ESTIMATION PROCEDURE OF *J* AND CTOD FRACTURE PARAMETERS FOR SE(T) FRACTURE SPECIMENS BASED ON THE η METHOD

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Abstract. *This work presents an evaluation procedure to determine the elastic-plastic J-integral and CTOD for pin-loaded and clamped single edge notch tension (SE(T)) specimens based upon the -method. The primary objective is to derive estimation equations applicable to determine J and CTOD fracture parameters for a wide range of a/W -ratios and material flow properties. Very detailed non-linear finite element analyses for plane-strain and full-thickness, 3D models provide the evolution of load with increased crack mouth opening displacement which is required for the estimation procedure. The present analyses, when taken together with previous studies provide a fairly extensive body of results which serve to determine parameters J and CTOD for different materials using tension specimens with varying geometries.*

Keywords: *integral J, CTOD on SE (T) specimens, eta methodology*

1. Introduction

Fracture assessment procedures for piping systems play a key role in design, fabrication and fitness-for-service (FFS) methodologies (such as, for example, repair decisions and life-extension programs) for oil and gas pipelines as well as marine risers. Fracture mechanics based approaches, also referred to as Engineering Critical Assessment (ECA) procedures, rely upon the notion that a single parameter which defines the crack driving force characterizes the fracture resistance of the material (Hutchinson, 1983). These approaches allow the severity of crack-like defects to be related to the operating conditions in terms of a critical applied load or critical crack size. In particular, assessments of cleavage fracture for pipeline steels and its weldments in the ductile-to-brittle transition (DBT) region are based on the one-parameter elastic-plastic characterization of macroscopic loading, most commonly the *J*-integral and the Crack Tip Opening Displacement also denoted as CTOD ou δ (Anderson, 2005).

Conventional testing standards to measure cleavage fracture resistance of structural steels, including pipeline and pressure vessel steels, most often employ three-point bend SE(B) specimens containing deep, through cracks ($a/W \geq 0.45 \sim 0.5$). However, structural defects (e.g., blunt corrosion, slag and nonmetallic inclusions, weld cracks, dents at weld seams, etc.) in pressurized piping systems are very often surface cracks that form during fabrication or during in-service operation (Eiber and Kiefner, 1986). These crack configurations generally develop low levels of crack-tip stress triaxiality which contrast sharply to conditions present in deeply cracked specimens. Recent defect assessment procedures advocate the use of geometry dependent fracture toughness values so that crack-tip constraint in the test specimen closely matches the crack-tip constraint for the structural component. In particular, fracture toughness values measured using single edge notch tension (SE(T)) specimens appear more applicable for characterizing the fracture resistance of pressurized pipelines and cylindrical vessels than standard, deep notch fracture specimens under bend loading. The primary motivation to use SE(T) fracture specimens in defect assessment procedures of cracked pipes is the strong similarity in crack-tip stress and strain fields which drive the fracture process for both crack configurations. (Cravero and Ruggieri, 2005).

This work presents an evaluation procedure to determine the elastic-plastic *J*-integral and CTOD for pin-loaded and clamped single edge notch tension (SE(T)) specimens based upon the η-method. The primary objective is to derive estimation equations applicable to determine *J* and CTOD fracture parameters for a wide range of a/W -ratios and material flow properties. Very detailed non-linear finite element analyses for plane-strain and full-thickness, 3-D

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2. Estimation Procedure for *J* and CTOD Based Upon the η Method

Evaluation of the *J*-integral from laboratory measurements of load-displacement records such as illustrated in Fig. 1 is most often accomplished by considering the elastic and plastic contributions to the strain energy for a cracked body under Mode I deformation (Anderson, 2005) as follows

$$J = J_e + J_p \tag{1}$$

where the elastic component, J_e, is given by

$$J_e = \frac{K_I^2 (1-\nu)^2}{E} \quad . \tag{2}$$

Here, K_I defines the elastic stress intensity factor and E and ν are the (longitudinal) elastic modulus and Poisson's ratio. For an SE(T) specimen, K_I is given by

$$K_I = \frac{P}{BW^{1/2}} f(a/W) \tag{3}$$

where $f(a/W)$ defines a nondimensional stress intensity factor dependent upon specimen geometry, crack size and loading condition. Here, *P* is the applied load, *B* denotes the specimen (net) thickness, *W* is the specimen width and *a* is the crack size. For any given value of *P*, *B* and *W*, calculation of K_I follows from evaluation of $f(a/W)$ for a given a/W -ratio and loading condition (pin-load or clamp). Cravero and Ruggieri (2007) provide a comprehensive set of the nondimensional function $f(a/W)$ for pin-loaded and clamped SE(T) fracture specimens with varying specimen geometry (different a/W and H/W -ratios – see specimen geometry given in Fig. 2).

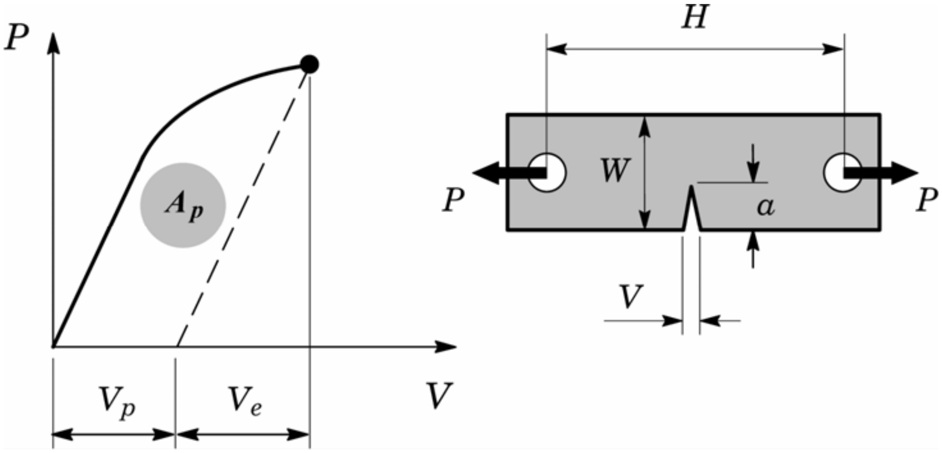

Figure 1. Schematic procedure to determine *J* and CTOD based upon the plastic area (equivalent to plastic work) under the load-displacement (CMOD) curve for a SE(T) fracture specimen

The plastic component, J_p, is derived from adopting the approach proposed by Sumpter and Turner (1976) to relate the *J*-integral to the area under the load versus load-crack mouth opening displacement (CMOD or *V*) – see Fig. 1. The approach simply relates the plastic contribution to the strain energy (due to the crack) and *J* in the form

$$J_p = \frac{\eta_J^{CMOD} A_p}{B(W-a)} \tag{4}$$

where A_p is the plastic area under the load versus CMOD, *B* is the net specimen thickness, $(W-a)$ is the initial uncracked ligament. Factor η_J introduced by Sumpter and Turner (1976) represents a nondimensional parameter which relates the plastic contribution to the strain energy for the cracked body with *J* and is assumed to be a function of the flawed configuration and independent of loading (Kanninen and Popelar, 1985).

Following the previous energy release rate interpretation of the *J*-integral and using the connection between *J* and δ (Anderson, 2005), a similar formulation also applies when the CTOD is adopted to characterize the material's fracture resistance in terms of its elastic, δ_e, and plastic, δ_p, components as

$$\delta = \delta_e + \delta_p \tag{5}$$

where

$$\delta_e = \frac{K_I^2(1-\nu)^2}{mE\sigma_{ys}} \tag{6}$$

and

$$\delta_p = \frac{\eta_\delta^{CMOD} A_p}{B(W-a)\sigma_{ys}} \tag{7}$$

In the above expression, σ_{ys} is the material's yield stress and factor η_δ represents a nondimensional parameter which describes the effect of plastic strain energy on the applied CTOD

The previous development based upon the η-factor retains strong contact with current standards to determine experimental *J*-values using common fracture specimens with deep cracks. Computation of η-factors for shallow cracked specimens is relatively straightforward and derives from plane-strain analyses as described in the next sections. Moreover, generalization of the η-methodology in estimation procedures for elastic-plastic fracture toughness (*J*, δ) involves two key benefits: 1) it provides a simpler and yet more accurate procedure to determine *J* and CTOD and 2) it imposes no restrictions on flow properties (essentially yield stress and hardening behavior) for the tested material. The following sections explore these issues and provide detailed analyses which yield η-factors applicable to determine *J* and CTOD in SE(T) specimens with a wide range of crack sizes, hardening properties and loading conditions.

3. Numerical Procedures

3.1. Finite Element Models

Detailed finite element analyses are performed on plane-strain models for a wide range of 1-T SE(T) specimens ($B = 25.4$ mm) and conventional geometry with $W = 2B$. The analysis matrix includes specimens with $a/W = 0.1$, 0.2, 0.3, 0.4, 0.5 and $H/W = 6$. Here, *H* is the distance between loading points (pin or clamp). Figure 2 shows the geometry and specimen dimensions for the analyzed crack configurations. The plane-strain finite element models constructed for the analyses of the SE(T) specimens employ a conventional mesh configuration having a focused ring of elements surrounding the crack front with a small key-hole at the crack tip; the radius of the key-hole, ρ_0, is 2.5 μm (0.0025 mm). Symmetry conditions permit modeling of only one-half of the specimen with appropriate constraints imposed on the remaining ligament. The half-symmetric model has one thickness layer of 1241 8-node, 3-D elements (2678 nodes) with plane-strain constraints imposed ($w = 0$) on each node. These finite element models are loaded by displacement increments imposed on the loading points to enhance numerical convergence. Cravero and Ruggieri (2007) provide additional details on these plane-strain models.

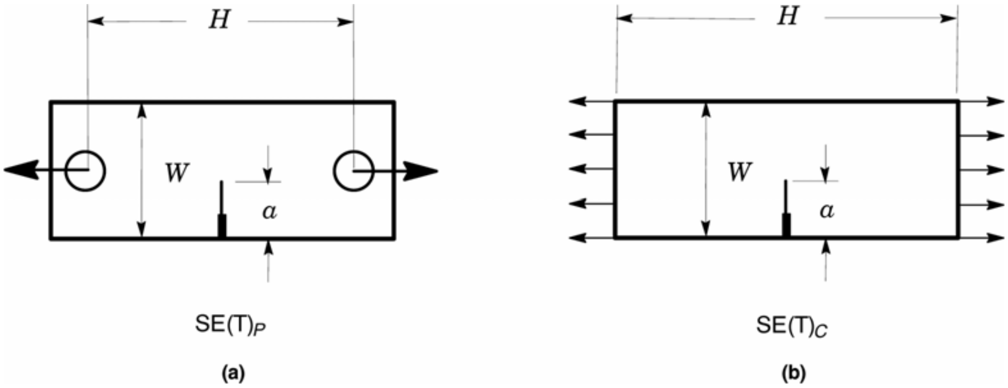

Figure 2. Geometries for analyzed SET fracture specimens: (a) Pin-loaded specimens; (b) Clamped specimens.

3-D finite element analyses are also conducted on plane-sided models for pin-loaded and clamped SE(T) specimens with thickness $B = 15$ mm and conventional geometry with $W = 2B$. The analysis matrix considers similar specimen configurations as adopted in the plane-strain analyses previously described with a shallow ($a/W = 0.2$) and a deep ($a/W = 0.5$) crack and $H/W = 6$. To further verify the effect of specimen geometry on the η-factors, 3-D computations are performed on finite element models for a clamped SE(T) specimen with a shallow ($a/W = 0.2$) and a deep ($a/W = 0.5$) crack having crack size to specimen with ratio, $a/W = 0.2$ and 0.5, thickness $B = 30$mm and $W = B/2$. In particular, such specimen geometry with $W = B/2$ is adopted in DNV-RP-F108 (DNV, 2006) to measure fracture toughness (*J*) in defect assessment procedures employed for pipeline design and installation analyses. Figure 3 displays the 3-D finite element for the pin-loaded SE(T) specimen with $a/W = 0.5$. The in-plane mesh details are very similar to the plane-strain models already described. Symmetry conditions enable analyses using one-quarter of the 3-D models with appropriate constraints imposed on the symmetry planes. The mesh has 15 variable thickness layers defined over the half-thickness ($B/2$); the thickest layer (width of 2 mm) is defined at $Z = 0$ with thinner layers (width of $0.5 \sim 0.2$ mm) defined near the free surface ($Z = B/2$) to accommodate strong Z variations in the stress distribution. Typical quarter-symmetric, 3-D models for these fracture specimens have approximately 18000 ~ 22000 elements. All finite element models are loaded by displacement increments imposed on the nodes which define the loading points along the thickness layers (see Fig. 3).

3.2. Computational Procedures and Material Laws

The finite element code WARP3D (Gullerud et al. 2004) provides the numerical solutions for the plane-strain and 3-D analyses reported here. Computation of the *J*-integral derives from a domain integral procedure (Moran and Shih, 1987) which yields *J*-values in excellent agreement with estimation schemes based upon η-factors for deformation plasticity while, at the same time, retaining strong path independence for domains defined outside the highly strained material near the crack tip. The numerical value of CTOD is determined based upon the 90° intercept procedure (Anderson, 2005) to the deformed crack flanks.

Evaluation of factor η requires nonlinear finite element solutions which include the effects of plastic work on J (CTOD) and the load-displacement response. These analyses utilize an elastic-plastic constitutive model with J_2 flow theory and conventional Mises plasticity in small geometry change (SGC) setting. The numerical solutions employ a simple power-hardening model to characterize the uniaxial true stress ($\bar{\sigma}$) *vs*. logarithmic strain ($\bar{\varepsilon}$) in the form

$$\frac{\bar{\varepsilon}}{\varepsilon_0} = \frac{\bar{\sigma}}{\sigma_0} \quad \bar{\varepsilon} \le \varepsilon_0; \quad \frac{\bar{\varepsilon}}{\varepsilon_0} = \left(\frac{\bar{\sigma}}{\sigma_0}\right)^n \quad \bar{\varepsilon} > \varepsilon_0 \tag{8}$$

where σ_0 and ε_0 are, respectively, the reference (yield) stress and strain and n is the material strain hardening. The finite element analyses consider material flow properties covering typical pipeline grade steels with $E = 206$ GPa and $\nu = 0.3$: $n = 5$ and $E/\sigma_0 = 800$ (high hardening material), $n = 10$ and $E/\sigma_0 = 500$ (moderate hardening material), $n = 20$ and $E/\sigma_0 = 300$ (low hardening material). These ranges of properties also reflect the upward trend in yield stress with the increase in strain hardening exponent characteristic of ferritic steels**.**

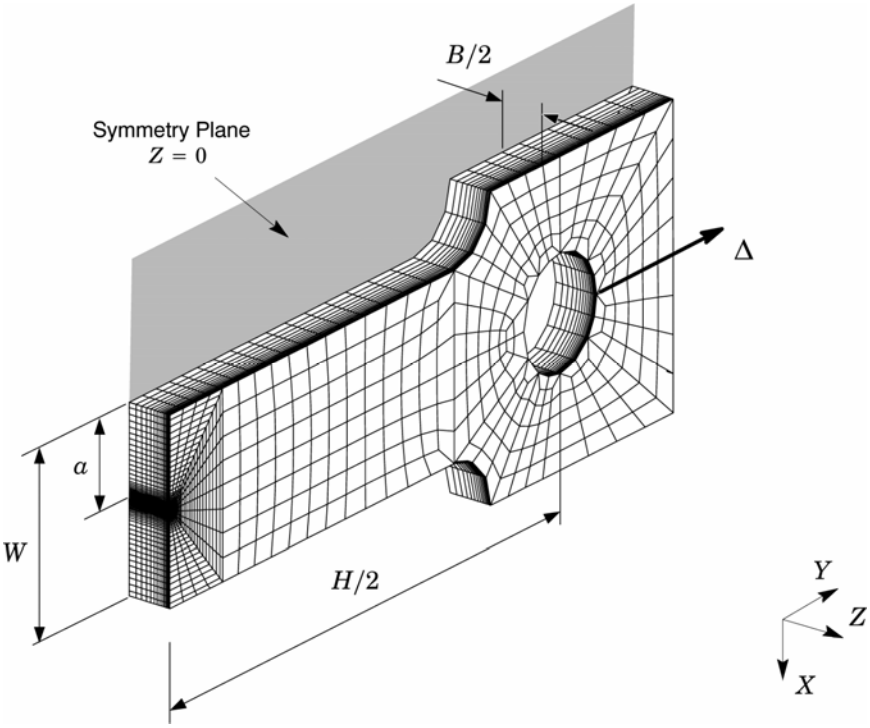

Figure 3. Quarter-symmetric, 3-D finite element model of pin-loaded SE(T) specimen with $a/W = 0.5$

4. Plastic η Factors

Evaluation of plastic η-factors for the analyzed crack configurations follows from solving Eqs. (4) and (7) upon computation of the plastic area, A_p**,** under the load-CMOD curve. A key question to resolve with the numerical procedure lies in the choice of the deformation level (CMOD) at which A_p (and consequently η) is evaluated. For very low deformation levels, the elastic component of the area under the load-deformation curve, A_e, has a magnitude which is comparable with the corresponding magnitude of the plastic component, A_p, thereby affecting the computed η-value. Guided by numerical experiences, the η-value is determined based upon an averaging procedure which computes the η-factor for the analyzed specimen as the least square value which lies within the deformation range given by $0.01 \le J/b\sigma_0 \le 0.05$. Here we note that such a procedure yields reasonably accurate and representative values for the eta-factor which are applicable for typical levels of experimentally measured *J*-values commonly observed in fracture testing. The research code CrackTool2D (Ruggieri, 2009) is employed to determine factors η_J and η_δ for the anayzed SE(T) fracture specimens.

Figures 4-5 provide the essential results from the plane-strain analyses needed to determine the elastic-plastic parameters *J* and CTOD for different material properties and crack configurations based on the η-factors for the pin-loaded and clamped SE(T) specimens. In the present context, these quantities are denoted $\eta_{J,P}^{CMOD}$, $\eta_{J,C}^{CMOD}$, $\eta_{\delta,P}^{CMOD}$ and $\eta_{\delta,C}^{CMOD}$ where it is understood that subscripts *P* and *C* refer to pin-loaded and clamped specimens**.** Figure 4(a-b) shows the variation of η_J with increased a/W -ratio for pin-loaded and clamped specimens. The results displayed in this plots reveal that factor η_J derived from CMOD exhibits little sensitivity to hardening properties for the entire range of a/W -ratio. Here, a strong linear relationship between factor η_J and a/W holds for all *n*-values. Figure 5(a-b) shows the variation of η_δ with increased a/W -ratio for pin-loaded and clamped specimens. In contrast to the previous results, the observed response displayed in these plots reveal that factor η_δ derived from CMOD exhibits a rather larger sensitivity to hardening properties for the entire range of a/W -ratio and for both loading conditions. Again, a strong linear relationship between factor η_δ and a/W holds for all *n*-values.

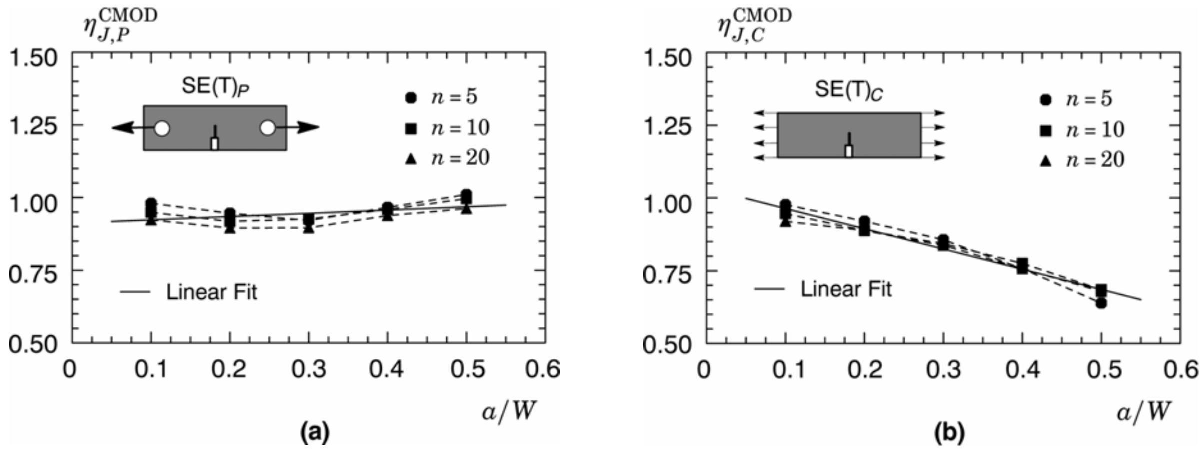

Figure 4. Variation of factor η_J with a/W -ratio for pin-loaded and clamped SE(T) specimens.

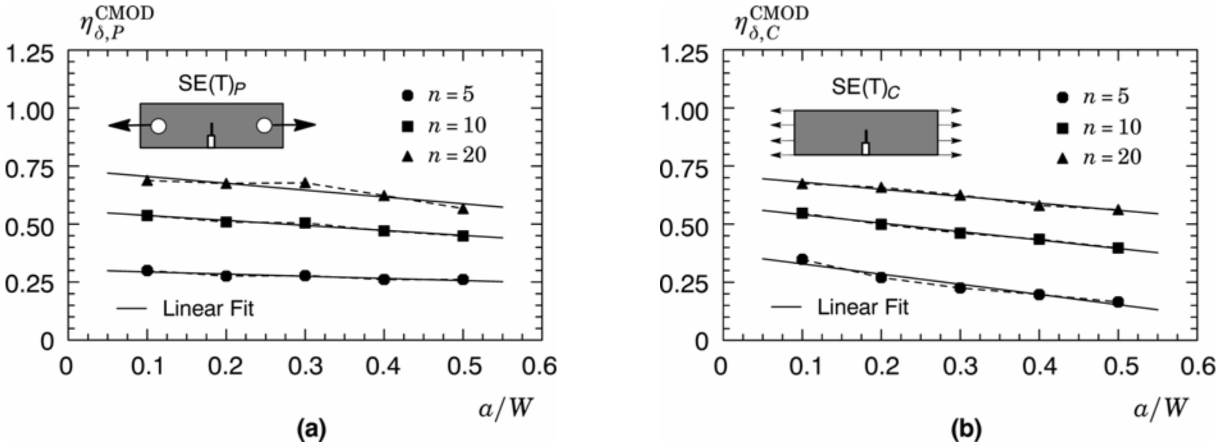

Figure 5. Variation of factor η_δ with a/W -ratio for pin-loaded and clamped SE(T) specimens.

Figure 6 compares the η_δ-factors for the pin-loaded and clamped SE(T) specimens based upon the plane-strain and full-thickness (3-D) analyses for the $n = 10$ (moderate hardening) material. The trends are clear. The computed 3-D η_δ-values are essentially similar to the corresponding plane-strain values. Further, the η_δ-factors for the conventional specimen configuration and the DNV geometry are practically indistinguishable from one another which entails using any of these two specimen configurations to characterize the material's fracture toughness provided the fracture resistance does not vary significantly with the plate orientation such as in fracture specimens extracted from a pipe. Overall, these results establish a strong support to use the extensive set of plane-strain eta-factors developed previously in fracture testing of SE(T) fracture specimens. Similar results are also observed for the η_J -values derived from the 3-D analyses; to conserve space, they are not shown here.

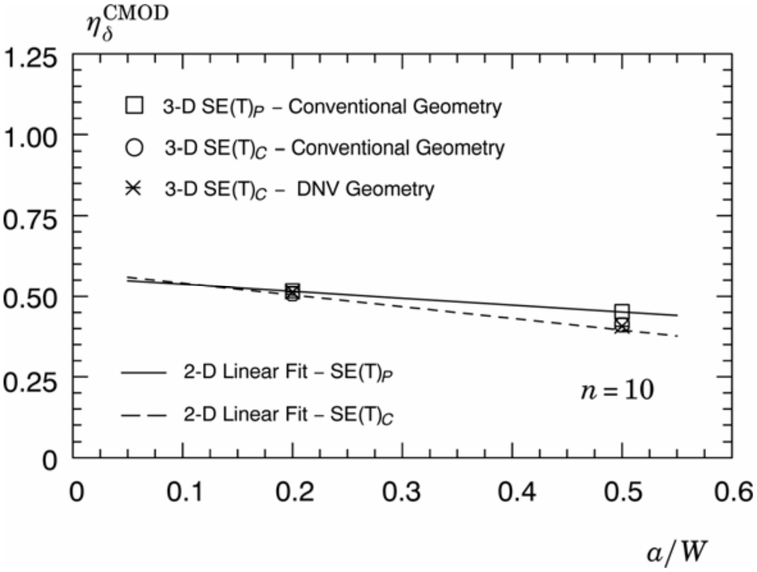

Figure 6. Comparison of factor η_δ with a/W -ratio for pin-loaded and clamped SE(T) specimens derived from plane-strain and full-thickness, 3-D analyses for the $n = 10$ material

5. Fitting Equations

To facilitate manipulation of the previous results to estimate J and CTOD in fracture testing, this section provides a polynomial fitting of the functional dependence of the corresponding η-factors on a/W for varying hardening properties based upon a least square scheme. The resulting curve fit for all cases agrees very well with the individual data points as displayed in the previous plots shown in Figs. 4 and 5.

- ***Factors $\eta_{J,P}^{CMOD}$ (J-integral for Pin-Loaded SE(T) Specimens)***

$$\eta_{J,P}^{CMOD} = 0.912 + 0.112(a/W) \quad ; \quad \text{all } n \tag{9}$$

- ***Factors $\eta_{J,C}^{CMOD}$ (J-integral for Clamped SE(T) Specimens)***

$$\eta_{J,C}^{CMOD} = 1.033 - 0.696(a/W) \quad ; \quad \text{all } n \tag{10}$$

- ***Factors*** $\eta_{\delta,P}^{CMOD}$ ***(CTOD for Pin-Loaded SE(T) Specimens)***

$$\eta_{\delta,P}^{CMOD} = 0.303 - 0.093(a/W) \quad ; \quad n = 5 \tag{11}$$

$$\eta_{\delta,P}^{CMOD} = 0.558 - 0.214(a/W) \quad ; \quad n = 10 \tag{12}$$

$$\eta_{\delta,P}^{CMOD} = 0.734 - 0.294(a/W) \quad ; \quad n = 20 \tag{13}$$

- ***Factors*** $\eta_{\delta,C}^{CMOD}$ ***(CTOD for Clamped SE(T) Specimens)***

$$\eta_{\delta,C}^{CMOD} = 0.372 - 0.439(a/W) \quad ; \quad n = 5 \tag{14}$$

$$\eta_{\delta,C}^{CMOD} = 0.577 - 0.364(a/W) \quad ; \quad n = 10 \tag{15}$$

$$\eta_{\delta,C}^{CMOD} = 0.710 - 0.302(a/W) \quad ; \quad n = 20 \tag{16}$$

6. Concluding Remarks

This work describes an estimation procedure to determine the elastic-plastic fracture parameters, *J* and CTOD, for single edge notch tension SE(T) specimens based upon the contribution of the plastic work to strain energy as characterized by the plastic area under the load-crack mouth opening displacement curve. An extensive set of plane-strain and 3-D analyses for SE(T) specimens with varying crack sizes and hardening properties enables very detailed computations of load-displacement data for these cracked configurations. These analyses then provide accurate values for η-factors which are applicable for a wide range of specimen geometries and material properties. The present analyses, when taken together with previous studies, provide a fairly extensive body of results which serve to determine parameters J and CTOD for different materials using bend specimens with varying geometries.

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