$$\frac{dy}{dx}=h'(x) \tag{4}$$

so,

$$g(y)=\frac{f(x)}{h'(x)} \tag{5}$$

where $h'(x)$ is the derivate of $h(x)$ (Papoulis, 1990).

Since $f(x)$ is a probability density function of white Gaussian noise of zero mean, the probability density function can be considered as a normal Gaussian distribution function of mean μ and standard deviation σ, Eq. (6).

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2} \tag{6}$$

The graphic of the normal Gaussian distribution function is shown below, Fig. 1.

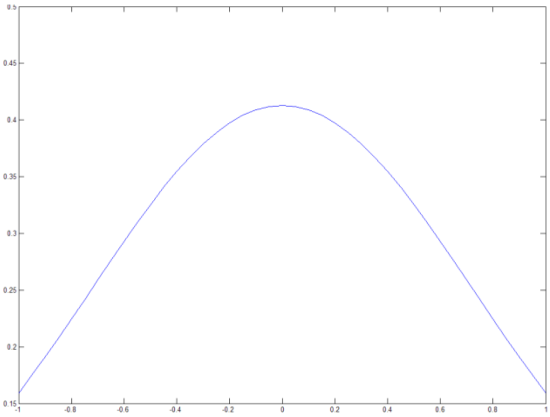


Figure 1. Gaussian distribution function

By using Eq. (4), one can estimate the probability density function for a component harmonic. For the case of a harmonic signal, of amplitude *a*, it gives:

$$g(y) = \frac{1}{\pi \cos(\arcsin(y / a))} \tag{7}$$

The behavior of the *pdf* for a harmonic component is very different in this case, as it shown in Fig. 2. It increase when *y* tend to + *a* and – *a* (region of the picks), presenting two distinct picks.

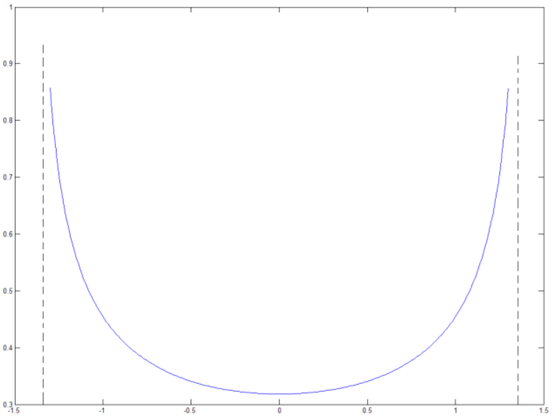


Figure 2. Probability density function of a component harmonic

This result is valid for any frequency and also for low frequency variation. Since there is a low variation, the amplitude is constant and the response is still harmonic. This concept of separation of the picks can be used as an indicator of harmonic component in the response of the model. It was initially proposed by Lago (1997). Brincker *et al.* (2000) also proposed to use the *pdf* to identify if the picks of the spectral density function of the response of a model were related with operational or structural modes as discussed in Silva (2008).

In the operational modal analysis, the effects of harmonics components are evaluated by analyzing the statistic behavior of the model response at those regions of frequency picks. The model response is passed through a narrow band filter and the new density function obtained with the separation of the picks of frequencies, which now contains the information only of pick of interest is analyzed. If the behavior of the estimated probability density function of the spectral density function obtained from the filtered response is the same of a white Gaussian noise, that pick is related with a structural mode. In the other case, if it has the same behavior of a probability density function of a harmonic function, it must be related to the operational mode.

3. EXPERIMENTAL TEST

This topic discusses de application of the approach to study the behavior of responses of a model obtained from its own operation conditions. The tests are conducted in an unbalance beam structure that presents an electric motor mounted at the free-end of the structure. The experimental set-up is shown below, Fig. 3. The model responses were captured by using the acquisition system from Data Physics, SignalCalc Ace®.

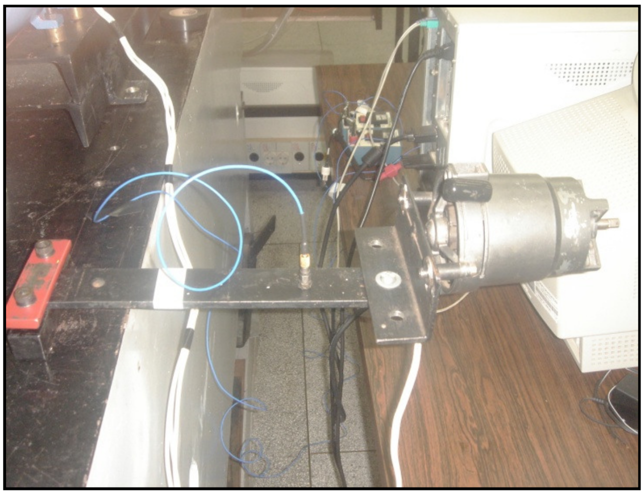


Figure 3. Experimental set up

The experimental tests were performed for two distinct conditions, initially for the system with the motor switched-off and later for the system in operating conditions, motor switched-on. Therefore, the modal parameters were estimated by using the classic modal analysis and the modal based only in response of the model. In the first case, it was used Ibrahim method and in the second it was used the Stochastic Subspace Identification (SSI) method (Freitas, 2008).

3.1. Analysis with the motor switched off

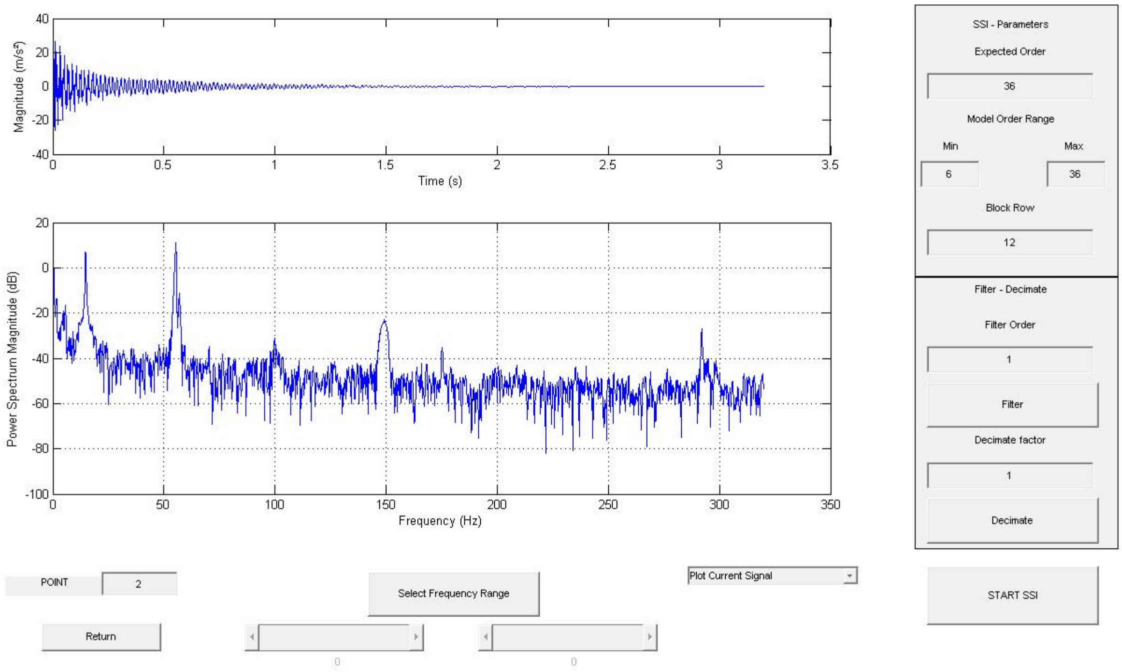
In this test the structural behavior of the model was evaluated for the system out of operation, and the modal parameters were estimated by using the conventional modal test. The input came from an impact hammer used to excite the structure in vertical direction, and the responses were measured in acceleration, by using PCB-accelerometers.

The responses were measured in 4 equal spaced points along of the beam, in a frequency range from a 0 to 300Hz, in the same direction of the excitation. The estimated modal parameters were taken as reference to compare with those obtained for the system in operating conditions.

In the frequency range analyzed were identified four picks of frequencies, corresponding to the first 4 natural frequencies of the model. Figure 4 shows the graphics of the response spectrum and response in time of the model. Also, table 1 gives the natural frequencies estimated.

Table 1. Natural frequencies - motor switched off.

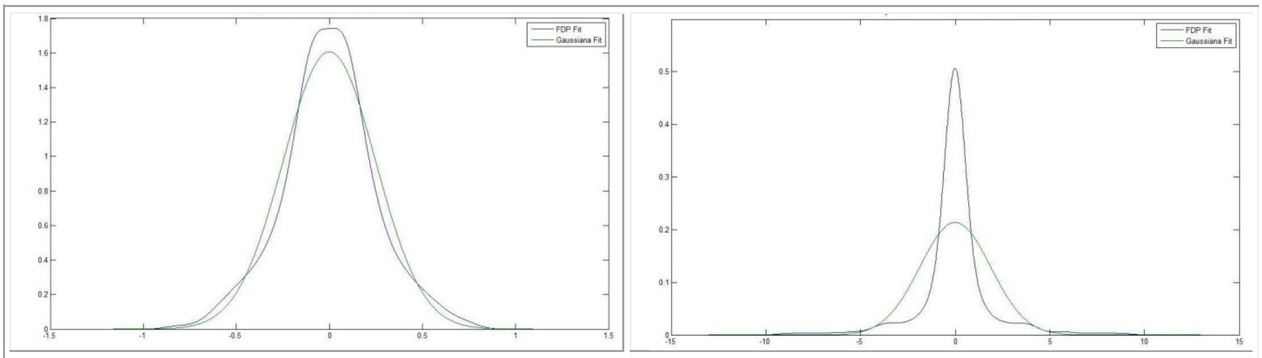
Mode	Frequency (Hz)
1st	14.7
2nd	55.0
3rd	149.2
4th	292.0



Expected Order: 36
Model Order Range: Min 6, Max 36
Block Row: 12
Filter Order: 1
Decimate factor: 1
POINT: 2

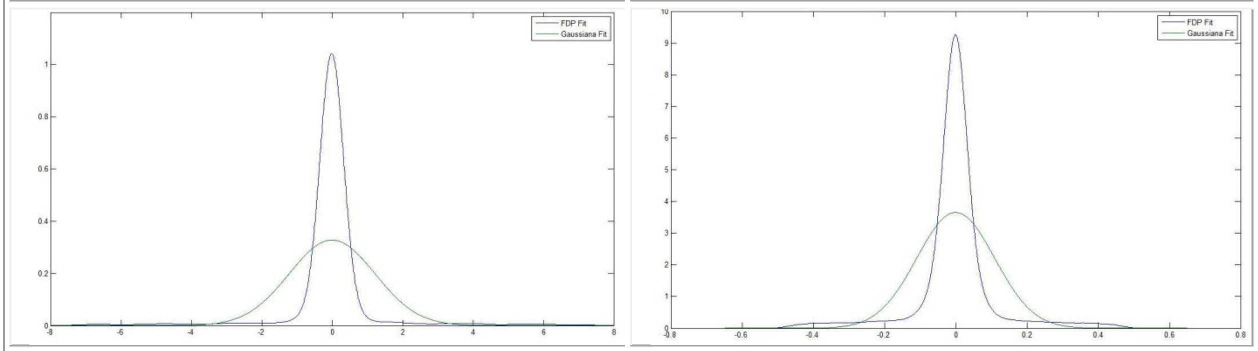
Figure 4. Response function and response in time of the model - motor switched off

The region of each frequency picks of the responses spectrum of the model were analyzed, as discussed previously in section 3. Figure 5 shows the behavior of the probability density function estimated from spectral density function of the model taking into account only the region of the frequency picks.



First Mode

Second Mode



Third Mode

Fourth Mode

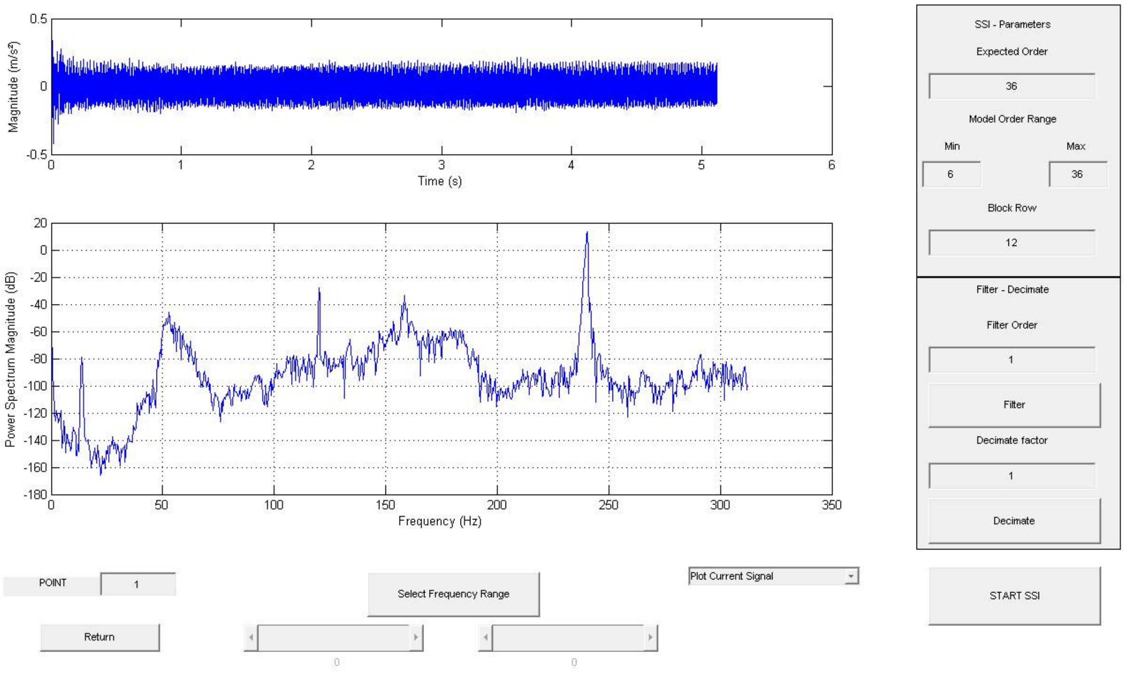
Figure 5. Estimated *pdf* of the response for each frequency pick- motor switched off

For the condition of the motor off the results obtained of the *pdf* analysis of the response of the system around the four picks of frequencies shows that the shape of the *pdf* for them approximates to the shape of the *pdf* of a normal function (Gaussian). That indicates according to what was discussed in the item 2 that the modes in analysis represent structural modes.

3.2. Analysis with the motor switched on

The structural behavior of the model, in this case, was evaluated for the system in operation, the motor was switched on and the modal parameters were estimated by using the SSI technique together with the proposed approach, the *pdf* concepts' was used to study the effects of operational modes. In this case, only the responses of the model were measure and the acquisition parameters as well as the measurement direction were the same of the previous test.

Two different situations were analyzed, in the first test the motor was operating in maximum rotation and in the second one, it was operating in a minimum rotation. Figure 6 shows the response spectrum and response in time of the model. Also, table 1 gives the natural frequencies estimated measured for the motor operating in a maximum speed.



SSI - Parameters

Expected Order: 36

Model Order Range: Min 6, Max 36

Block Row: 12

Filter - Decimate

Filter Order: 1

Decimate factor: 1

POINT: 1

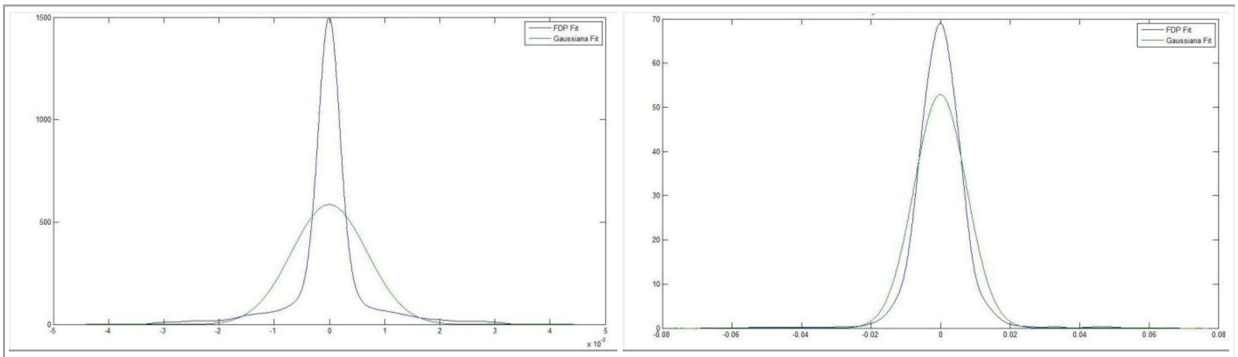
Figure 6. Response function and response in time of the model - motor switched on

The estimated parameters of the model based on the only-input modal analysis by using the SSI technique, without to evaluate the effect of harmonic components, table 2, shows a considerable difference in relation to the previous one. There exists an increase in the number of estimated frequencies of the model.

Table 2. Natural frequencies - motor switched on.

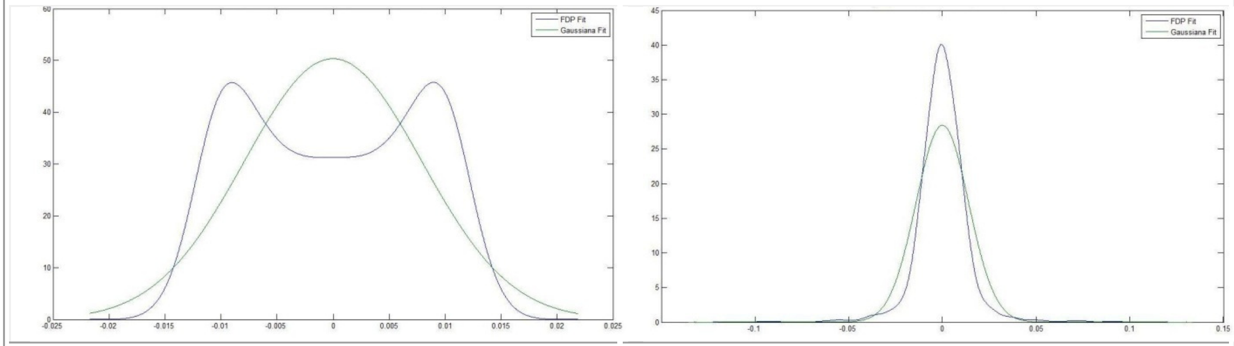
Mode	Frequency (Hz)
1st	13.7
2nd	52.7
*3rd	120.0
4th	158.6
*5th	241.4
6th	291.4

In order to confirm the evaluated parameters, the probability density function for each region of each frequency picks of the responses spectrum of the model was analyzed. Figure 7 shows the probability density functions obtained each pick of frequency, the results show that some identified modes do not behaves as structural modes and are operational modes.



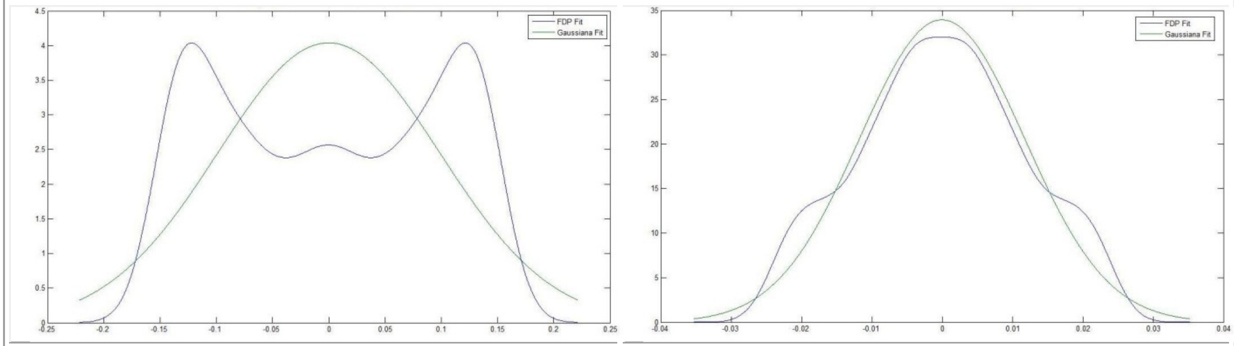
First Mode

Second Mode



Third - Harmonic

Fourth Mode



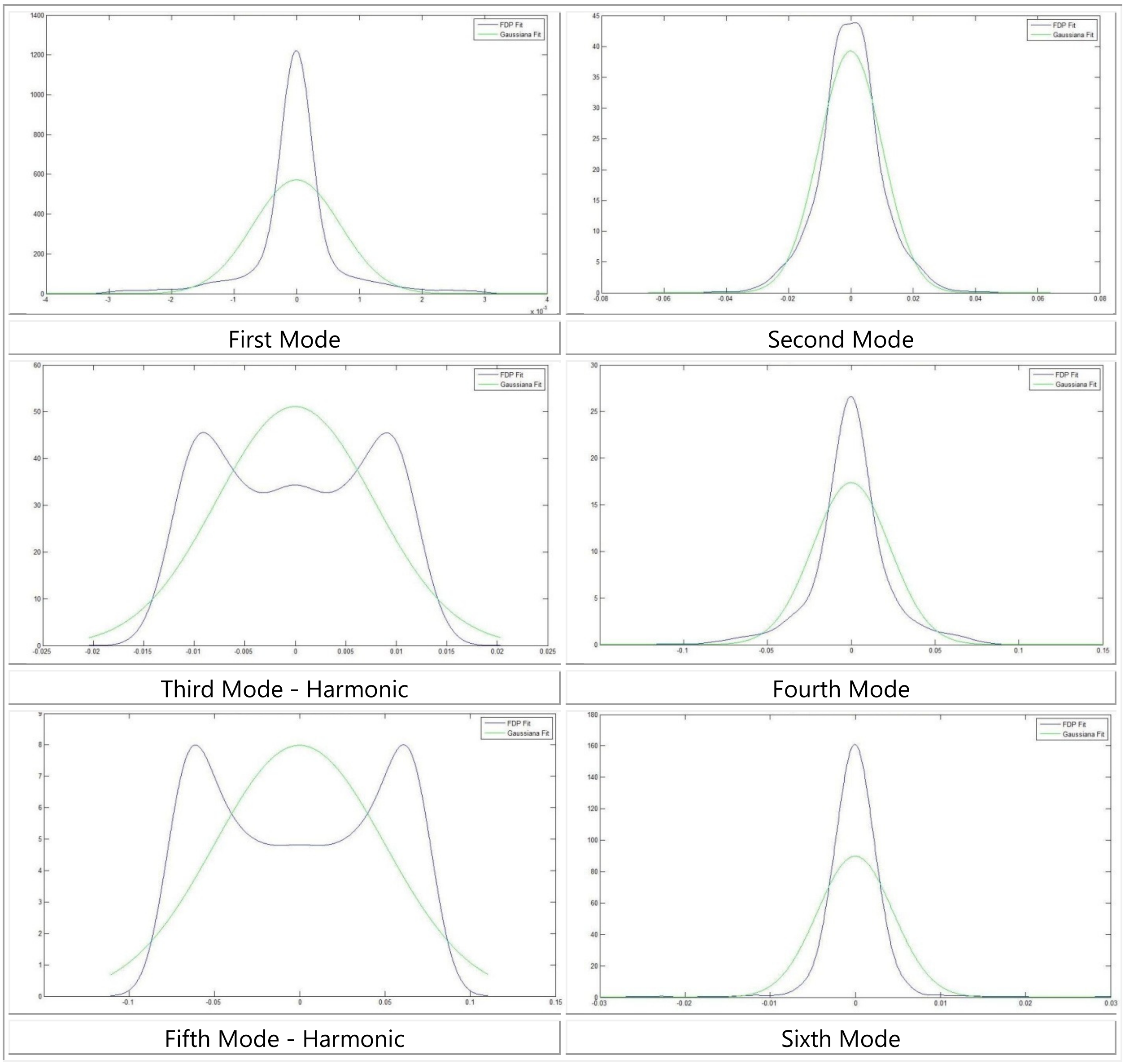
Fifth Mode - Harmonic

Sixth Mode

Figure 7. Estimated *pdf* of the response for each frequency pick - motor switched on, high rotation

The analysis of the probability density function shows that the third and fifth modes are not structural, since their *pdf* does not present a Gaussian behavior that characterizes the behavior of structural modes. On the other side, they present a form of the probability density of a harmonic function that is an indication that such modes are operational modes. Therefore, those modes would harm the correct identification of the modal parameters (natural frequencies, mode shape and damping ratio) of the system, using directly the routines of modal analyzes.

For the second situation, motor operating in a low rotation, the results are similar to the first case, however the third and fifth frequencies decreased, since there are operational. Figure 8 shows the estimated *pdf* for the respective picks of frequencies of the spectrum response of the model. The results confirm that the third and fifth modes are operational modes and the others four modes are structural modes.



First Mode

Second Mode

Third Mode - Harmonic

Fourth Mode

Fifth Mode - Harmonic

Sixth Mode

Figure 8. Estimated *pdf* of the response for each frequency pick - motor switched on, low rotation

4. CONCLUSIONS

This work shows a study of the influence of harmonic components in the Operational Modal Analysis. Experimental tests were accomplished in an unbalance beam structure containing an electric motor mounted at the free-end of the structure. It shows the effects of the harmonic components in the spectrum response of the model and their influence in the modal parameters identification, as well as, how this effect could be minimized by using the concept of probability density function applied to the spectrum response of the model.

The results for the analyzed cases showed that the approach could be a good indicator of harmonic components in the response and its application could improve the identification process, leading to the correct structural parameter of the model. They also evidenced its potentiality for real applications.

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6. RESPONSIBILITY NOTICE

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