COMPOSITE PIPING DYNAMIC BEHAVIOR

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Abstract. *This work presents the simulation of a composite piping system under different flow velocities. The fluid and structural domain are modeled as unidimensional problem while the fluid behavior is described in velocity formulation. For the coupled fluid-structure problem the Finite Element Method is employed to solve the final linearized system. Some remarks are made about structural reinforcements using simplified models for both composite and sandwiched structures.*

Keywords: *Composite piping, fluid-structure interaction, finite elemente method*

1. INTRODUCTION

The structural behavior of a piping network is usually treated employing unidimensional beam formulation. An important problem in this subject is the transient behavior of the fluid produced by valve and pump operations or by fluid-structure interaction. This fluid transients will induce structural transients or vibrations.

This problem has been treated with different formulations. The fluid behavior only may be analysed with method of characteristics or finite differences as in Jennings et al. (2005), but fluid-structure interaction is treated with Finite Elements using beam simplest models. On the fluid domain the pressure is the primary variable but some recents works as (Kochupillai et al., 2005) and Sreejith et al. (2004), to cite some of them, use velocity as fluid variable. There are works using Spectral Elements (Lee and Park, 2006) to describe the uniform straight pipelines. The objective of this different formulations is to overcome some numerical instabilities as its increase with large problems.

If the piping material is isotropic there is no problem with the structural formulation as the ratio length to diameter is very large and the Euler Bernoulli model can be employed. If the piping material is a composite material as fiberglass reinforced epoxy the same procedure is employed using the Bending Modulus (Smith Project Guide, 2001). A composite made aerial pipeline might be more sensitive to transients as it has low mass and low mechanical properties than the conventional piping materials. Some piping network have used a ballast in the gap between the internal and external pipe forming a external tube reinforcement to improve the inertia.

There is a vast bibliography on the composite and sandwiched beam mainly on the property determination. A short review on formulation can be found on Hu et al. (2008) and there are a lack of recent works on piping with external reinforcement like a sandwiched piping.

The cilindrical and shell composite formulation are well known Qatu (2004) and Soedel (2004) and there is a large literature available for modeling the coupled fluid structure interaction involving the transient flow through pipes or shells Kadoli and Ganesan (2004). All this formulations are at least bidimensional and not aplicable to unidimensional transient pipeline published work.

The objective of this work is to present a simplified model that can be used on unidimensional Finite Element for network models.

2. PROBLEM FORMULATION

The governing dynamic equations for the pipeline structure with internal flow are classical and can be found in many authors Kochupillai et al. (2005) and Sreejith et al. (2004) or Lee and Park (2006). The instantaneous transverse motion is described by:

$$E_f I_t w'''' - T_o w'' + m_t \ddot{w} - \tau S w' = 0 \tag{1}$$

where E_f is the Bending Modulus for composite reinforced pipe, I_t is the area moment of the section reinforced by fibers, m_t is the linear density of the tube or mass per length, T_o is the axial force, τ is the flow induced shear tension, S internal perimeter of the tube, $w(x,t)$ is the transverse displacement. As usual w' is the derivative $\partial w/\partial x$, x is the tube axis coordinate and a point over the displacement is the temporal derivative, velocity $\dot{w}$ and two points acceleration $\ddot{w}$. The instantaneous longitudinal displacement $u(x,t)$ on axial position x of the tube is described as:

$$(E_t A_t + T_o)\, u'' - m_t \ddot{u} + \tau S = 0 \tag{2}$$

A_t fiber reinforced transverse area, E_t is the traction modulus, m_t, T_o, τ and S as defined for Eq. (1). The term u' is $\partial u/\partial x$, where x is the axis coordinate, the velocity is $\dot{u}$ and acceleration $\ddot{u}$.

The continuity equation of the fluid taking in the piping wall elasticity, can be expressed as (Lee and Park, 2006), (Sreejith et al., 2004):

$$\dot{p}A_e + m_f a^2 \left(c' - 2\nu\dot{u}'\right) = 0 \tag{3}$$

where p is the instantaneous pressure and $\dot{p}$ its time derivative, A_e is the flow area, m_f is the fluid mass per length of the tube, c is the instantaneous fluid velocity ($c(c,t)$), c' is $\partial c/\partial x$, the Poisson's coefficient for the pipe material is ν and u the axial local pipe displacement. The sound or perturbation propagation velocity a depends on the fluid properties, piping dimensions and its mechanical properties:

$$a^2 = \frac{E_v E_t t}{\rho_f \left(E_v D + E_t t\right)} \tag{4}$$

on the Eq. (4) E_v is the fluid bulk modulus and ρ_f its density, D is the internal diameter and t the wall thickness.
Applying the force balance on fluid element the axial momentum can be obtained and expressed as:

$$(pA_e)' + \tau S + m_f \left(\ddot{u} + \dot{c} + cc' + c\dot{u}'\right) = 0 \tag{5}$$

in transverse direction:

$$(pA_e w')' + \tau S w' + m_f \left(\ddot{w} + 2c\dot{w}' + \dot{c}w' + c^2 w'' + c\dot{c}'w'\right) = 0 \tag{6}$$

The shearing force can be express as a function of the fluid variable c

$$\tau S = m_f \frac{f}{2D} c^2 \tag{7}$$

where f is the friction factor and is function of the Reynold's number and of the wall rugosity. Substituting on Eq. (2) the structural behavior can be expressed as:

$$\left(E_t A_t + T_o\right) u'' - m_t \ddot{u} + m_f \frac{f}{2D} c^2 = 0 \tag{8}$$

and substituting Eq. (5) and Eq. (7) on Eq. (1).

$$E_f I_t w'''' + \left(pA_e - T_o + m_f c^2\right) w'' + p' A_e w' + m_f \left(2c\dot{w}' + \dot{c}w' + c\dot{c}'w'\right) + m_t \ddot{w} = 0 \tag{9}$$

and substituting Eq. (6) and Eq. (5) on Eq. (3) it is possible to express the fluid motion with only one equation:

$$c'' - \frac{1}{a^2}\dot{c}c' - \frac{1}{a^2}c\dot{c}' - \frac{1}{a^2}\ddot{c} - \frac{f}{a^2 D}c\dot{c} - 2\nu\dot{u}'' - \frac{1}{a^2}\dot{c}\dot{u}' - \frac{1}{a^2}c\ddot{u}' = 0 \tag{10}$$

The system formed by Eq. (8) to Eq. (10) can be linearized using the following equations:

$$c(x,t) = c_o + c_t(x,t) \tag{11}$$

$$p(x,t) = p_o + p_t(x,t) \tag{12}$$

where p_o and c_o are the steady pressure and velocity, c_t and p_t the velocity and pressure fluctuations. If this perturbations are small, as usually it happen on pump and valve operations, it is possible to consider $|p_t| < p_o$ and $|c_t| < c_o$ so the small nonlinear terms can be taken out and the piping dynamic equations can be written as:

$$E_f I_t w'''' + \left(pA_e - T_o + m_f c_o^2\right) w'' + 2m_f c_o \dot{w}' + m_t \ddot{w} = 0 \tag{13}$$

$$\left(E_t A_t + T_o\right) u'' - m_t \ddot{u} + m_f \frac{f}{2D} c_o c_t + m_f \frac{f}{2D} c_o^2 = 0 \tag{14}$$

$$c_t'' - \frac{c_o}{a^2}\dot{c}_t - \frac{\ddot{c}_t}{a^2} - \frac{f}{a^2 D} c_o c_t - 2\nu\dot{u}'' - \frac{1}{a^2} c_o \ddot{u}' = 0 \tag{15}$$

$$\dot{p}A_e + m_f a^2 \left(c_t' - 2\nu\dot{u}'\right) = 0 \tag{16}$$

2.1 Formulation for distributed stiffiness coupling for reinforced piping

The reinforced piping is made by two concentric tubes coupled by a material that filled up the gap. The first consideration is that the pipeline ratio length per diameter is very high and the shear effect is probably small. So is realistic suppose that the filling material has a equivalent linear elastic stiffness k_d. The dammping effect will be included in a next work.

In the equation of motion for the external pipe, or reinforcement tube, Eq.(1), the flow shear term is dropped and a distributed stiffiness k_d is added. The resulting coupling force and its direction depends on local displacement from the axis w for external and v for the internal pipe.

$$E_{fe} I_t w'''' - T_o w'' + m_{te}\ddot{w} + k_d(w - v) = 0 \tag{17}$$

The coupling force will be generated only if w and v have opposite directions or different displacement value. The equation of motion for the internal tube holds the fluid shear term as the fluid flows inside and the stiffness force term is added too:

$$E_{fi} I_t v'''' + \left(pA_e - T_o + m_f c_o^2\right) v'' + 2m_f c_o \dot{v}' + m_t \ddot{v} + k_d(v - w) = 0 \tag{18}$$

The coupled system of equations Eq(17), Eq(18), Eq(14) and Eq(15) describes the behavior of the reinforced piping system.

2.2 Solution Method

From the linearized coupled system that is given by equations Eq(17), Eq(18), Eq(14) and Eq(15) and following the Finite Element procedure the matricial system is obtained:

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{C}]\{\dot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{F}\} \tag{19}$$

In the matricial equation the matrix $[\mathbf{C}]$ is unsymmetrical and can not be treated as a proportional damping matrix. To avoid this problem the matricial Eq. (19) is written in a state space form. Defining $z_i = \dot{x}_i$, the Eq. (19) can be written as:

$$[\mathbf{M}]\{\dot{\mathbf{z}}\} + [\mathbf{C}]\{\mathbf{z}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{F}\} \tag{20}$$

or

$$\{\dot{\mathbf{z}}\} + [\mathbf{M}]^{-1}[\mathbf{C}]\{\mathbf{z}\} + [\mathbf{M}]^{-1}[\mathbf{K}]\{\mathbf{x}\} = [\mathbf{M}]^{-1}\{\mathbf{F}\} \tag{21}$$

redefining the vector z as:

$$\mathbf{z} = \left\{ \begin{array}{c} x \\ \dot{x} \end{array} \right\} \tag{22}$$

and the equation of motion on space state form is given by:

$$\{\dot{\mathbf{z}}\} = [\mathbf{A}]\{\mathbf{z}\} + \{\mathbf{B}\} \tag{23}$$

where

$$[\mathbf{A}] = \left[\begin{array}{cc} 0 & [\mathbf{I}] \\ -[\mathbf{M}]^{-1}[\mathbf{K}] & -[\mathbf{M}]^{-1}[\mathbf{C}] \end{array} \right] \qquad \{\mathbf{B}\} = \left\{ \begin{array}{c} 0 \\ [\mathbf{M}]^{-1}[\mathbf{F}] \end{array} \right\} \tag{24}$$

As there is no external force acting on the system, $[\mathbf{F}] = 0$ on Eq.(24), the eigenvalue problem is solved for each flow velocity.

3. NUMERICAL RESULTS AND DISCUSSION

The first case is a simulation of the behavior of the fiberglass reinforced piping under steady flow for different velocities. The second case presented here is the fiberglass with external pipe reinforcement. To avoid misunderstanding the conventional fiberglass reinforced epoxy is treated as fiberglass piping and the fiberglass with external pipe reinforcement is treated as fiberglass with reinforcement.

3.1 Internal Flow in Fiberglass piping

The fiberglass reinforced piping has the following properties from the reference Smith Project Guide (2001). The main dimensions are: external and internal diameter D_e=34,1 mm, D_i=30,4 mm, length L=5,8 m, transverse section of the tube reinforced by fibers A_t=134,8 mm^2, area moment $I_t = 1,7898 \times 10^{-08}$ m^4. Bending modulus $E_f = 1,4755 \times 10^{10}$ N/m^2, traction modulus $E_t = 1,2213 \times 10^{10}$ N/m^2, Poisson's coefficient $\nu = 0,38$, mass by length m_t=0,213 kg/m. Water is

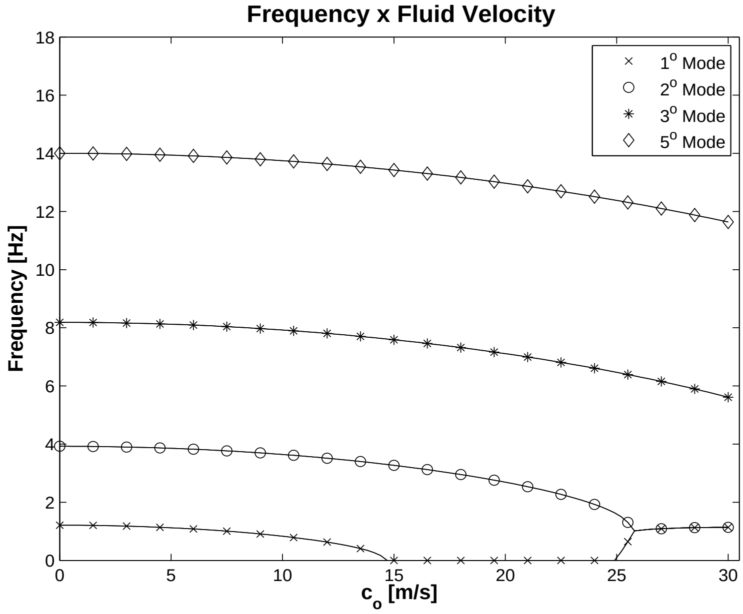

Figure 1. Frequency x Flow Velocity. Internal flow in fiberglass piping.

used as fluid, the density ρ_f=1000 kg/m^3 bulk modulus E_v=2,24$\times 10^9$ N/m^2, cinematic viscosity υ=1,06$\times 10^{-6}$ m^2/s, and mass of water by length of the tube m_f=0,7450 kg/m.

The eigenvalue problem given by Eq. (24) is solved for each flow velocity and 100 unidimensional elements are used in all results presented in Fig.(1) and Fig.(2). In the Fig. (1) is plotted the frequencies for the first 4 bending vibration modes as function of the steady flow velocity c_o.

In this Fig.(1) the critical steady velocity c_o is about 14,7 m/s, at this velocity and above there is no dynamic stiffness as the natural frequency is zero. This critical velocity is a high velocity and it is no used even for transportation of low density fluids like gases or vapors in pipelines. This critical velocity does not depends only on the piping material and fluid, but its dimensions like diameter thickness and length, pipeline supports and its spacing. In this work the pipe is clamped-supported.

For c_o at about 25,8 m/s takes place a coupling between the first and second modes this merging mode is named coupled-mode flutter or flutter. But this phenomena takes place at a highest velocity than the critical one and it is of little interest in this work as it is on unstable flow region and the main objective of this work is to improve the stability of the dynamical system.

On Fig. (2) is ploted the receptance Frequency Response Function, FRF (w/F), for the fiberglass piping without flow (c_o=0) and with flow velocity closed to the critical velocity c_o=14,0 m/s. In both cases the applied force is on x=4,64m, in the w direction, with calculated displacement in the same point. So there is only bending modes on the response and this is the criterion to plot the modes on Fig.(1).

The 4th mode does not appear on Fig.(2) and probably it is associated to fluid column vibration mode. On Tab.(1) are showed the real and imaginary part of the eigenvalues to this mode or eigenvector. The imaginary number is the damped angular frequency and it is almost constant ($\sim$7,8 Hz) for all velocities. The real part is the damping factor and its negative sign means the system solution is stable or not divergent.

Table 1. Real and imaginary parts for 4th eigenvalue associated to Fluid Mode.

real	imaginary [rad/s]	fluid velocity c_o [m/s]
-0.0018	48.7940	0.00
-0.2161	48.7934	13.50
-0.2903	48.7929	18.90
-0.3266	48.7927	21.60
-0.3624	48.7923	24.30
-0.3978	48.7920	27.00

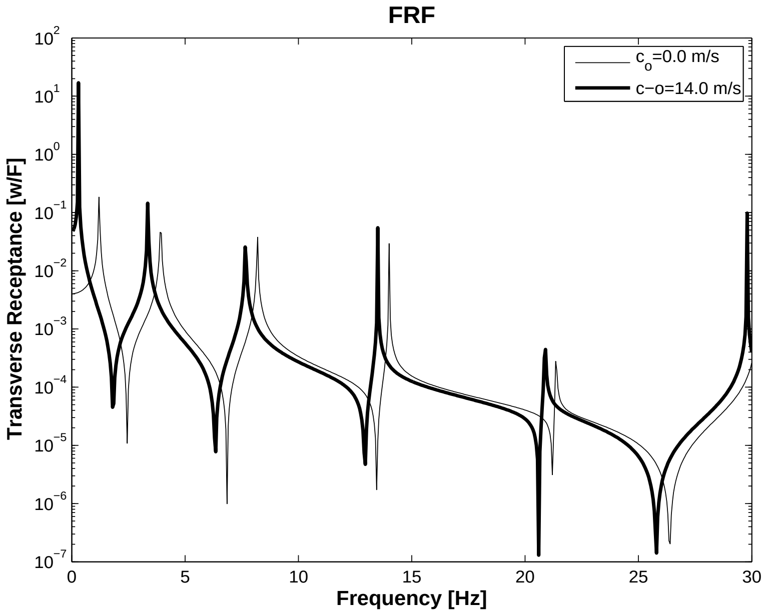

Figure 2. Frequency Response Function, FRF (w/F), for fiberglass piping for c_o=0,0 and c_o=14,0 m/s.

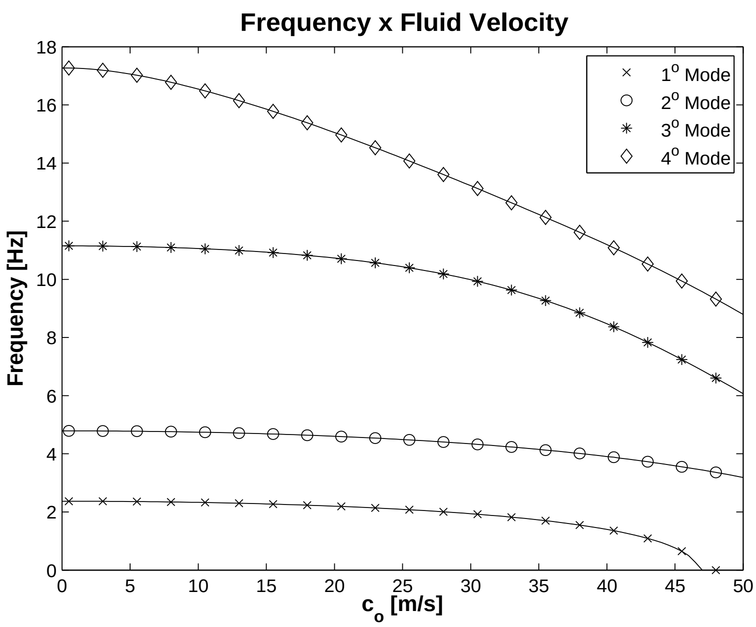

Figure 3. Frequency x Flow Velocity. Internal flow in fiberglass reinforced piping.

3.2 Internal Flow in fiberglass with reinforcement

The dimensions and properties of the internal fiberglass piping are the same as used in the previous case. The reinforcement or external pipe has the same properties as the internal tube, the dimensions are D_i^r=84,1 mm, D_e^r=100,1 mm and has the same lenght as the internal pipe. The equivalent stiffness for the gap filling is k_d=10 N/mm.

The eigenvalue for the fiberglass with reinforcement given by Eq. (24) is solved for each flow velocity and 400 unidimensional elements are used in all results presented in Fig.(3), Fig.(4) and Fig. (5). In the Fig. (3) is plotted the frequencies only for the first 4 bending vibration modes as function of the steady flow velocity c_o.

In this Fig.(3) the critical steady velocity c_o is about 47,0 m/s, and this velocity is highest than 14,7 m/s the previous calculated critical velocity for the fiberglass piping. As in the anterior case the pipe is clamped-supported by the internal pipe, the external pipe is supported by the gap filled material. This layout is maintained for all cases ploted on figures.

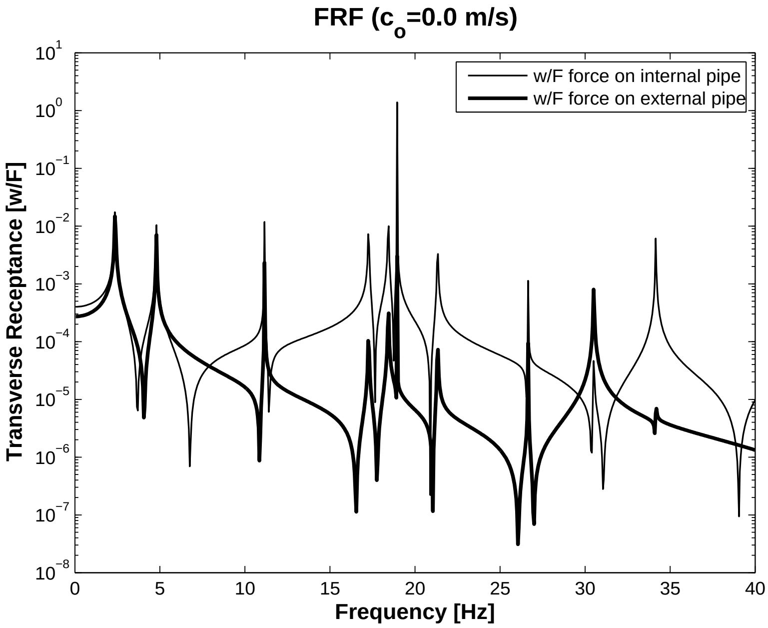

Figure 4. Frequency Response Function, FRF (w/F), fiberglass with reinforcement piping for c_o=0,0 and c_o=47,0 m/s.

On the Fig. (4) is ploted the receptance Frequency Response Function, FRF (w/F), for the fiberglass with reinforcement without flow (c_o=0). The force is applied on position x=4,64m, on the external or reinforcement pipe in the w direction and the calculated displacement response on the same point is ploted on Fig. (4). On same Fig. (4) is ploted the response in v direction for a force applied at the same position, x=4,64 m, on the internal pipe. As both excitation forces are applied on transverse direction there is only bending modes.

On the Fig. (4) is shown that the pipe behave as a uniform body for the first 3 bending modes. For the 4^{Th} mode the amplitude of the internal pipe is more pronounced but there are others as the mode associated to a frequency near 35 Hz where the external pipe has a small response.

In the last figure Fig. (5) the receptance Frequency Response Function, FRF (w/F), for the fiberglass with reinforcement without flow (c_o=0) and with flow velocity closed to the critical velocity c_o=45,0 m/s. In both cases the applied force is on x=4,64m, in the v direction on the internal pipe, with calculated displacement in the same point. There are the same behavior observed for the fiberglass piping.

4. FINAL REMARKS

The results presented in this work are encouraging. Different boundary conditions can be imposed and are of pratical interest as it can save material and improve the stiffness.

A elastic axial distributed stiffness coupling both pipes on longitudinal direction and a distributed damping probably will produce more realistic behavior. But experimental tests are still necessary to validate the formulation, to obtain coefficients and to establish valid limits.

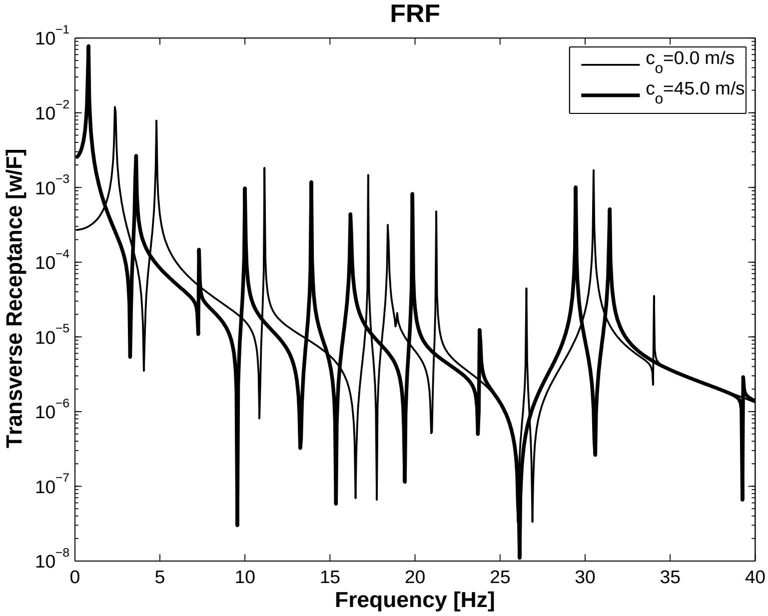

Figure 5. Frequency Response Function, FRF (w/F), fiberglass with reinforcement piping for c_o=0,0 and c_o=47,0 m/s.

5. ACKNOWLEDGEMENTS

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