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NUMERICAL ANALYSIS OF SELF-HEATING EFFECTS IN VISCOELASTIC DAMPERS

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Abstract. *Viscoelastic materials have been frequently used for passive vibration control. It is well-known that the dynamic behavior of such materials is strongly influenced by environmental and operational factors, among which the excitation frequency and temperature play a significant role. As a result, the influence of those factors must be properly accounted for in the modeling of viscoelastic dampers in order to ensure accurate predictions and effective design. It has been frequently assumed, in most of previous research studies, uniform, time-independent temperature distribution over the volume of the viscoelastic material. This is believed to be a rough simplifying assumption as, in actual situations, self-heating of the viscoelastic material occurs as the result of energy dissipation during vibration cycles. Since the strain distributions are more frequently non-uniform and the damper geometry favors non-homogeneous or even anisotropic heat transfer, self-heating leads to non-uniform temperature distributions over the volume of the viscoelastic material. As a result, the material becomes non-homogeneous in terms of mechanical properties and such properties also vary as time evolves, which characterizes a non-linear problem. This paper is devoted to a finite-element-based methodology intended to perform the thermoviscoelastic analysis of viscoelastic dampers accounting for self-heating. After presenting the underlying theoretical aspects, the numerical results for a rotational viscoelastic damper are presented, emphasizing the procedure conceived to compute the internal temperature distribution of the viscoelastic material and the frequency response functions. The numerical results are discussed in terms of the influence of the internal temperature changes on the performance of the viscoelastic damper.*

Keywords: *vibrations, viscoelastic damping, finite element, thermoviscoelasticity*

1. INTRODUCTION

Viscoelastic materials (VEM) have been long used in a wide range of industrial applications over the past few decades, due to their ability to dissipate mechanical energy and thus reduce the vibration level in structures submitted to dynamic excitations. In the case of viscoelastic polymers, inter-molecular friction mechanisms result in the dissipation of a fraction of the total vibration energy that is converted into heat, alterating the values of local temperature field. The relative importance of the temperature changes that may occur into the viscoelastic parts of a damped structure depends on several parameters such as the excitation amplitude and frequency, and on the material loss factor η, which can be wieved as the ratio of dissipated energy over total mechanical energy per cycle. It is well-known that the viscoelastic material parameters are strongly temperature-dependent since that the temperature of glass transition of most polymers is relatively low. As a consequence, even a small heat generation may cause a significant alteration of the damped structure's response, and such an effect may result in an uncontrolled temperature increase if the glass transition temperature is reached. Lesieutre and Godwinswamy (1995) described this phenomenon called "thermal runaway".

The application of damping polymers to structures include various possible configurations that can roughly be classificated into two main categories, respectively thin layers and discrete devices. Thin layers's thickness generally is not greater than 1 mm, and the layers are bonded by adhesion to structural, more rigid and elastic surfaces. Depending on the device's configuration, the viscous energy dissipation can occur when the viscoelastic layer is directly bonded to the structure's surface (unconstrained configuration) or when the layer is set up to form a "sandwich structure" being bonded between the base structure and a thin metal cover sheet, in such a way that relative motion generate a shear stress-strain state into the viscoelastic layer (constrained configuration). In both cases, since the dimensions of the exchange interfaces between structural and viscoelastic parts are important, and owing to the thermal conduction capabilities of most metals and alloys, most of the generated heat is evacuated by conduction and natural convection, and the VEM average temperature tends to be determined by the ambient conditions since self-heating effects can be neglected.

Self-heating effects appear to be more critical when occuring in discrete devices which have smaller dimensions and are generally placed at the keypoints of the global vibrating structures. These devices are also subjected, in most cases, to shear stress-strain states, but they include thicker viscoelastic parts. Last but not least, it should be reminded that the thermal conduction VEM polymers is low when compared to those of metals and alloys, which can limitate the thermal evacuation and leading to higher temperature field values once thermal equilibrium has been reached. Gopalakrishna and Lai (1998) pointed out that the local temperature values in the viscoelastic heart of damping substructures used in building bases can increase by up to 20°C in a few seconds when a large storm occurs. These are the reasons for which developping metodologies to model self-heating effects in discrete devices are considered to be important. In particular, it becomes very convenient in order to create a predictive FE-based model which can be used to evaluate temperature variations due to self-heating and help designers ensure that the temperature changes will remain small enough to not jeopardize the structure damping performance.

After the presentation of the main theoretical aspects, the FE-based iterative scheme will be explained and applied to the simulation of the thermomechanical behaviour of a simple 2-D rotational damper model. Its implementation whitin the commercial finite element software ANSYS^{TM}and its results will be discussed.

2. DYNAMIC MODELLING OF LINEAR VISCOELASTICITY AND DISSIPATED ENERGY

2.1 The complex modulus approach

The stress-strain relationship for linear viscoelastic materials under uniaxial sollicitation is given by:

$$\sigma(t) = E_0\varepsilon(t) + \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau \tag{1}$$

In Eq. (1), E_0 represents the elastic part of the response and E is the relaxation function. The resulting stress depends both on the strain value at corresponding time and on the loading history since the initial loading has been applied. In the case of multiaxial dynamic loading, this approach becomes rapidly complicated since the relaxation tensor $\mathcal{R}(t)$ has no simple expression in the time domain. The best-suited method is the complex modulus approach, which describes the response of the VEM in the frequency domain both in term of elastic response and energy dissipation. It can be defined as the Laplace transform of the relaxation function. Using $s = i\omega$ as the Fourier variable, we obtain a complex, frequency-dependent modulus (Nashif et al., 1985):

$$E^*(\omega) = E^{'}(\omega) + iE^{''}(\omega) = E^{'}\left[1 + \eta(\omega)\right] \tag{2}$$

In Eq. (2), $E^{'}(\omega)$ and $E^{''}(\omega)$ are known as storage and loss moduli, respectively, while $\eta(\omega) = \dfrac{E^{''}(\omega)}{E^{'}(\omega)}$ is the loss factor. In the case of a harmonic response due to a ω-periodic excitation, the viscous energy dissipation leads to a phase shift ϕ between the applied stress and the resulting strain. Thus, the complex modulus can be written in term of stress-strain amplitudes and phase angle as follows (Nashif et al., 1985):

$$E^* = \frac{\sigma_0 e^{i\omega t+\phi}}{\varepsilon_0 e^{i\omega t}} = \frac{\sigma_0}{\varepsilon_0}\left(\cos\phi + i\sin\phi\right) = \frac{\sigma_0}{\varepsilon_0}\cos\phi\left(1 + i\tan\phi\right) \tag{3}$$

Since $\eta(\omega) = \tan\phi$, the loss factor is often referenced to in the litterature as *tangent modulus*.

Viscoelastic polymers are generally assumed to be isotropic and governed by a frequency-independent Poisson ratio. Nashif et al. (1985) described the various factors that affect the storage modulus and loss factor values, namely the temperature, the humidity rate, the deformation amplitude and the age of the material. The response of such materials to mechanical sollicitations is determined by inter-molecular reorganization mechanisms which are highly sensitive to temperature changes, whereas in general conditions of use the other factors only have a limited incidence on the VEM behaviour. Thus temperature and frequency will be further considered to be the only variables on which depends the complex modulus:

$$E^*(\omega,T) = E^{'}(\omega,T) + iE^{''}(\omega,T) \tag{4}$$

It should be mentioned that, for commercially available VE materials, charts representing the storage and loss moduli and the loss factor, as functions of frequency and temperature, are frequently provided by manufacturers, as obtained from experimental tests.

2.2 Damping formulation in FE codes

In the frequency domain, the representation of linear viscoelasticity using the complex modulus will result in a complex, temperature and frequency-dependent matrix $[K^*(\omega,T)]$.

$$-\omega^2[M]\{x\} + i\omega[C]\{x\} + [K^*(\omega,T)]\{x\} = \{f\} \tag{5}$$

where $[M]$ and $[C]$ are the structure's mass and viscous damping matrices, respectively.

The complex stiffness matrix $[K^*(\omega,T)]$ can be decomposed into its real and imaginary parts, as follows:

$$[K^*(\omega,T)] = [K(\omega,T)] + i[H(\omega,T)] \tag{6}$$

The components of $[K(\omega,T)]$ represent the elastic behaviour of both the elastic and viscoelastic parts of the structure whereas the $[H(\omega,T)]$ matrix contains only the imaginary parts of the complex stiffness terms associated to viscoelastic elements. For fixed values of ω and T, the reaction force vector due to the $[H(\omega,T)]$ matrix is complex and proportional to the displacement; thus, $[H(\omega,T)]$ can be considered a form of **histeretic damping** model of the system. In viscoelastically damped systems such as translation joints, damping effects are essentially due to the viscoelastic dissipation which is adequately represented by the histeretic damping matrix: it can therefore be assumed that the viscous matrix $[C]$ components can be neglected. Taking into account this assumption and substituting Eq.(6) into Eq.(5) we obtain the equation of motion:

$$\left(-\omega^2[M]+[K(\omega,T)]+i[H(\omega,T)]\right)\{x\}=\{f\} \tag{7}$$

Alternatively, it is possible to express the damping matrix in the equation of motion using conventional viscous damping model. This is done by defining an equivalent viscous damping matrix $[C_{eq}(\omega,T)]=\dfrac{1}{\omega}[H(\omega,T)]$ so that Eq. 7 becomes:

$$\left(-\omega^2[M]+i\omega[C_{eq}(\omega,T)]+[K(\omega,T)]\right)\{x\}=\{f\} \tag{8}$$

The main interest in using the mathematical models (7) or (8) is that dynamic response of the system containing viscoelastic materials can be performed in the frequency domain using directly the numerical procedures integrated in commercial FE codes.

2.3 Heat generation from viscous dissipation

The formulation presented in this section follows the procedure suggested by Merlette (2005). Considering a damped system at time t, the damping force vector $\{f_d\}$ can be expressed as:

$$\{f_d(t)\}=[C_{eq}(\omega,T)]\{\dot{x(t)}\} \tag{9}$$

and the power dissipated by the viscous effect is given by:

$$\dot{W}_d=\{\dot{x}\}^T\{f_d(t)\}=\{\dot{x}\}^T[C_{eq}(\omega,T)]\{\dot{x(t)}\} \tag{10}$$

Rittel (2000) points out that only a fraction of the dissipated energy is actually converted into heat, and introduces the parameter β, which is the ratio of the energy converted into heat over the total amount of dissipated energy per cycle. The thermal generated heat per unit of time can now be rewritten as:

$$\dot{W}_{th}=\beta\dot{W}_d=\beta\{\dot{x(t)}\}^T[C_{eq}(\omega,T)]\{\dot{x(t)}\} \tag{11}$$

The parameter β depends both on the strain amplitudes and frequency, and on the VEM. In most cases, $0.1\leq\beta\leq 1$. When the thermal dilatation effects and the strain rate amplitudes remain weak, β can be considered constant as a first approximation. In the case of an ω-periodic sinusoidal excitation, $\{\dot{x(t)}\}=i\omega\{x(t)\}$ can be substituted into (11), leading to:

$$\dot{W}_{th}=-\beta\omega^2\{x(t)\}^T[C_{eq}(\omega,T)]\{x(t)\} \tag{12}$$

Remembering that $[C_{eq}(\omega,T)]=\dfrac{1}{\omega}[H(\omega,T)]$, Eq. (12) can be rewritten as follows:

$$\dot{W}_{th}=-\beta\omega\{x(t)\}^T[H(\omega,T)]\{x(t)\} \tag{13}$$

2.4 The complex modulus as a function of frequency and temperature

Nashif et al. (1985) describe the general trends for the evolution of the storage modulus and loss factor over temperature for classical, thermorheologically simple viscoelastic polymers. Since the complex modulus in a function of both temperature and frequency, its numerical implementation in the context of finite element simulations appears to be easier when using the *Frequency-Temperature Superposition Principle*, which consists in introducing a temperature-dependent coefficient α_T that links the current angular frequency ω to a reference, reduced angular frequency ω_r so that $\omega_r=\alpha_T\omega$, and:

$$E^*(\omega,T)=E^*(\omega_r)=E^*(\alpha_T\omega) \tag{14}$$

A function providing the values of the complex modulus for the material 3M ™*ISD112* for a given range of frequencies between 1 and 1×10^6 Hz and for a given temperature T was implemented by Lima, according to the analytical expressions of α_T and $E^*(\omega_r)$ described by Lima and Rade (2005) according to the reference given by Drake and Soovere (1984). The *ANSYS* software allows the definition of temperature-dependent structural material properties. During the pre-processing phase, the modulus of elasticity and damping material ratio can be defined by a 2-dimension table where each temperature value matches the corresponding value of $E^{''}(\omega_r,T)$ and $\eta(\omega_r,T)$.

3. Finite-element modelling of the thermomechanical problem

3.1 Iterative process

As the strain rates determine the temperature variations, which, on their turn, influence the material properties, the thermoviscoelastic problem is characterized as a nonlinear coupled problem. As a result, special iterative procedures are required for its numerical resolution, based on the formulation previously presented.

The global heat generated from material dissipation is given by Eq. (13) and corresponds to an amount of mechanical energy transformed into heat at the level of the global structure. However, this phenomenon occurs only into the viscoelastic parts of the device. In the case of thermomechanical calculations, it can be necessary to express the generated heat for each viscoelastic finite element, using element matrices:

$$\dot{W}_{th\,i} = -\beta\omega\{x(t)\}_i^T[H(\omega,T)]_i\{x(t)\}_i \quad (15)$$

In Eq. (15), $\dot{W}_{th\,i}$, $\{x(t)\}_i$ and $[H(\omega,T)]_i$ are respectively the heat generated power, the nodal displacements vector and the equivalent histeretic damping matrix of element i. Taking into account the fact that the element i is made of viscoelastic material, $[H(\omega,T)]_i$ is the imaginary part of the complex damping matrix $[K^*(\omega,T)]_i = [K(\omega,T)]_i + i[H(\omega,T)]_i$. Since the VEM is assumed to be isotropic and considering the Poisson ratio as frequency-independent, it is possible to factor out the complex modulus, so that $[K^*(\omega,T)]_i = E^{'}(\omega,T)(1+i\eta(\omega,T))[\bar{K}]_i$.

Therefore, $[H(\omega,T)]_i = \eta(\omega,T)[K(\omega,T)]_i$ and Eq. (15) becomes:

$$\dot{W}_{th\,i} = -\beta\omega\eta(\omega,T)\{x(t)\}_i^T\left([K(\omega,T)]_i\right)\{x(t)\}_i \quad (16)$$

As shown by Eq. (16), the generated heat depends both on the nodal displacements and the stiffness matrix. To perform a time integration in order to obtain the transient thermal solution for a time increment Δt, it is necessary to determine the vector of nodal generalized thermal loads $\{q\}$. This vector depends on both the convection fluxes at the boundaries of the structure and the generated heat, calculated from the structural matrices. Since the structural matrices are temperature-dependent, the harmonic structural solution has to be performed at various stages of the global analysis to update the heat generation rate. The material non-linearity with respect to the temperature field values results in the necessity of implementing a 2-way sequentially coupled resolution process to obtain the thermomechanical solution.

Figure (1) represents the solution flow diagram which has been implemented using the *ANSYS Parametric Design Language* (*APDL*). This language offers the opportunity to define the whole finite element model, that consists in two distinct solution environments, namely the transient thermal and harmonic structural environments, which share the same geometry. The table defining the VEM properties with respect to temperature values is read from a text file generated from the complex modulus function, implemented in the *Matlab* ™environment. At each iteration, a harmonic analysis is performed, and the strain energy $\frac{1}{2}\{x(t)\}_i^T\left([K(\omega,T)]_i\right)\{x(t)\}_i$ for each viscoelastic element is post-processed to compute the heat generation according to Eq. (16). Afterwards, the thermal solution is obtained between previous time t and time $t+\Delta t$. The new nodal temperature values at $t+\Delta t$ are subsequently applied as body loads, which are taken into account for the structural matrix formulation in the next iteration.

3.2 Numerical application

Figure (2) depicts a rotational joint made of a 5mm thick viscoelastic hollow cylinder held between two structural parts: an external 3mm thick tube and an internal cylindric rod which is 15 mm in diameter. Both the tube and the rod are made of steel whereas the viscoelastic cylinder is made of the 3M ™ISD112 material. The purpose of this device is to attenuate structural vibrations when the inner structural part is submitted to a harmonic torque $T = T_0 e^{i\omega t}$ in the z out-of-plane direction and the external structural part remains clamped. The dispositive is 20 mm high. Since the application of the torque results in only one nonzero stress component ($\tau_{r\theta}$ in the cylindrical coordinate system associated to the device) it is possible to model the structural behaviour of the damper using 2D plane stress elements.

The structural and thermal simulation environments were modeled using 2D elements *PLANE42* and *PLANE55*, which are plane 4-node elements having structural and thermal degrees of freedom, respectively. *PLANE42* element can be used in the plane stress configuration. In both cases the element key options took into account the thickness value to formulate the element matrices. Natural convection heat fluxes were applied as thermal boundary conditions, in axial and radial directions with respect to the damper geometry. The film coefficient and bulk temperature are, respectively, $h = 20$ W.m^{-2}.K^{-1} and $T_\infty = 25^\circ$ C.

4. DISCUSSION OF THE SIMULATION RESULTS

This section presents a comparative study of the influence of two distinct parameters, namely the thermal dissipation fraction and the torque amplitude, over the self-heating effect in the rotational damper modeled with the *ANSYS* ™finite element software. The frequency of the harmonic sollicitation is 10 Hz, and the approximate tangential stress value

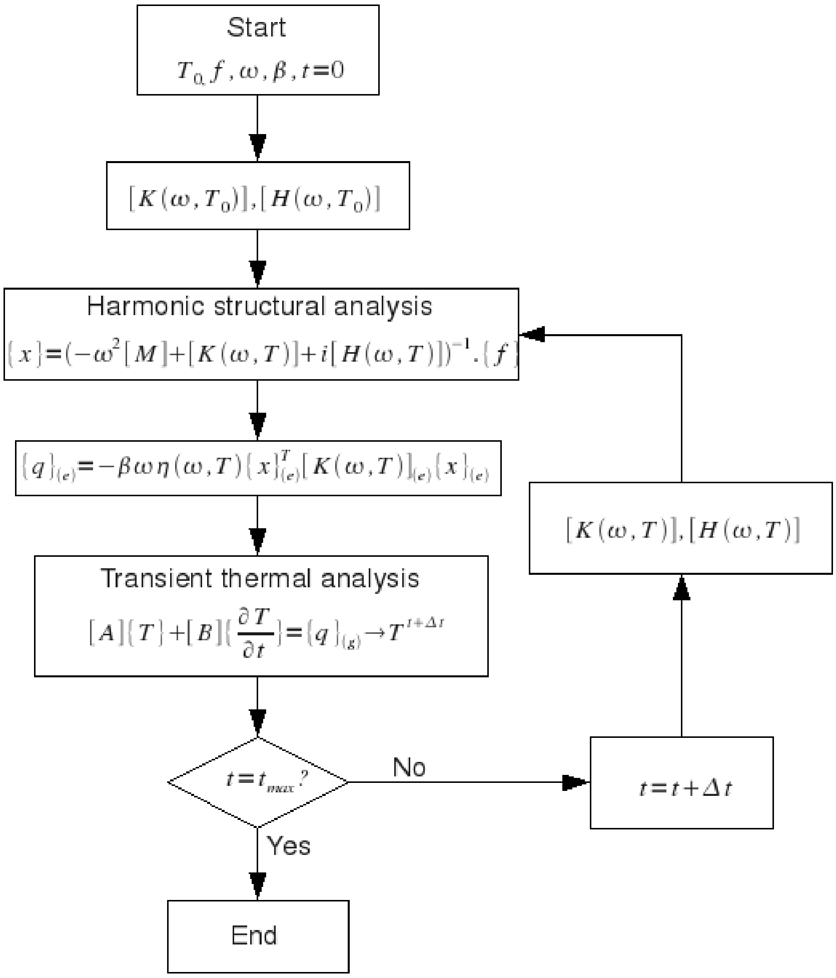


Figure 1. Iterative thermomechanical solution flow diagram

resulting of the application of a 0.1 N.m torque value is 100 KPa. Although the frequency may seem to be low, it has been proved to be sufficient to generate significant self-heating effects over a large time range when associated to a 100 KPa stress (Merlette, 2005).

4.1 Influence of the thermal dissipation fraction β

A series of 2-way coupled thermomechanical self-heating numerical simulations have been performed using the iterative procedure presented in Section (3.1), with three distinct β values: 0.1,0.5 and 1. The harmonic loading was applied during 1000 seconds, which corresponds to 10 000 cycles at a 10 Hz frequency. Afterwards, a simple thermal transient analysis was performed during 1000 more seconds, to describe the cooling process after the removal of the harmonic load.

Figure (3) shows that at the end of the harmonic loading, the temperature in the heart of the viscoelastic part can increase up to 6.87°C. During the whole cycling, the temperature increase is roughly proportional to the thermal dissipation fraction as it is suggested by Eq. (15). Therefore, the maximum β value of 1 generates the greatest self-heating effect. It should also be noticed that, the less dissipative the material is, the sooner the equilibrium temperature is reached.

Table (1) shows the time values at which thermal equilibrium was reached for each case, and the corresponding equilibrium temperature values.

4.2 Influence of the torque amplitude

The heat generation rate per element is directly proportional to the strain energy, which is roughly proportional to the square of the harmonic nodal displacements. To evaluate the influence of the torque magnitude over the self-heating effects, a new series of simulations was performed, this time involving three values of T_0 torque magnitude values, namely

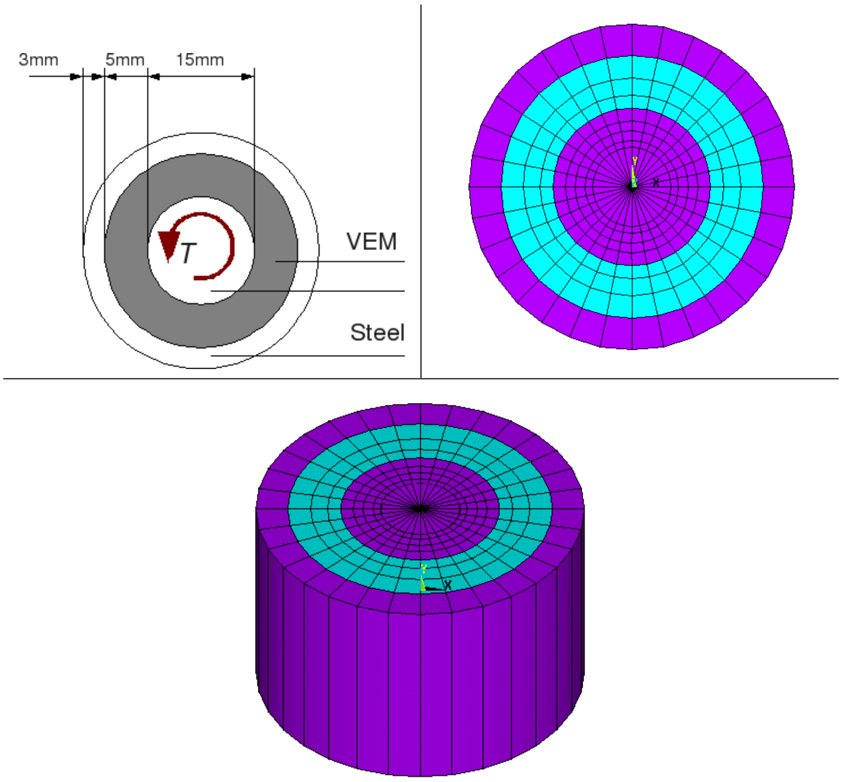


Figure 2. Sketch of the rotationnal joint and its FE model

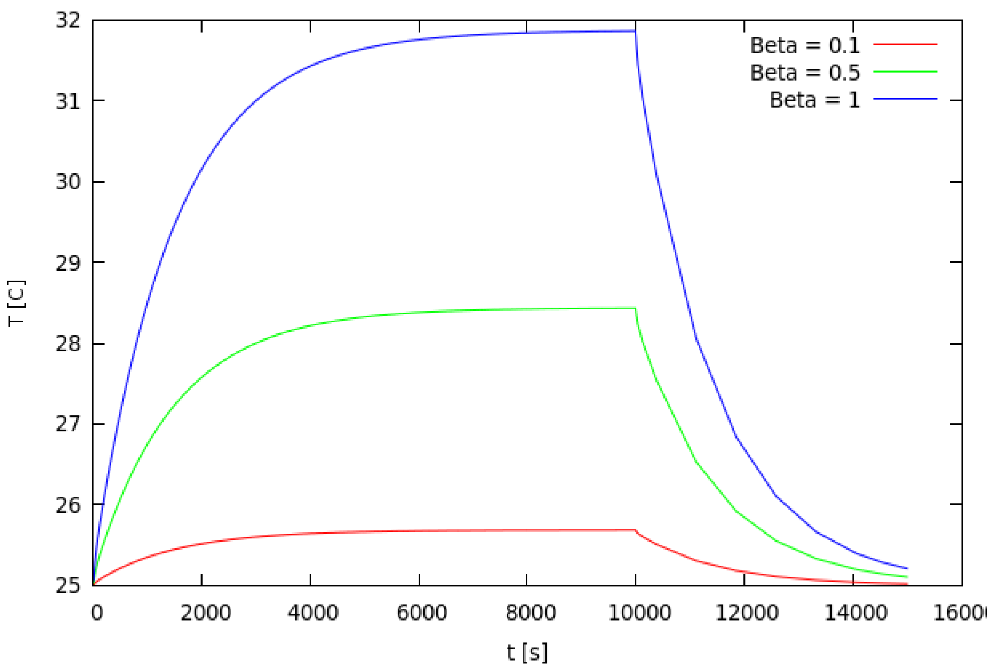


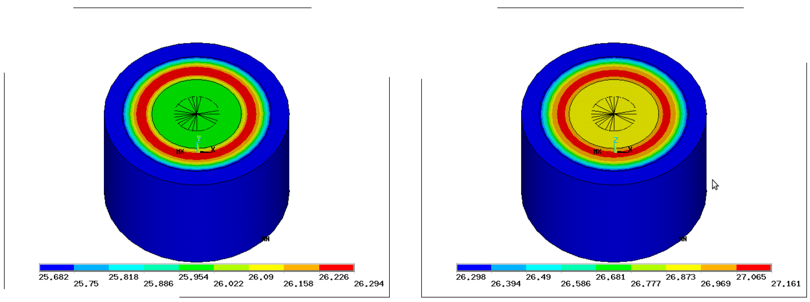
Figure 3. Time evolution of temperature in the heart of the viscoelastic joint, for a 0.11 N.m torque amplitude at a 10 Hz frequency for $\beta = 0.1$, $\beta = 0.5$ e $\beta = 1$

0.11, 0.19 and 0.26 N.m. The thermal dissipation fraction β is held at a constant value of 0.5. This value is also close to the results of self-heating studies performed by Merlette (2005) on translation joints made of material 3M4910, at similar frequency and strain rate conditions.

Table (2) show the time values at which thermal equilibrium was reached for each case, and the corresponding equilibrium temperature values.

Table 1. Equilibrium temperatures for rotational joints submitted to a harmonic 0.11 N.m force at a 10 Hz frequency

β	T_e [°C]	t_e [s]
0.1	25.68	5820
0.5	28.43	8210
1	31.87	9570



$t = 250$ s $t = 500$ s

Figure 4. Temperature distribution for $t = 250$ s and $t = 500$ s

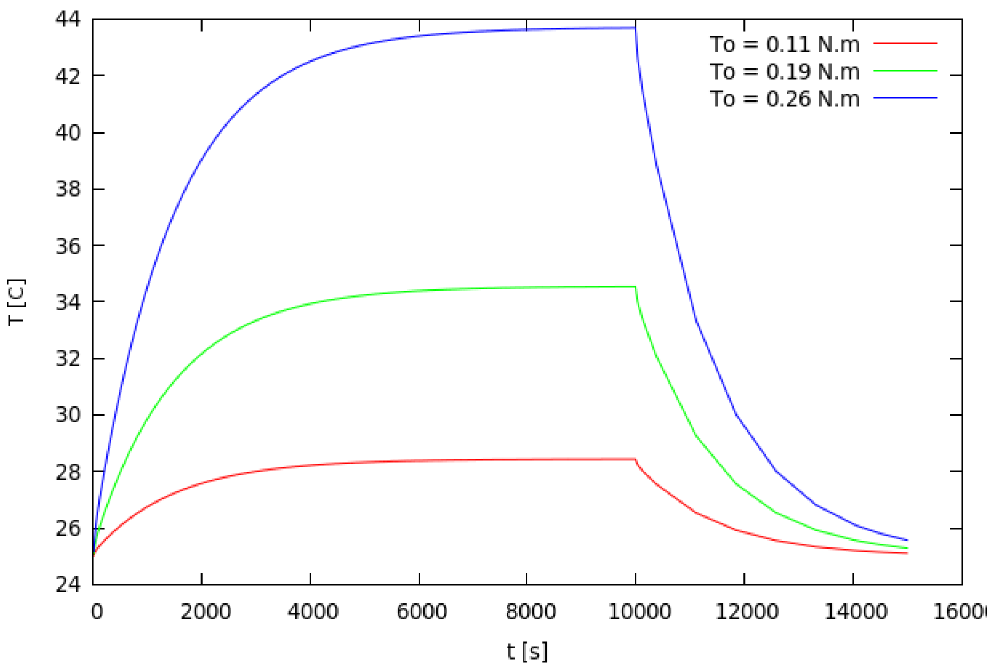


Figure 5. Time evolution of temperature in the heart of the viscoelastic joint, for $\beta = 0.5$ at a 10 Hz frequency and considering $T_0 = 0.11$ N.m, $T_0 = 0.19$ N.m and $T_0 = 0.26$ N.m

Table 2. Equilibrium temperatures for rotational joints submitted to harmonic forces at a 10 Hz frequency

T_0 [N.m]	T_e [°C]	t_e [s]
0.11	28.43	8210
0.19	34.53	8810
0.26	43.69	9460

Figure (4) shows the temperature distribution at two time instants ($t = 250$ s and $t = 500$ s) in the rotational damper, for $\beta = 0.5$ and $T_0 = 0.19$ N.m, obtained from the *ANSYS* results visualizer. From this figure, it can be seen that the maximal temperature is reached in the viscoelastic cylinder whereas the temperature of the external face of the outside steel cylinder remains constant at a 25řC value, due to external convection.

Figure (5) shows that increasing the torque magnitude results in a significant augmentation of self-heating effects. Such temperature elevations lead to turn the VEM less rigid and, as a consequence, result in an even higher displacement level. The next simulation aims at showing that self-heating effects truly modify the VEM damper performances.

4.3 Evolution of the angular displacement as a function of time

From the previous results, it is obvious that self-heating effets, if uncontrolled, may have a great influence on the damping performances of discrete viscoelastic devices submitted to cyclic loading. The temperature local increases which occur inside the viscoelastic parts until the load is removed are responsible for the decrease of both the storage modulus and the loss factor, turning the whole structure less rigid and less damped. This phenomenon has been explicited by the results presented in this subsection, whose goal is to show the evolution of the angular displacement of a point located in the viscoelastic heart between $t = 0$ s and $t = 200$ s, while the structure is submitted to a 0.26 N.m amplitude moment at 10 Hz.

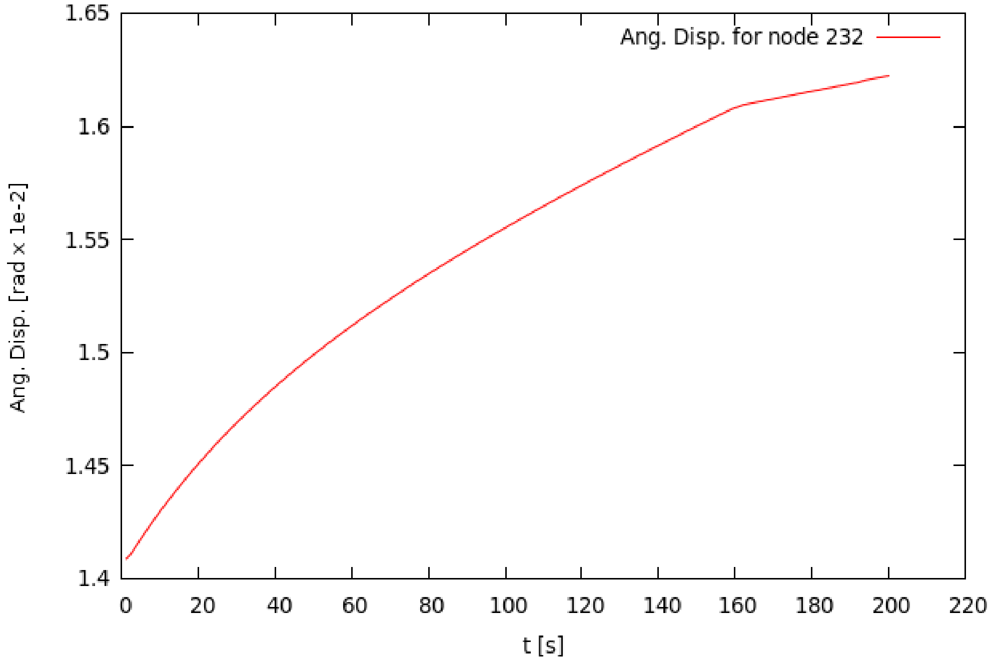


Figure 6. Diagram of θ displacement of node 232 versus time, for a 0.26 N.m torque amplitude at a 10 Hz frequency for $\beta = 1$

As shown by Fig. (6), a 200 seconds interval is long enough for the angular displacement amplitude to increase by to 15% due to the effect of self-heating. The angular displacement amplitudes increase from 1.41×10^{-2} radian at $t = 0$ s to 1.62×10^{-2} radian at $t = 200$ s. Such a modification in the structure response is prone to cause important changes in the resulting noise level, turning the viscoelastic damper less efficient.

5. CONCLUSION

The iterative process developped in order to simulate the thermomechanical behaviour of a viscoelastic damper has lead to the obtention of characteristic self-heating curves which can help predicting the importance of maximal temperature elevations within the viscoelastic material, according to the geometry, material and excitation properties of the model. In all the cases that have been studied, at a 10 Hz frequency, the equilibrium temperature is reached after approximately 5 000 and 10 000 seconds, which corresponds to 50 000 to 100 000 cycles. The value of the equilibrium temperature appears to be highly conditionned by the thermal dissipation fraction β and by the torque amplitude T_0, since the difference between equilibrium and initial temperature is roughly proportionnal to the square of the torque magnitude. Although some assumptions available in the litterature give approximate values for the β fraction associated to 3M^{TM}materials, there is no theoretical study about the dependence of this parameter in respect to the temperature and mechanical conditions. An identification procedure may be developped using an experimental damper submitted to cyclic loading as a reference, in which the present algorithm may be used as a fitting tool to ajust the β parameter. Afterwards, the present method may also be applied to more complex, three-dimensional structures. Finally, it should be reminded that the determination of the transient values of the temperature field enables the evaluation of the harmonic displacement and strain magnitudes over time. In the case of significant self-heating effects, the maximal displacement magnitude when the equilibrium temperature has been reached can be used as a criterium to determine the maximum available load for harmonic cycling.

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