MODELING SUPERPLASTICITY PHENOMENOM IN METALLIC MATERIALS USING A CONTINUUM DAMAGE THEORY

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Abstract: *The present paper is concerned with the modeling of superplasticity phenomenon in metallic materials using a continuum damage theory. The goal is to propose a one-dimensional phenomenological damage model, as simple as possible, able to perform a mathematically correct and physically realistic description of plastic deformations, strain hardening, strain softening, strain rate sensitivity and damage (nucleation and growth of voids) observed in tensile tests performed at different strain rates. Only two tensile tests at different controlled strain rates are necessary to obtain all the material parameters that appear in the theory. Examples concerning the modeling of tensile tests of a magnesium alloy at different strain rates are presented and analyzed. The results obtained show a very good agreement between experimental results and model prevision.*

Keywords: *superplasticity, strain rate sensitivity, magnesium alloy, tensile test, continuum damage mechanics*

1. Introduction

A wide class of materials - metals, ceramics, intermetalics, nanocrystaline, etc - show superplastic behavior under special processing conditions. Although, up to now, there is no precise physical definition of superplasticity phenomenon in metallic materials, from a phenomenological point of view, superplasticity can be defined as very high deformations prior to local failure. In the case of tensile tests under controlled strain rate, this means very high elongations of the specimens before rupture. The deformation process is generally conducted at high temperature and the strain can be 10 times the obtained under room temperature. Superplastically deformed material in tensile tests gets thinner in a very uniform manner, rather than forming a 'neck' (a local narrowing) which leads to fracture. The most important characteristic of a superplastic material is its high strain rate sensitivity of flow stress that confers a high resistance to neck development and results in the high tensile elongations characteristic of superplastic behavior. Superplasticity is used to form directly complex objects, by the application of gas pressure or with a tool, and often with the help of dies, avoiding complicated and costly joining and machine steps. The applications of superplastic formations were originally limited to the aerospace industry, but it has recently been expanded to include the automobile industries as a result of breakthroughs in the range of materials that can be made superplastic.

The present paper is concerned with the modeling of such phenomenological behavior using a continuum damage theory. It is not the goal here to discuss the microscopic mechanisms of superplastic deformation. Most of the studies presented up to now in the literature are concerned with micro-structural aspects of the phenomenon. In the case of superplasticity, the damage is mainly due to nucleation and growth of voids in the material. An interesting analysis of cavity initiation and growth can be found in Khaleel et al, 2001. Other experimental works about superplastic behavior in magnesium alloys can be found in Kim et al 2001; Xin Wu and Yi Liu., 2002; Tan, 2002 ; Somekawa et al, 2003; Lin et al, 2005; Takuda et al, 2005; Yin, $2005_{a,b}$; Lee, et al, 2005.

In this work, the material parameters that appear in the proposed constitutive equations are identified for magnesium alloy AZ31 and the model previsions are compared with experimental test performed at different strain rates. Magnesium alloys have recently attracted significant interest due to their excellent specific properties that make them potentially suitable candidates for replacing heavier materials in some automobile parts. Superplastic forming of Mg alloys is an alternative way of shaping these materials into complex geometries in one single operation. Thus, significant efforts are being lately devoted to understanding the underlying physical processes that take place during superplastic deformation of Mg alloys in order to improve their formability. The experimental results considered on this paper are taken from Del Valle et al. (2005).

2. Basic definitions

Let's consider a simple tension test in which the specimen has a gauge length L and cross section A_o submitted to a prescribed elongation $\Delta L(t)$. The force applied on the specimen is noted $F(t)$. The so-called engineering strain ε and the engineering stress σ are defined as

$$\varepsilon(t) = \Delta L(t)\,/\,L \quad ; \quad \sigma(t) = F(t)\,/\,A_o \tag{1}$$

The so-called true strain ε_t and true stress σ_t are defined as

$$\varepsilon_t = ln(1+\varepsilon) \quad ; \quad \sigma_t = \sigma(1+\varepsilon) \tag{2}$$

From definitions (1) and (2) it is possible to obtain the following relations

$$\varepsilon_t = ln(1+\varepsilon) \;\Rightarrow\; \dot{\varepsilon} = exp(\varepsilon_t)\,\dot{\varepsilon}_t \quad ; \quad \dot{\varepsilon}_t = \dot{\varepsilon}\,/(1+\varepsilon) \tag{3}$$

The ASTM Standard E 2448 – 05 “Standard Test Method for Determining the Superplastic Properties of Metallic Sheet Materials” describes the procedure for determining the superplastic forming properties (SPF) of a metallic sheet material. It includes tests both for the basic SPF properties and also for derived SPF properties. The test for basic properties encompasses effects due to strain hardening or softening.

3. Modeling the true stress against true strain curve without damage

The main idea of the model is to propose a very simple expression for the true stress σ_t *vs* true strain ε_t curve :

HIP 1: $\sigma_t = a\,\left[1 - exp(-b\varepsilon_t)\right]$, with a and b being positive functions of ε_t and $\dot{\varepsilon}_t$ (4)

The dependency of the parameters a , b on ε_t and $\dot{\varepsilon}_t$ is the key to the definition of a physically realistic model. From experimental observations (see next section), it is possible to propose the following expression:

$a(\varepsilon_t, \dot{\varepsilon}_t)$ is such that $\left[e^{(a\text{-}N_a)} - e^{(a_o\text{-}N_a)}\right] = \left[\dot{\varepsilon}_t\,exp(\varepsilon_t)\right]^{K_a}$ (5)

with a_o , K_a , N_a , being temperature dependent positive parameters. Since $\dot{\varepsilon} = \dot{\varepsilon}_t\,exp(\varepsilon_t)$, a is constant in tensile tests with fixed value of the engineering strain rate $\dot{\varepsilon}$. Furthermore, it is easy to verify that $a = a_o$ when $\dot{\varepsilon} = 0$. If we define $\hat{a} = ln(e^{a}\text{-}e^{a_o})$, then, from (5) it is possible to obtain the following relations:

$$a = ln\left(e^{\hat{a}} + e^{a_o}\right) \qquad and \qquad \hat{a} = K_a\,ln(\dot{\varepsilon}) + N_a \tag{6}$$

To simplify the model, it will be assumed from now on that $a_o = 0$, hence from (6):

$$a(\varepsilon_t, \dot{\varepsilon}_t) = ln\left(e^{\hat{a}} + 1\right) \qquad with \qquad \hat{a} = K_a\,ln(\dot{\varepsilon}) + N_a \tag{7}$$

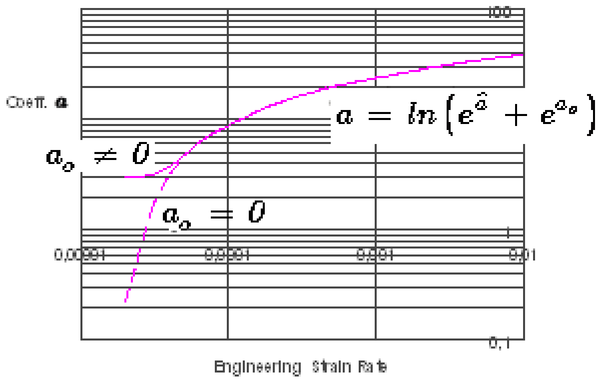

Figure 1: Variation of coefficient a with $a_0 = 0$ and $a_0 \neq 0$ for a magnesium alloy AZ31 at 375 °C

The relation between coefficient a and the engineering strain rate $\dot{\varepsilon}$ is plotted in Fig. 1 for the two different cases $a_o = 0$ and $a_o \neq 0$. It is evident that the obtained results differ only for lower strain rates.

Also from experimental observations, it is possible to propose:

$b(\varepsilon_t, \dot{\varepsilon}_t)$ is such that $\left[e^{(ab\text{-}N_{ab})} - e^{(a_o b_o\text{-}N_a b)}\right] = \left[\dot{\varepsilon}_t\,exp(\varepsilon_t)\right]^{K_{ab}}$ (8)

with b_o, K_{ab}, N_{ab}, being temperature dependent positive parameters. Since $\dot{\varepsilon} = \dot{\varepsilon}_t \, exp(\varepsilon_t)$, as shown for a, ab is constant in tensile tests with fixed value of the engineering strain rate $\dot{\varepsilon}$ and $ab = a_o b_o$ when $\dot{\varepsilon} = 0$. If we define $\widehat{ab} = ln\left(e^{ab} - e^{a_o b_o}\right)$, from (8) it is also possible to obtain the following relations:

$$ab = ln\left(e^{\widehat{ab}} + e^{a_o b_o}\right) \qquad and \qquad \widehat{ab} = K_{ab}\, ln(\dot{\varepsilon}) + N_{ab} \tag{9}$$

To simplify the model, it will be assumed from now on that $a_o b_o = 0$, hence:

$$ab(\varepsilon_t, \dot{\varepsilon}_t) = ln\left(e^{\widehat{ab}} + 1\right) \qquad with \qquad \widehat{ab} = K_{ab}\, ln(\dot{\varepsilon}) + N_{ab} \tag{10}$$

Thus parameter $b(\varepsilon_t, \dot{\varepsilon}_t)$ can now be expressed as ab and a ratio:

$$b(\varepsilon_t, \dot{\varepsilon}_t) = \frac{ab(\varepsilon_t, \dot{\varepsilon}_t)}{a(\varepsilon_t, \dot{\varepsilon}_t)} = \frac{ln\left(e^{\widehat{ab}} + 1\right)}{ln\left(e^{\hat{a}} + 1\right)} \;\; with \;\; \hat{a} = K_a\, ln(\dot{\varepsilon}) + N_a, \; \widehat{ab} = K_{ab}\, ln(\dot{\varepsilon}) + N_{ab} \tag{11}$$

Considering relation (11), it is easy to verify that b increases with increasing $\dot{\varepsilon}$ for $K_{ab} > K_a$, while b decreases with increasing $\dot{\varepsilon}$ for $K_{ab} < K_a$. Although expressions (5) and (8) are strongly non-linear, all parameters K_a, N_a, K_{ab}, N_{ab} can be identified from two tensile tests with constant engineering strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$. In a tensile test with constant engineering stress rate $\dot{\varepsilon}_i$, from (4), true stress σ_t can be expressed as

$$\sigma_t = a_i\left[1 - exp(-b_i \varepsilon_t)\right] \tag{12}$$

with

$$\hat{a}_i = K_a\, ln\left[\underbrace{exp(\varepsilon_t)\;\dot{\varepsilon}_t}_{\dot{\varepsilon}_i}\right] + N_a \qquad , \qquad \widehat{a_i b_i} = K_{ab}\, ln(\underbrace{exp(\varepsilon_t)\;\dot{\varepsilon}_t}_{\dot{\varepsilon}_i}) + N_{ab} \tag{13a-13b}$$

Parameters a_i and b_i can be identified from the true stress *vs* true strain curve obtained in a tensile test with constant engineering strain rate $\dot{\varepsilon}$ using a minimum squares curve fitting technique or using the following simpler procedure:

3.1. Identification of a_i

Parameter a_i can be identified from the true stress *vs* true strain curve. From (12), it is possible to obtain:

$$\lim_{\varepsilon_i \to \infty} (\sigma_t) = a_i \tag{14}$$

Hence, a_i is the maximum value of the stress σ_t (Fig.2).

3.2. Identification of b_i

From (12) it is also possible to verify that $\left.\frac{d\sigma_t}{d\varepsilon_t}\right|_{\varepsilon_t = 0} = a_i b_i$. Hence, once a_i is known, b_i can be identified from the initial slope of the true stress *vs* true strain curve. From (12) it is even possible to obtain

$$exp(-b_i \varepsilon_t) = \left(\frac{a_i - \sigma_t}{a_i}\right) \quad \text{or,} \quad -b_i \varepsilon_t = ln\left(\frac{a_i - \sigma_t}{a_i}\right)$$

Hence , b_i can be approximated from the following expression:

$$b_i = -\left(\frac{1}{\varepsilon_t}\right) ln\left(\frac{a_i - \sigma_t}{a_i}\right) \tag{15}$$

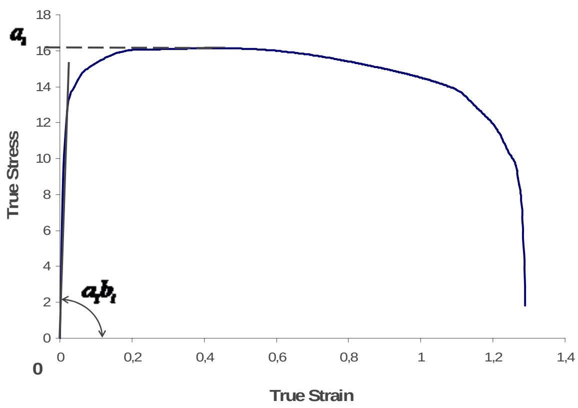

Figure 2: Identification of parameters a_i and b_i from the true stress *vs* true strain curve

3.4. Identification of K_a , N_a , K_{ab} , N_{ab}

Once a_1, b_1 and a_2, b_2 are identified from two tensile tests with different engineering strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$, the correspondent values of $\widehat{a_1}$, $\widehat{a_2}$ and $\widehat{a_1b_1}$, $\widehat{a_2b_2}$ are calculated from the definitions: $\widehat{a_i} = ln(e^{a_i}\text{-}1)$; $\widehat{a_ib_i} = ln\left(e^{a_ib_i}\text{-}1\right)$. These values permit to identified the parameters K_a, N_a, K_b, N_b as shown below. From (13a) it is possible to obtain

$$\widehat{a_1} - K_a ln(\dot{\varepsilon}_1) = N_a \quad \text{and} \quad -[\widehat{a_2} - K_a ln(\dot{\varepsilon}_2) = N_a] \tag{16), (17}$$

Hence, combining these equations it is possible to obtain

$$\widehat{a_1} - \widehat{a_2} = K_a\left[ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)\right] \Rightarrow K_a = \frac{\widehat{a_1} - \widehat{a_2}}{ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)} \tag{18}$$

The parameter K_a can be obtained from the following equation

$$N_a = \widehat{a_1} - K_a ln(\dot{\varepsilon}_1) \tag{19}$$

Once parameters K_a, N_a are known, it is possible to calculate $\widehat{a_i}$ for different strain rates and then obtain the correspondent a_i values from equation (7):

$$a_i = ln\left(1 + e^{\widehat{a_i}}\right) \tag{20}$$

With a similar procedure, from (13b), it is possible to verify that

$$\widehat{a_1b_1} - K_{ab} ln(\dot{\varepsilon}_1) = N_{ab} \quad \text{and} \quad -\left[\widehat{a_2b_2} - K_{ab} ln(\dot{\varepsilon}_2) = N_{ab}\right] \tag{21), (22}$$

Hence, combining these equations it is possible to obtain

$$\widehat{a_1b_1} - \widehat{a_2b_2} = K_{ab}\left[ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)\right] \Rightarrow K_{ab} = \frac{\widehat{a_1b_1} - \widehat{a_2b_2}}{ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)} \tag{23}$$

Parameter N_{ab} can be obtained from the following equation

$$N_{ab} = \widehat{a_1b_1} - K_{ab} ln(\dot{\varepsilon}_1) \tag{24}$$

Once parameters K_{ab}, N_{ab} are known, it is possible to calculate $\widehat{a_ib_i}$ for different strain rates and then obtain the correspondent a_ib_i values from relation (10):

$$a_i b_i = ln\left(1 + e^{\widehat{a_i b_i}}\right) \tag{25}$$

Finally, it is possible to calculate b_i as $a_i b_i$ and a_i ratio.

$$b_i = \frac{a_i b_i}{a_i} \Rightarrow b_i = \frac{ln\left(1 + e^{\widehat{a_i b_i}}\right)}{ln\left(1 + e^{\widehat{a}_i}\right)} \tag{26}$$

3.5. Determination of material parameters for magnesium alloy AZ31 at 375 °C with initial grain size d =17µm

In order to identify the parameters that appear in the previous sections, two different series of experimental results referred to two tensile tests carried out at different strain rates ($\dot{\varepsilon}_1 = 0{,}0003\ (1/\mathrm{sec})$ and $\dot{\varepsilon}_2 = 0{,}01\ (1/\mathrm{sec})$) have been considered.

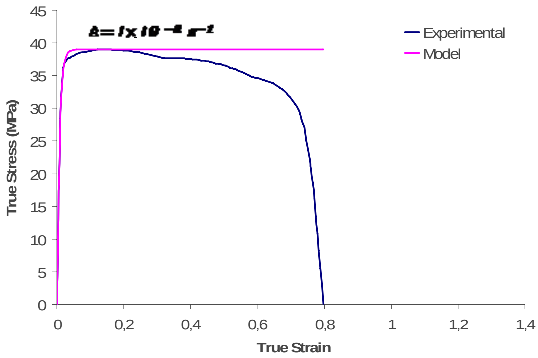

Figure 3: Stress-strain curve and model curve for $\dot{\varepsilon}_2 = 0{,}01\ (1/sec)$

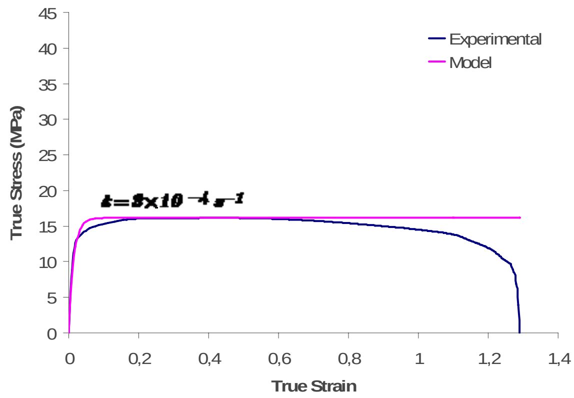

Figure 4: Stress-strain and model curve for $\dot{\varepsilon}_1 = 0{,}0003\ (1/sec)$

Considering these strain rates we have $a_1 = 16{,}15\ MPa$; $a_1 b_1 = 1130{,}5\ MPa$; $a_2 = 38{,}99\ MPa$; $a_2 b_2 = 5068{,}7\ MPa$. From (18), (19), (23) and (24) it is possible to obtain $N_a = 0{,}251$; $K_a = 124{,}06\ MPa$; $N_{ab} = 68.98$; $K_{ab} = 6{,}51\ MPa$

Figs. 3 and 4 show the model curves considering these parameters.

4. Modeling the true stress against true strain curve with damage

Only a few damage models were proposed for superplastic alloys, such as Chandra (2002). In the present paper it is introduced an auxiliary variable D that accounts for the nucleation and growth of voids observed in tensile tests performed at different strain rates.

HIP 2: $$\sigma_t = (1 - D)\ a\ [1 - exp(-b\varepsilon_t)] \quad with \quad 0 \leq D \leq 1 \tag{27}$$

HIP 3: $$D = \begin{cases} 0, \ if\ AUX > 1 \\ -\left[\dfrac{1}{b_d}\right] ln(AUX), \ if\ 0 < AUX < 1 \\ 1, \ if\ AUX < 0 \end{cases} \quad with \quad AUX = 1 - \left(\frac{\varepsilon_t - K_d/b}{a_d}\right) \tag{28}$$

Where

$$a_d = -K_{a_d}(\dot{\varepsilon}) + N_{a_d} \quad and \quad a_d b_d = K_{a_d b_d}[\dot{\varepsilon}]^{-N_{a_d b_d}} \tag{29a-29b}$$

All parameters K_d, N_d, a_d, b_d can be identified from two tensile tests with constant engineering strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$. Considering HIP 2. For a tensile test with constant stress rate $\dot{\varepsilon}_i$, the damage variable D can be expressed as $\dfrac{\sigma_t}{a\left[1-exp(-b\varepsilon_t)\right]} = (1 - D) \Rightarrow D = 1 - \dfrac{\sigma_t}{a\left[1-exp(-b\varepsilon_t)\right]}$, after the softening behavior. Hence, the experimental curve D *vs* ε_t can be easily obtained (Fig.5).

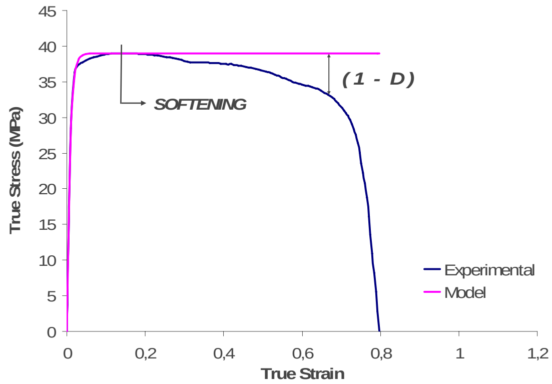

Figure 5: Experimental identification of the auxiliary variable D.

Since $\dot{\varepsilon} = exp(\varepsilon_t)\ \dot{\varepsilon}_t$, it is possible to obtain from HIP 3

$$\varepsilon_t = a_d\left[1 - exp(-b_d D)\right] + K_d/b \tag{30}$$

Parameters a_i and b_i can be identified from the true stress *vs* true strain curve obtained in a tensile test with constant engineering strain rate $\dot{\varepsilon}$ using a minimum squares curve fitting technique or using the following simpler procedure. If $D = 0$, from (22) it is possible to obtain $\varepsilon_t = K_d / b$ A "corrected curve" is obtained by eliminating the viscous term K_d/b from this curve.

$$(\varepsilon_t)_{corrected} = a_d\left[1 - exp(-b_d D)\right] \tag{31}$$

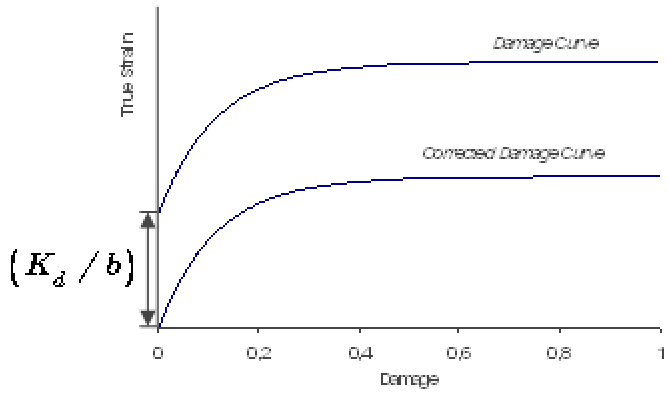

Figure 6: Damage curve and corrected damage curve obtained from a tensile test with constant engineering stress rate.

Parameters a_i and b_i can be identified from the true stress *vs* true strain curve using a minimum squares curve fitting technique or using the following simpler procedure.

4.1. Identification of a_d

From (31), it is possible to obtain:

$$\lim_{D\to 1}\left((\varepsilon_t)_{corrected}\right) = a_d \tag{32}$$

Hence, a_d is the value of the corrected strain $(\varepsilon_t)_{corrected}$ when $D \to 1$

4.2. Identification of b_d

From (31) it is possible to verify that $\left.\dfrac{d(\varepsilon_t)_{corrected}}{dD}\right|_{D=0} = a_d b_d$ Hence, once a_d is known, b_d can be identified from the initial slope of the true corrected damage curve. From (30) it is possible to obtain $exp(-b_d D) = \left(\dfrac{a_d - (\varepsilon_t)_{corrected}}{a_d}\right)$ or, $-b_d D = ln\left(\dfrac{a_d - (\varepsilon_t)_{corrected}}{a_d}\right)$

b_d can be approximated from the following expression:

$$b_d = -\left(\frac{1}{D}\right) ln\left(\frac{a_d - (\varepsilon_t)_{corrected}}{a_d}\right) \tag{33}$$

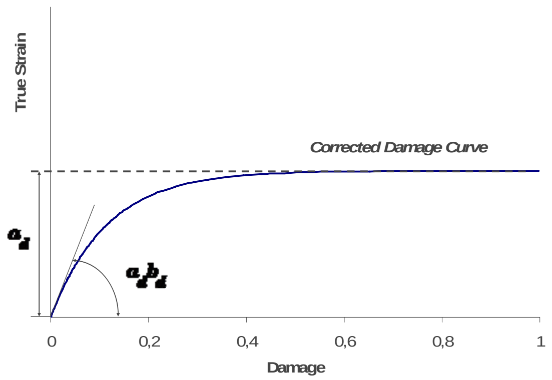

Figure 7: Identification of the parameters a_d and b_d from the true stress *vs* true strain curve

4.3. Identification of K_{a_d}, N_{a_d}, $K_{a_d b_d}$, $N_{a_d b_d}$

Once a_{d_1}, b_{d_1} and a_{d_2}, b_{d_2} are identified from two tensile tests with different engineering strain rates $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$, the parameters K_{a_d}, N_{a_d}, $K_{a_d b_d}$, $N_{a_d b_d}$ can also be identified. From (29a) it is possible to obtain

$$-K_{a_d}\dot{\varepsilon}_1 + N_{a_d} = a_{d_1} \text{ and } -\left[-K_{a_d}\dot{\varepsilon}_2 + N_{a_d} = a_{d_2}\right]$$

Hence, combining these equations it is possible to obtain

$$K_{a_d}(\dot{\varepsilon}_2 - \dot{\varepsilon}_1) = a_{d_1} - a_{d_2} \Rightarrow K_{a_d} = \frac{a_{d_1} - a_{d_2}}{\dot{\varepsilon}_2 - \dot{\varepsilon}_1} \tag{34}$$

The parameter N_{a_d} can be obtained from the following equation

$$N_{a_d} = a_{d_1} + K_{a_d}\dot{\varepsilon}_1 \tag{35}$$

While from (29b) we have

$$ln(K_{a_d b_d}) + N_{a_d b_d}\, ln(\dot{\varepsilon}_1) = ln(a_{d_1} b_{d_1}) \text{ and } -\left[ln(K_{a_d b_d}) + N_{a_d b_d}\, ln(\dot{\varepsilon}_2) = ln(a_{d_2} b_{d_2})\right]$$

which combined give $N_{a_d b_d}$

$$N_{a_d b_d}\left[ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)\right] = \left[ln(a_{d_1} b_{d_1}) - ln(a_{d_2} b_{d_2})\right] \Rightarrow N_{a_d b_d} = \frac{\left[ln(a_{d_1} b_{d_1}) - ln(a_{d_2} b_{d_2})\right]}{\left[ln(\dot{\varepsilon}_1) - ln(\dot{\varepsilon}_2)\right]} \tag{36}$$

Parameter $K_{a_d b_d}$ can be obtained from the following equation

$$K_{a_d b_d} = a_1 / (\dot{\varepsilon}_1)^{N_{a_d b_d}} \tag{37}$$

4.5. Identification of K_d

Parameter K_d can be identified from the strain *vs* damage curves obtained in a tensile test with constant strain rates $\dot{\varepsilon}_1$. and $\dot{\varepsilon}_2$. The value $\eta = K_d / b$ can be obtained from both experimental damage curves, considering an average value. Hence, we have

$$K_d = (b_1\eta_1 + b_2\eta_2)/2 \quad and \quad \eta_i = K_d / b_i \tag{38}$$

4.6. Determination of material parameters for magnesium alloy AZ31 at 375 °C with initial grain size d =17µm

The experimental damage curves shown in Figs. 8 and 9 have been obtained for the investigated strain rates $\dot{\varepsilon}_1 = 0{,}0003\ (1/sec)$ and $\dot{\varepsilon}_2 = 0{,}01\ (1/sec)$

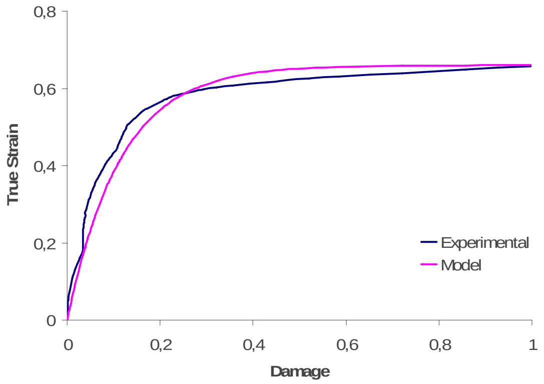

Figure 8: Corrected damage curve. Model and experiment. $\dot{\varepsilon}_1 = 0{,}0003\ (1/sec)$.

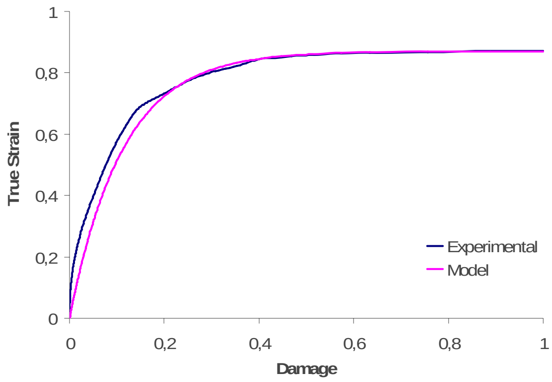

Figure 9: Corrected damage curve. Model and experiment. $\dot{\varepsilon}_2 = 0{,}01\ (1/sec)$.

For $\dot{\varepsilon}_1 = 0{,}0003\ (1/sec)$ the model curve which fits the experimental corrected damage curve gives $\eta_1 = 0{,}067$, $a_{d_1} = 1{,}12$ and $a_{d_1}b_{d_1} = 13{,}44$ while for $\dot{\varepsilon}_2 = 0{,}01\ (1/sec)$ the model curve gives $\eta_2 = 0{,}12$, $a_{d_2} = 0{,}72$ and $a_{d_2}b_{d_2} = 7{,}92$ (Fig 8 and 9). Thus, from (34), (35), (36) and (37) it is possible to obtain $N_{a_d} = 1{,}16$; $K_{a_d} = -43{,}29$; $N_{a_d b_d} = -0{,}13$; $K_{a_d b_d} = 4{,}28$ while relation (38) gives $K_d = 8{,}71$. Hence, the following coefficients were identified for the examined magnesium alloy while the related model curves shown in Fig. 10 have been obtained: $a_1 = 16{,}20\ MPa$; $a_1 b_1 = 1130{,}5\ MPa$; $a_2 = 38{,}99\ MPa$; $a_2 b_2 = 5068{,}7\ MPa$; $N_a = 68{,}92$; $K_a = 6{,}50\ MPa$; $N_{ab} = 10236$; $K_{ab} = 1122{,}1\ MPa$; $K_d = 8{,}71$ $a_{d_1} = 1{,}15$; $a_{d_1}b_{d_1} = 12{,}65$; $a_{d_2} = 0{,}73$; $a_{d_2}b_{d_2} = 7{,}92$; $N_{a_d} = 1{,}16$; $K_{a_d} = -43{,}29$; $N_{a_d b_d} = -0{,}13$; $K_{a_d b_d} = 4{,}28$

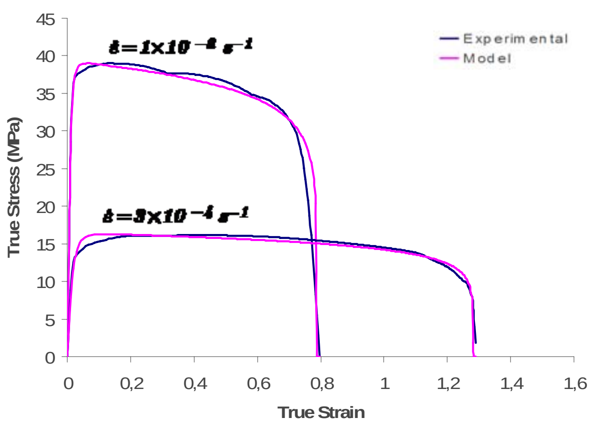

Figure 10: Stress-strain curve for $\dot{\varepsilon}_1 = 0{,}0003(1/sec)$ and $\dot{\varepsilon}_2 = 0{,}01(1/sec)$ for a magnesium alloy AZ31B-F at 375 °C

5. Proposed model

From section (3) and (4) the model equations can be summarized as follows:

$$\sigma_t = (1 - D)\ a\ [1 - exp(-b\varepsilon_t)] \quad with \quad 0 \le D \le 1$$

Where

$$D = \begin{cases} 0, \ if\ AUX > 1 \\ -\left[\dfrac{1}{b_d}\right]\ ln\,(AUX),\ if\ 0 < AUX < 1 \\ 1,\ if\ AUX < 0 \end{cases} \qquad with \qquad AUX{=}1{-}\left(\frac{\varepsilon_t\ -\ K_d/b}{a_d}\right)$$

and

$$ln\,(e^a{-}1) = K_a\,ln(\dot\varepsilon) + N_a \ , \quad ln\left(e^{ab}{-}1\right) = K_{ab}\,ln(\dot\varepsilon) + N_{ab} \ , \quad AUX{=}1{-}\left(\frac{\varepsilon_t\ -\ \left[K_d\,/\,b\right]}{a_d}\right) \ ,$$

$$a_d = -K_{a_d}(\dot\varepsilon) + N_{a_d} \qquad , \qquad a_d b_d{=}K_{a_d b_d}\,[\dot\varepsilon]^{-N_{a_d b_d}}$$

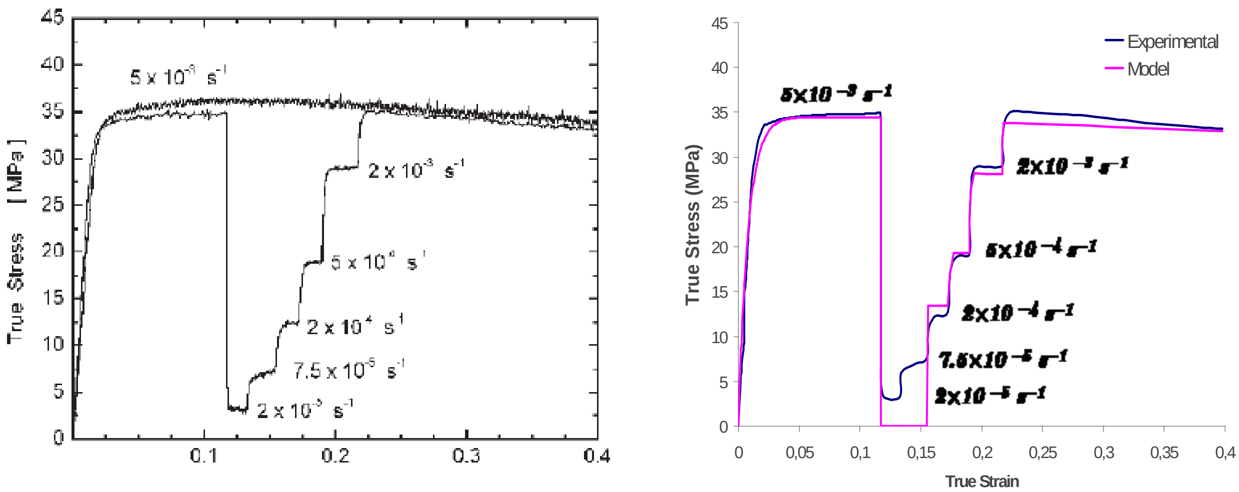

Figure 11a-b: Step Test curve from Del Valle 2005 and model for a magnesium alloy AZ31 at 375 °C
$(a_0 = a_0 b_0 = 0)$

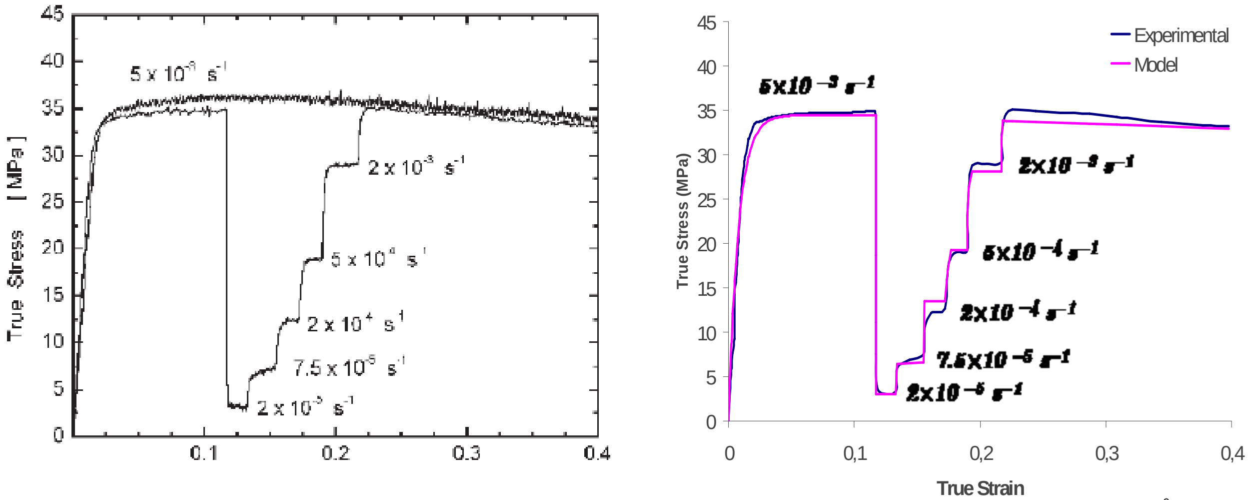

Figure 12a-b: Step Test curve from Del Valle 2005 and model curve for a magnesium alloy AZ31 at 375 °C
$(a_0{=}3\ ,\ a_0 b_0 = 135)$

Using the coefficient previously identified, it is now possible to predict the mechanical behavior of the same magnesium alloy deformed at the same conditions but at a different value of engineering strain rate. Good results have been observed applying the model to the step test experimental data taken from Del Valle 2005 and referred to the same material tested in the previous tensile tests (Fig. 11). Nevertheless, as shown in Fig. 11b, unrealistic previsions of the superplastic behavior can be observed for the lowest strain rates as $\dot{\varepsilon} = 0,000075\ (1/sec)$ $\dot{\varepsilon} = 0,00002\ (1/sec)$. Such limitation can be circumvented by taking the parameters a_o and b_o different from zero. When nonzero values are assumed for a_0 and b_0, good fitting curves can be obtained even for low values of strain rate as it can be seen in Fig. 12a-b. The hypothesis of nonzero values for a_0 and b_0 suggests, as reported by literature, an evidence of some form of threshold stress for superplastic flow since dislocation activity is not normally observed at lower strain rates.

6. Concluding remarks

The one-dimensional phenomenological damage model proposed on this paper is able to perform a mathematically correct and physically realistic description of plastic deformations, strain hardening, strain softening, strain rate sensitivity and damage observed in tensile tests performed at different strain rates and temperatures.

The identification of parameters that appear in the theory is discussed in detail and examples concerning the modeling of tensile tests of a magnesium alloy at different strain rates and temperatures are presented and analyzed. It is necessary to perform only two tensile tests at different strain rates in order to identify the parameters that appear in the theory. The results obtained show a good agreement between experimental results and model prevision for different strain rates. Finally, it is important to observe that it is necessary to adapt such model to account for compression loading, what can be done through the introduction of a new auxiliary variable related to the cumulated plastic strain. It is also necessary an adequate thermo mechanical framework in order to extend the proposed model to a tri-dimensional context, which is essential since one of the main practical motivation to study such alloys is the superplastic forming (SPF) of sheet metals. In a tri-dimensional context, the adequate choice of the measure of strain and of the objective time derivative is essential to build a physically realistic and mathematically correct model (Costa-Mattos, 1998).

7. References

ASTM Standard E 2448 – 05. Standard Test Method for Determining the Superplastic Properties of Metallic Sheet Materials.

Chandra, N., 2002. Constitutive behavior of superplastic materials. International Journal of Non-Linear Mechanics 37, 461-484.

Costa-Mattos, H., 1998. A Thermodynamically Consistent Constitutive Theory for Fluids. International Journal of Non-linear Mechanics. Vol. 33, no. 1, pp. 97 – 110.

Del Valle, J.A., Pérez-Prado, M.T., Ruano, O.A., 2005. Deformation mechanisms responsible for the high ductility in a Mg AZ31 alloy analyzed by electron backscattered diffraction. Metallurgical And Materials Transactions A 1427-1438—Volume 36A.

Lee, B.H., Shin, K.S., Lee, C. S., 2005. High temperature Behavior of AZ31 Mg Alloy. Materials Science Forum Vols 475-479 pp. 2927-2950.

Lin, H.K., Huang, J.C., Langdon, T.G., 2005. Relationship between texture and low temperature superplasticity in an extruded AZ31 Mg alloy processed by ECAP. Materials Science and Engineering A 402, 250–257.

Khaleel, M.A., Zbib, H.M., Nyberg, E.A., 2001. Constitutive modeling of deformation and damage in superplastic materials. International Journal of Plasticity 17, 277-296.

Kim, W. J., Chung, S. W., Chung, C.S., Kum, D., 2001. Superplasticity in thin magnesium alloy sheets and deformation mechanism maps for magnesium alloys at elevated temperatures. Acta Mater. 49, 3337–3345.

Somekawa, H., Hosokawa, H., Watanabe, H., Higashi, K., 2003. Diffusion bonding in superplastic magnesium alloys. Materials Science and Engineering A339, 328–333.

Takuda, H., Morishita T., Kinoshita, T., Shirakawa, N., 2005. Modelling of formula for flow stress of a magnesium alloy AZ31 sheet at elevated temperatures. Journal of Materials Processing Technology 164–165, 1258–1262.

Tan, J.C., Tan, M.J., 2002. Superplasticity in a rolled Mg–3Al–1Zn alloy by two-stage deformation method. Scripta Materialia 47, 101–106.

Xin, W., Yi, L., 2002. Superplasticity of coarse-grained magnesium alloy. Scripta Materialia 46, 269–274.

Yin, D.L., Zhang, K.F., Wang, G.F., 2005. Microstructure evolution and fracture behavior in superplastic deformation of hot-holled AZ31 Mg alloy sheet. Materials Science Forum Vols 475-479 pp. 2923-2926

Yin, D.L., Zhang, K.F., Wang, G.F., Han , W.B., 2005. Superplasticity and cavitation in AZ31 Mg alloy at elevated temperatures. Materials Letters 59, 1714– 1718.