PARAMETRIC EXCITATION IN ENGINEERING SYSTEMS

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Abstract. *For a long time parametric excitation in engineering systems was associated mainly with parametric resonances and harmful vibrations. Parametric excitation seemed to have only negative effects on the dynamics of systems and therefore research was focused on how to avoid or at least minimize the adverse consequences of parametric excitation.*
However, recent research results have shown that parametric excitation may cause not only harmful instabilities in a dynamical system but can also improve the capability of a system to suppress vibrations. In particular it is possible to avoid the onset of an instability by introducing parametric excitation to the system. These findings are quite new and the numerous possibilities of making use of it still need to be explored and discussed.
In this article the basics of parametric excitation as a means to suppress vibrations in engineering systems are presented and several theoretical and experimental studies are reviewed. The potential of this novel design concept is discussed and directions for further research and future practical applications are outlined.

Keywords: *Parametric excitation, self-excitation, vibration damping, stability*

1. INTRODUCTION

If an excitation is applied to a mechanical system it will, in general, vibrate. Most of the time these vibrations are unwanted and require countermeasures if the vibration amplitudes exceed a certain level. Depending on the nature of the excitation, different strategies and methods of vibration reduction are in use. This contribution focuses on a rather new idea to reduce, and in some cases even cancel vibrations in mechanical systems by means of parametric excitation.

In mechanical systems different types of excitation are observed. A frequently encountered type is the *external excitation*, also named *forced excitation*. Unbalance excitation as known from rotating machinery is one example, the excitation of a structure which is attached to a vibrating foundation is another. Vibration amplitudes are determined by the dynamical properties of the system and the amplitude of excitation. In the case of a linear system, vibration amplitudes are proportional to excitation amplitudes and damping properties of the system. Vibration reduction for the system in general is achieved mainly by either choosing these parameters appropriately or by tuning the system as to avoid resonances.

Although mentioned in first place, the reduction of vibrations caused by external excitation will only play a minor role in this paper. We will almost exclusively deal with mechanical systems where two other types of excitation mechanisms are prevailing: Self-Excitation (SE) and Parametric Excitation (PE). As in most practical cases, self-excitation will be the source of unwanted vibrations. It will be shown that parametric excitation can be employed as a mean to suppress self-excited vibrations, as well as free vibration.

2. PARAMETRIC EXCITATION - PAST AND PRESENCE

Parametrically excited systems have been of interest since a long time, and research dates back as far as to the 19th century, when M.Faraday investigated sloshing liquids in a container and, about 30 years later, when E.Mathieu established the famous equation given his name. Since then, parametric excitation (PE) has attracted much interest, mainly because it may lead to a unique type of resonances, called *parametric resonances*. Given the available space, it is virtually impossible to even give a brief overview and just name the important contributions in this field of ongoing active research.

From an application point of view and focussing on mechanical engineering two different aspects of parametric excitation can be extracted from the numerous references: how to avoid or reduce the effect of parametric excitation in a system and how to take advantage of PE, especially *PE-resonances*. The second aspect is much less popular and not very many applications of PE in this sense are known. Within this area, the idea prevails, to make use of the large amplitudes which will occur when a system is operated at a parametric resonance. For instance, recent works focus on micro-electromechanical systems (MEMS) as a possible application, see e.g. [Shaw et al., 2004]. However, not all PE-resonances lead to large amplitudes, since some of them may be *non-resonant*. This special case has not been studied at all, until Tondl found out about an interesting phenomenon associated with non-resonant parametric resonances [Tondl, 1998] .

In his paper Tondl shows early results obtained from analog computer simulations of an unstable, non-linear, parametrically excited system, see Fig.1. The surprising detail in this result is a frequency interval of the PE, where the self-excited vibration amplitudes of the system are completely suppressed. Since this occurs at the frequency of a parametric resonance, the phenomenon was named *parametric anti-resonance*.

This pioneering work triggered research efforts at various places. It led to a growing number of contributions related to this phenomenon, only some of them shall be mentioned here. Analytical methods and bifurcation analysis have been applied by Verhulst and his students [Fatimah, 2002], [Abadi, 2003]. Very comprehensive investigations, both analytically and numerically were carried out by [Dohnal, 2005] and this author [Ecker, 2005]. Parametric excitation of a more general

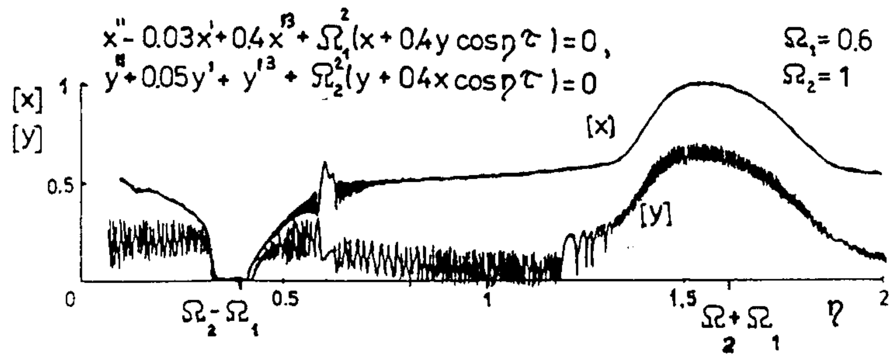


Figure 1. Simulation result (obtained with an analog computer) of a self-excited system exhibiting vibration suppression near the parametric combination resonance frequency $\eta_0 = \Omega_2 - \Omega_1$. From [Tondl, 1998]

type has been applied by Makihara in im Tokio [Makihara,Ecker,Dohnal,2005] and valuable contributions have been made also by Nabergoj from Trieste [Nabergoj,Tondl,2001]. Last but not least, Tondl himself has continued to study the effect of vibration quenching by parametric excitation [Tondl, Nabergoj, Ecker, 2005] and e.g. has also investigated parametric damping and parametric mass excitation, see [Tondl, 2001].

3. MODELLING SYSTEMS WITH PARAMETRIC STIFFNESS EXCITATION

The generic equations of motion of a mechanical system with parametric stiffness excitation can be written in a rather general matrix form as

$$\mathbf{M}\ddot{\mathbf{x}} + \left[\mathbf{C} + \mathbf{G}(\nu) + \mathbf{C}^{\mathbf{Z}}(\mathbf{x})\right]\dot{\mathbf{x}} + \left[\mathbf{K} + \mathbf{N}(\nu) + \mathbf{K}^{\mathbf{Z}}(\mathbf{x})\right]\mathbf{x} + \mathbf{K}_{\mathbf{PE}}(t)\mathbf{x} = \mathbf{F}_{\mathbf{ex}}. \tag{1}$$

The vector of deflections is denoted $\mathbf{x}$. For a linear, homogeneous system with constant system matrices, only the following matrices would be needed and therefore non-zero: mass matrix $\mathbf{M}$, damping matrix $\mathbf{C}$, stiffness matrix $\mathbf{K}$. Parametric stiffness excitation (PSE) is introduced by matrix $\mathbf{K}_{\mathbf{PE}}(t)$ with time-periodic coefficients according to harmonic functions $\cos(\omega t + p_{ij})$. Only single-frequency PSE with frequency ω is considered for this system but multi-location parametric excitation is not excluded. Phase relations between different locations of PE are introduced by phase angles p_{ij}

$$\mathbf{K}_{\mathbf{PE}}(t) = \cos(\omega t + p_{ij})\mathbf{P}_{\mathbf{E}}. \tag{2}$$

The number of degrees of freedom of the system determines the size of the system matrices. It is pretty obvious how to establish these matrices for simple two or three mass chain systems, as used in [Tondl, Nabergoj, Ecker, 2005] and several other references by the author.

Self-excitation can be introduced to the system by setting elements of the damping matrix $\mathbf{C}$ to a negative value. Negative damping is one of the common methods to represent the effect of flow-induced self-excitation [Blevins, 1977].

Basic non-linear behavior can be represented by the additional stiffness and damping matrices $\mathbf{C}^{\mathbf{Z}}(\mathbf{x})$ and $\mathbf{K}^{\mathbf{Z}}(\mathbf{x})$, which may depend in an arbitrary way on vector $\mathbf{x}$ and also, if required, on $\dot{\mathbf{x}}$.

Matrix $\mathbf{G}(\nu)$ is a function of a system parameter ν and is needed in mechanical systems to represent gyroscopic forces, which would depend on a rotational speed ν. Also frequently encountered in rotor systems are non-conservative forces $\mathbf{N}(\nu)\mathbf{x}$, created by bearings and seals. Such forces usually increase with increasing rotor speed and may ultimately destabilize such a system. It is of particular interest to investigate rotor system, since the effect of a parametric anti-resonance could improve the performance of rotating machinery quite significantly. Finally, to take into account the effect of external forces, $\mathbf{F}_{\mathbf{ex}}$ appears on the right hand side of Eq.(1).

4. PARAMETRIC RESONANCE FREQUENCIES

It is widely known, see e.g. [Cartmell, 1990], that a system with parametric excitation may exhibit *Principle Parametric Resonances* at frequencies $\eta^{pr}_{j/n}$ and *Parametric Combination Resonances* at frequencies $\eta^{cr}_{j\pm k/n}$ for the PSE-frequency ω equal to:

$$\eta^{pr}_{j/n} = \frac{2\Omega_j}{n}, \qquad \eta^{cr}_{j\pm k/n} = \frac{|\Omega_j \pm \Omega_k|}{n}, \qquad (j \neq k),\ (j,k,n = 1,2,3,...). \tag{3}$$

Symbols Ω_j and Ω_k denote the j-th (k-th) natural frequency of the system. The denominator n represents the order of the parametric resonance. Most of the time only first order resonances $n = 1$ of the lowest frequencies Ω_1, Ω_2 are significant. The effect of a parametric anti-resonance can only occur for parametric combination resonances. It depends on the system, whether the difference type or the summation type is *non-resonant* and can be used to achieve vibration suppression. It can be shown that for a symmetric stiffness matrix $\mathbf{K} = \mathbf{K}^T$ parametric vibration suppression will occur for the difference-type combination resonance $\eta^{cr}_{(j-k)/1} = (\Omega_j - \Omega_k)$ and that an interval of instability will be observed at the summation-type combination resonance $\eta^{cr}_{(j+k)/1} = (\Omega_j + \Omega_k)$.

To predict the appropriate PE-frequency for vibration suppression it is necessary to know the natural frequencies of the system. Therefore, the lower undamped natural frequencies $\Omega_{1,2,3,...}$ have to be calculated from system Eq.(1) by solving the eigenvalue problem for $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}$.

5. ANALYSIS OF SYSTEMS WITH PARAMETRIC EXCITATION

In its most general version Eq.(1) defines a set of non-homogeneous, non-linear, time-periodic differential equations of second order. Also because of its basically unlimited complexity with regard to the size of the system there does not exist a single method to suit all kind of problems. Depending on the actual size of the problem, the presence of non-linearities and inhomogeneous terms, different methods are advantageous to be applied. Another factor is also if time series of system states of the original problem are sought or if only the local stability of such a solution is of interest.

5.1 Numerical simulation method

The most direct method, which can be applied to virtually any kind of such problems is numerical simulation. By integrating the system equations in the time domain, starting from initial conditions, the solution is computed. Nonlinearities and large matrices only affect the computational speed, but would not prevent using simulation. Of course, appropriate integration methods have to be applied, to balance computational effort and accuracy. Nevertheless, CPU-time may become still a problem, when the stability of a system near the stability threshold shall be investigated and very slowly changing transients have to be followed.

5.2 Analytical methods

A number of analytical and semi-analytical methods have been developed to deal with time-periodic systems. Even trying to briefly introduce the most interesting ones would exceed the length of this overview by far. Therefore, only one method is explicitly mentioned, since it has been used quite successfully in this context. This method is nowadays mostly called *Method of Averaging* (MoA). However, based on being promoted by Krylov, Bogoliubov and Mitropolski, in the past the method is also associated with these names. A rather detailed comparison of three distinctively different methods is presented in [Ecker, 2005].

The Method of Averaging is applicable to a linear(ized) and homogeneous subset of Eq.(1) and will primarily provide information about the stability of the system. It can be implemented as a first order method, as well as for higher orders. However, deriving a first order solution can be already cumbersome for a low-dimensional system. This, and the need to identify a small parameter in the system, are the major disadvantages of this method. But to be fair, a price has to be paid with practically every of the analytical methods. A very recent and detailed presentation of MoA is found in [Verhulst, 2006]. The application to PE-systems of various complexity is thoroughly discussed in [Dohnal, 2005].

The advantages of analytical methods can be seen easily by the following example. Equations (3) and (4) are a simplified and normalized version of Eq.(1), which have been used by Tondl and others to investigate the stability of PE-excited two-degree of freedom systems.

$$\boldsymbol{u}'' + \boldsymbol{\Omega^2 u} = -\varepsilon\left(\boldsymbol{\Theta u}' + \cos\eta\tau \boldsymbol{Q^c u}\right), \tag{4}$$

$$\begin{aligned} \boldsymbol{u} &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \qquad \boldsymbol{\Theta} = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix}, \\ \boldsymbol{\Omega^2} &= \begin{bmatrix} \Omega_1^2 & 0 \\ 0 & \Omega_2^2 \end{bmatrix}, \quad \boldsymbol{Q^c} = \begin{bmatrix} Q^c_{11} & Q^c_{12} \\ Q^c_{21} & Q^c_{22} \end{bmatrix}. \end{aligned} \tag{5}$$

By application of the Method of Averaging two necessary conditions are obtained for stability at a parametric excitation frequency $\eta = (\Omega_2 - \Omega_1)$:

$$\Theta_{11} + \Theta_{22} > 0, \tag{6}$$

$$\Theta_{11}\Theta_{22} + \frac{1}{4\Omega_1\Omega_2} Q^c_{12} Q^c_{21} > 0. \tag{7}$$

Not only the stability of the system can be examined for a certain PE-frequency. It is also possible to calculate the frequency interval of stability in the vicinity of this frequency. The interval is defined as

$$\eta_0 + \varepsilon\sigma_{lo} < \eta < \eta_0 + \varepsilon\sigma_{hi}. \tag{8}$$

with

$$\sigma_{lo,hi} = \mp\frac{(\Theta_{11}+\Theta_{22})}{2}\sqrt{-\frac{Q^c_{12}Q^c_{21}}{4\Omega_1\Omega_2\Theta_{11}\Theta_{22}} - 1}. \tag{9}$$

It is interesting to note that first order averaging leads to exactly the same results as obtained in (Tondl, 1998) by a different method based on Floquet theory.

In exchange for compact results, one has to accept that accuracy is degraded as soon as the parameter $\varepsilon < 1$ cannot be considered as small anymore, at least if only a first order approximation is used.

5.3 Numerical stability analysis

The stability of the trivial solution $\mathbf{x} = \mathbf{0}$ of system Eq.(4),(5) can be investigated also numerically by means of Floquet-theory, see [Verhulst, 2006]. Floquet's theorem postulates that for a system of first order differential equations

$$\dot{\mathbf{y}} = \mathbf{A}(t)\,\mathbf{y}, \qquad \mathbf{A}(t) = \mathbf{A}(t+T), \tag{10}$$

with a T-periodic matrix $\mathbf{A}(t)$ each fundamental matrix $\mathbf{M}(t)$ of the system can be represented as a product of two factors

$$\mathbf{M}(t) = \mathbf{Q}(t)\exp(t\mathbf{C}), \tag{11}$$

where $\mathbf{Q}(t)$ is a T-periodic matrix function and $\mathbf{C}$ is a constant matrix.

Stability of the time-periodic system can be determined either from the eigenvalues of the *Floquet exponent matrix* $\mathbf{C}$ or from the *monodromy matrix* $\mathbf{M}(T)$, which is in fact the state transition matrix evaluated after a period T. The monodromy matrix can be calculated numerically by repeated integration of the system equations over one period T, starting from independent sets of initial conditions. It is convenient to use the columns of the identity matrix $\mathbf{I}$ as initial vectors to start from. By solving n initial value problems over one period T

$$\dot{\mathbf{y}} = \mathbf{A}(t)\mathbf{y}, \quad [\mathbf{y}(0)_1, \mathbf{y}(0)_2, ..., \mathbf{y}(0)_n] = \mathbf{I}, \quad t = [0, T], \tag{12}$$

and by arranging the results as follows

$$\mathbf{M}(T) = [\mathbf{y}(T)_1, \mathbf{y}(T)_2, ..., \mathbf{y}(T)_n] \tag{13}$$

the monodromy matrix is obtained. Finally the eigenvalues of the monodromy matrix

$$\mathbf{\Lambda} = \mathrm{eig}(\mathbf{M}(T)), \tag{14}$$

are calculated numerically. The system is unstable if any of the eigenvalues are larger than one in magnitude

$$\max(|\Lambda_1|, |\Lambda_2|, ..., |\Lambda_n|) \quad \begin{cases} < & 1 \quad \text{stable system} \\ > & 1 \quad \text{unstable system.} \end{cases} \tag{15}$$

This procedure leads to a very efficient numerical method, which allows to examine the stability of a parametrically excited system much faster than with direct numerical integration. This method, however, cannot be used when nonlinearities are present and will not give vibration amplitudes.

6. EXAMPLES

In this section several examples of mechanical systems with parametric excitation are presented, to demonstrate the capabilities of the proposed method. Due to limited space, not all the details for every model can be included and therefore references are given where to find the full documentation for each example.

6.1 Two-mass system

In Figure 2 a schematic is shown of a two-mass system with one stiffness element being periodically changed according to $k_{01}(t) = k_{01}(1 + e^c_{01}\cos(\omega t)$. It is assumed that the damping element $c_{02} < 0$ and therefore the system may exhibit self-excited vibrations. To demonstrate the stabilizing effect of parametric excitation, the system was investigated for the critical value of parameter c_{02} at the stability threshold. In Fig. 3 the stable area is filled white and the unstable area is grey. This result was obtained by a numerical stability analysis. One can see easily that significantly lower values of the critical parameter c_{02} are possible at the combination resonance frequency of the system $\Omega_2 - \Omega_1$. This indicates that PE

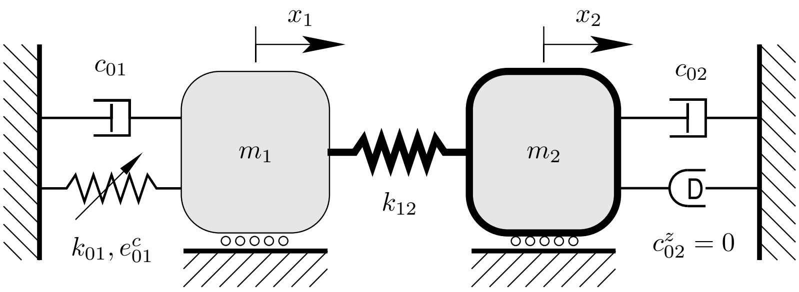


Figure 2. Two-mass system with self-excitation because of negative damping $c_{02} < 0$ and parametric stiffness excitation at $k_{01}(t)$.

at this frequency and with a sufficiently high amplitude will allow for an increase of the self-excitation parameter without becoming unstable. If the wrong frequency is chosen, however, then the effect is turned upside down, as one can see from the result in the vicinity of the parametric combination resonance of the summation type.

The stability threshold, separating stable and unstable regions, is also plotted as a broken line, near the parametric resonances. These two lines have been obtained by analytical formulas for the interval of stability, similar to Eqs.(8) and (9). Note that also the interval of instability near $\Omega_1 + \Omega_2$ can be obtained with rather high accuracy, compared to the numerical solution. The full set of data for this example can be found in [Ecker, 2005].

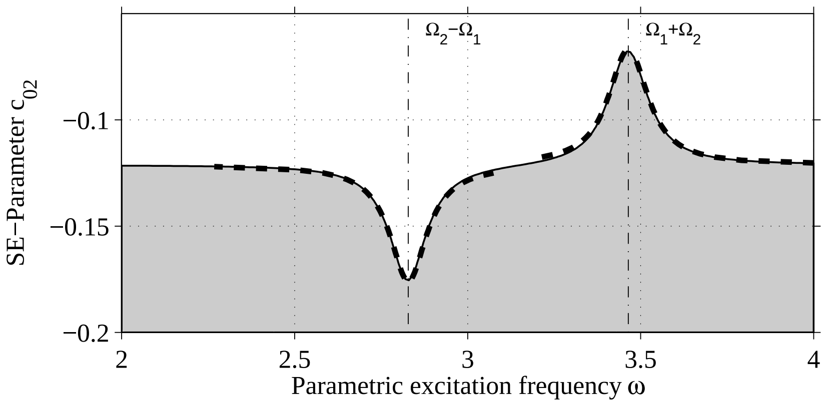


Figure 3. Typical stability map for a system as shown in Fig.2. White areas indicate a stable system, grey areas instability due to self-excitation. Stability threshold is calculated numerically (solid line) and by analytic formulas (broken lines).

6.2 Vibration suppression by PE in rotor systems

In this section numerical studies and also one experimental study will be reviewed and discussed, that demonstrate how parametric excitation could be employed in a rotor system to enhance it's performance. At present these design concepts may seem to be futuristic and not readily applicable in a real world machine. Since this is a well known scenario for an emerging new technology, it is likely worth the time and effort to investigate and explore the benign aspects of parametric excitation.

All the following studies have two facts in common which we can put in front of this discussion. (1) Parametric excitation is introduced by *open-loop control* of a system parameter. (2) The *frequency* of the harmonic parameter variation where the *best performance* is achieved will be near the difference type of the *combination resonance frequency* $\eta^-_{21/1} = \omega_2^e - \omega_1^e$.

Note that the available space does not allow to fully document the models and parameters used in the examples below. The reader is referred to the references where the models are explained comprehensively and details are listed.

6.3 Rigid rotor and time-periodic bearing mounts

There is a multitude of rotor systems, where the rotor can be considered as a rigid body and the flexibility of the system is located within the rotor bearings and the bearing mounts. Especially air-bearings, but also fluid-film bearings can create destabilizing forces at high rotor speeds and the rotor may become unstable beyond a speed threshold. By introducing a

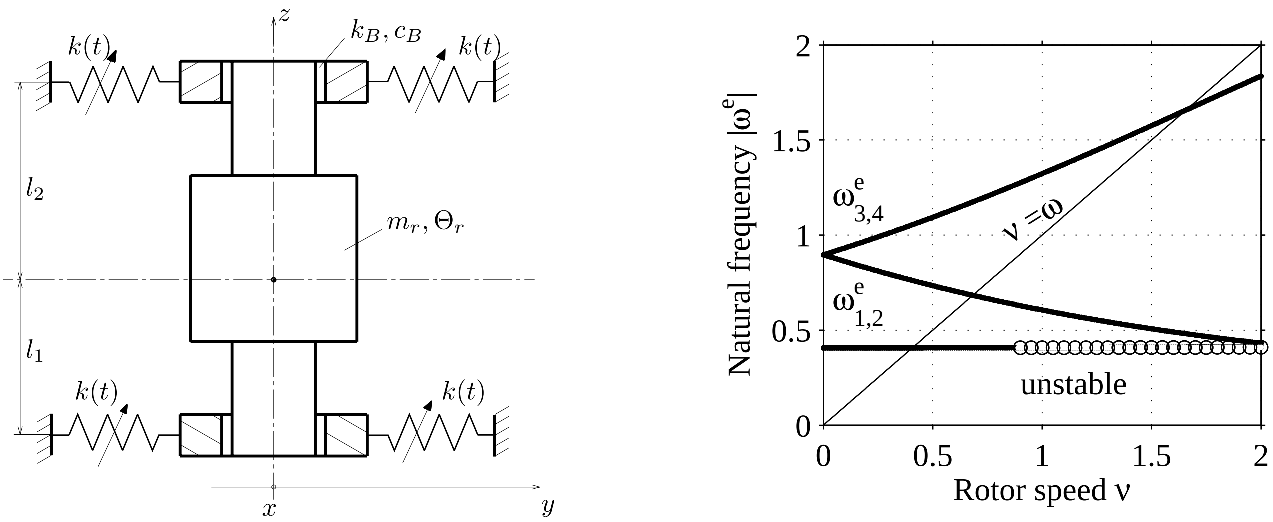


Figure 4. Rigid rotor m_R, θ_R supported by flexible bearing mounts with time-periodic stiffness component $k(t)$. Bearing parameters k_B, c_B lead to instability of first vibrational mode beyond stability threshold $\nu \cong 0.8$, see speed map to the right.

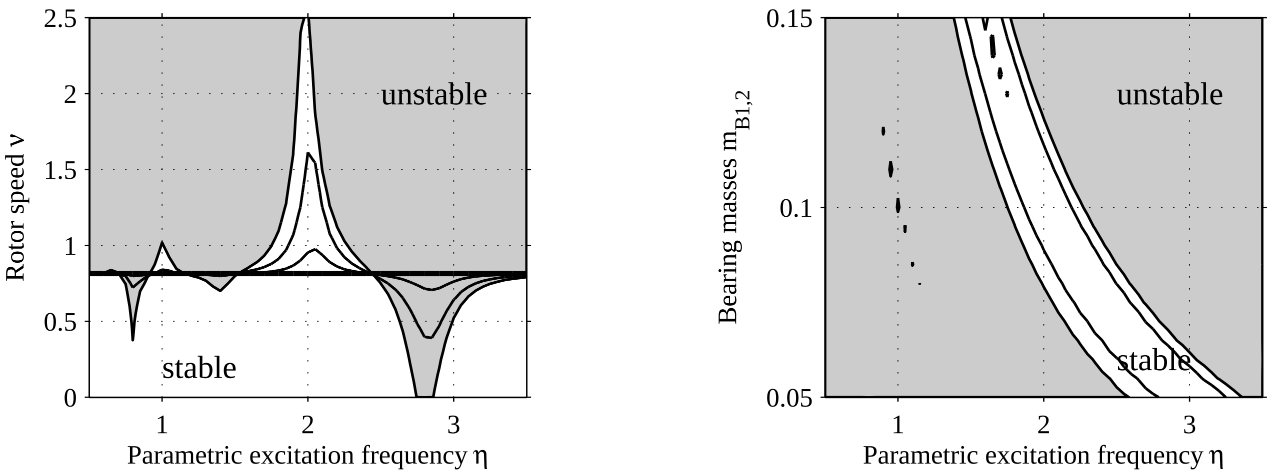


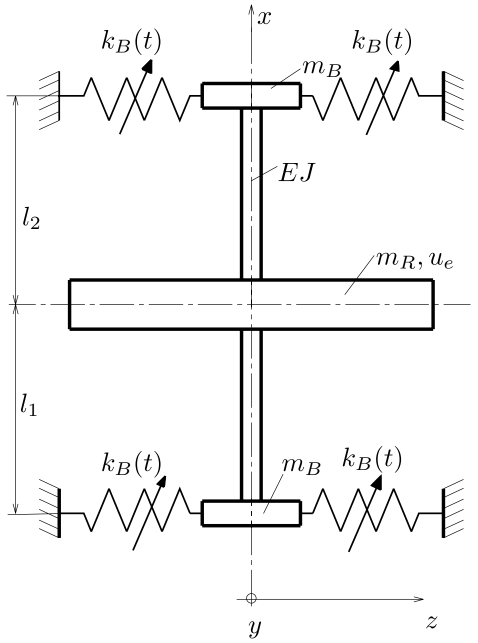
Figure 5. Stability charts for rotor system with PSE of bearing mounts. Left: Increase of stability threshold at $\eta = 2.0$ for different levels of PSE amplitude (0, 20, 40, 60% of k_{avg}). Right: Change of optimal PSE-frequency and stability interval as a function of bearing masses due to change of natural frequencies. See [Ecker, Tondl, 2004] for details and system data.

time-varying stiffness of the bearing support the system becomes parametrically excited and can benefit from an increased stability limit. In first studies [Ecker, Tondl, 2004] and [Ecker, Tondl, 2005] this concept was investigated.

Figure 4 shows a sketch of a simple rotor model with parametric stiffness excitation of the bearing mounts and the associated critical speed map for the rotor without PSE. Due to bearing instabilities the rotor becomes unstable beyond a scaled speed of about $\nu \cong 0.8$. With harmonic stiffness variation of the bearing mounts at $\eta^-_{21/1} = \omega^e_2 - \omega^e_1$ this speed limit can be increased significantly. Figure 5 (left) is a stability map for various PSE-amplitudes and shows the beneficial effect of PSE at $\eta \cong 2.0$. The diagram to the right demonstrates how the optimal PSE-frequency η depends on the bearing masses $m_{B1,2}$ as they also determine the dynamic properties (natural frequencies) of the system.

6.4 Rotor with flexible shaft and time-periodic bearing stiffness

The onset of instability can be improved for flexible rotors as well. This has been confirmed by the detailed studies [Ecker, Pumhössel, Tondl, 2002] and [Ecker, 2005] of a Jeffcott/Laval-rotor with parametric stiffness excitation inside the bearings. See the sketch of a vertical rotor with elastic shaft and open-loop controlled bearings in Fig. 6. The technology needed to realize stiffness control is already available. PSE can be achieved by either active magnetic bearings or by pressure-controlled fluid bearings. Similar to the previous and also to the next example, one can increase the stability threshold easily by a factor of two, if the necessary amplitude of the PSE can be provided. The bearing arrangement of



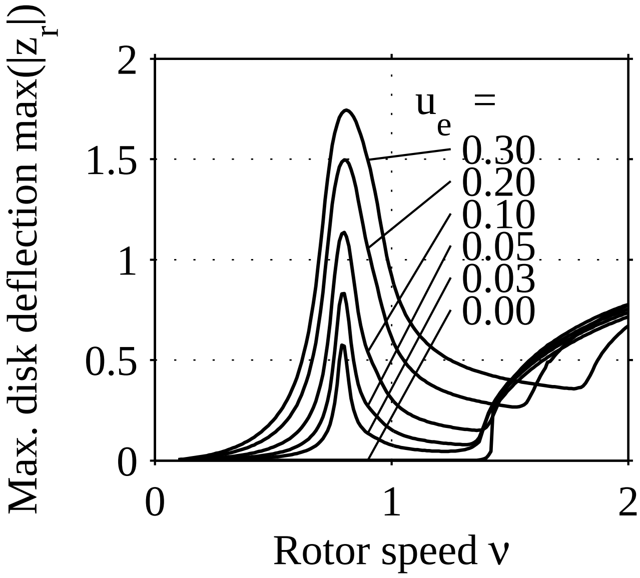


Figure 6. Symmetric flexible rotor m_R with mass unbalance u_e and external non-linear damping. Bearings with time-periodic stiffness component $k(t)$. Internal damping of shaft EJ leads to instability of first vibrational mode. Right: Vibration amplitudes at resonance speed and onset of instability for unbalance parameter u_e. See [Ecker, 2005] for system data.

Fig. 6 is only suitable for vertical rotors, since the static weight of the rotor must not be supported or transmitted by the time-periodic stiffness element.

Another important issue is rotor unbalance and how it possibly interacts with parametric excitation. In a numerical study in [Ecker, 2005] this question has been addressed and answered. In Fig. 6 (right) the vibration amplitudes at the disk station are plotted. As one can easily see, the first resonance speed appears at a scaled rotor speed of $\nu \cong 0.8$. The onset of instability is at $\nu \cong 1.4$ due to parametric stiffness excitation. Without PSE the instability region starts at about $\nu \cong 1.0$. As the unbalance excitation, which is represented by symbol u_e, is increased, no adverse interaction between forced excitation and parametric excitation is observed. Quite the contrary, the onset of instability moves to higher rotor speeds as the unbalance eccentricity is increased. This figure and result was not obtained by a stability investigation but by numerical integration of the non-linear and inhomogeneous set of differential equations of the rotor system.

Let us briefly touch also the question of a time-periodic damping variation, since it might be impossible to completely separate stiffness and damping properties with certain PSE-devices. In general, only energy-conserving system parameters may create enhanced damping, if they are changed periodically. According to a recent study [1], however, it seems to be possible to further increase the damping by additional time-periodic damping parameters.

6.5 Flexible multi-station rotor with local time-periodic stiffness

In the previous examples it was assumed that PSE is introduced at the bearing station of a rotor, either by a special type of bearing or an open-loop controlled bearing mount. To incorporate an additional device in a rotor design is quite difficult, no matter whether the system is open-loop or closed-loop controlled. Consequently, the bearing stations might not be the best positions for a device to create PSE. Moreover, in other positions there would be the advantage that one does not have to deal with static loads being transmitted by the PSE device.

Therefore, it is also important to study various positions along the rotor axis as possible and feasible stations for PSE. Such an investigation is under way, with a simple model in use as depicted in Fig. 7. Note that for reasons of simplicity this model assumes rigid bearings at both shaft ends and a rather flexible rotor shaft. At station (1) a small lumped mass is assumed and parametric stiffness excitation $k(t)$ is introduced. Instability of the rotor is caused by internal damping of the shaft, which leads to a speed threshold at $\nu = 1.25$. The stability map on the right hand side demonstrates, that also such a system can be stabilized at $\eta = \omega_2^e - \omega_1^e$ up to twice the original speed threshold and even higher, if large PSE-amplitudes are permitted.

There are of course PE-frequencies, at which the system is destabilized. In this numerical example, the scaled frequencies in the range between $1.5 < \eta < 2.0$ would result in a completely unstable system at any speed. This instability is caused by the primary parametric resonance for $2\omega_1^e$. Note that there is no need to select this dangerous frequency. Also there is no passing or running through this frequency. Therefore, such PE-resonance frequencies can be avoided easily

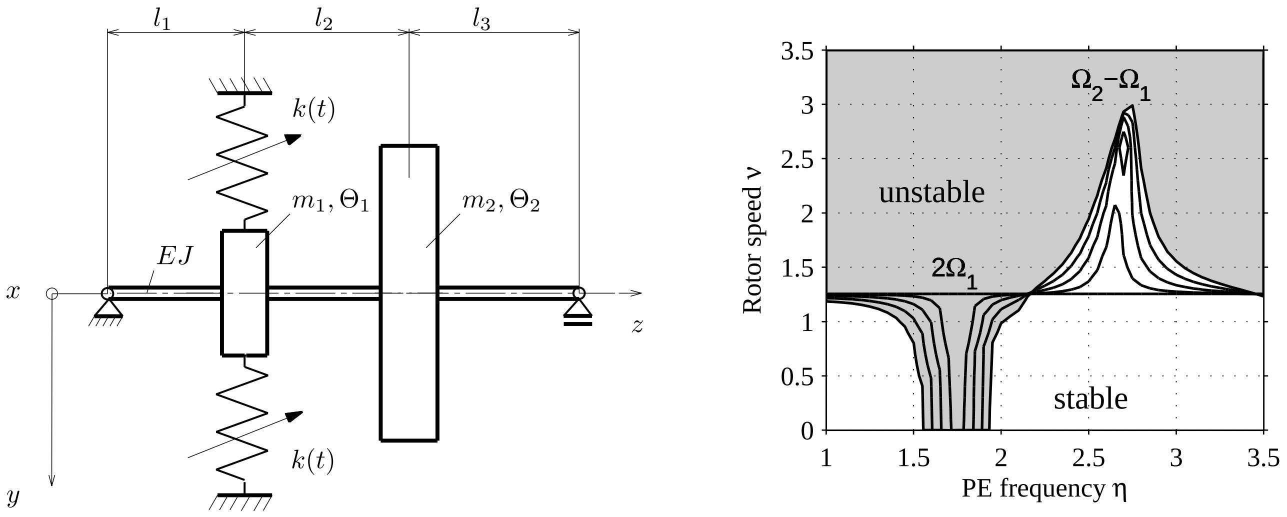


Figure 7. Flexible rotor with PSE-device attached to the rotor shaft at a station along the shaft. Right: Exemplary stability map with increased rotor stability for PSE-frequency $\eta \cong 2.6$ and parametric resonance at $\eta = 1.8$. Contour lines hold for increased PSE-amplitudes.

and are not a limiting factor to the basic idea.

6.6 Flexible rotor blade with axial time-periodic forcing

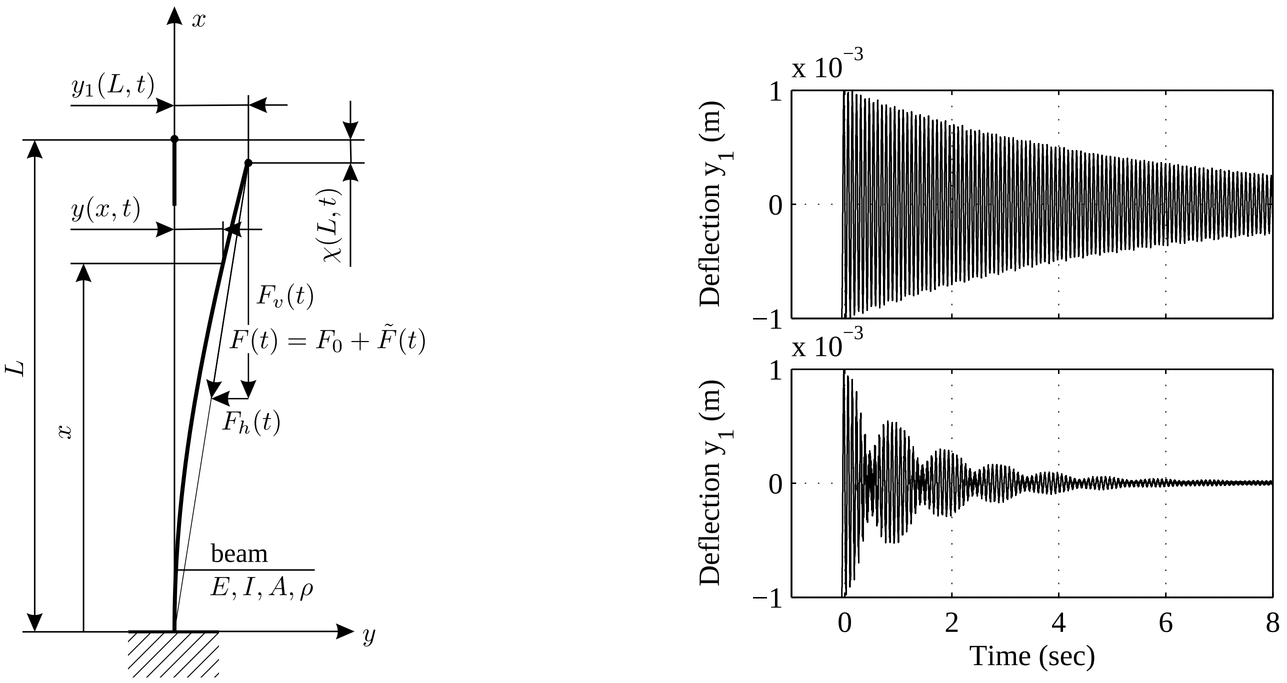


Figure 8. Cantilever beam with time-periodic tip force oriented to the root of the beam. Top right: Measured signal of free lateral vibration $y_1(t)$ at the tip of the beam, $\tilde{F}(t) = 0$. Bottom right: Signal $y_1(t)$ with time-periodic forcing $F(t) = F_0 + \tilde{F}(t)$ at $\eta = \omega_2^e - \omega_1^e$, see [Ecker, Pumhössel, 2009] for details.

The last example to demonstrate beneficial aspects of parametric excitation deals with a cantilever beam, which is loaded axially by a time-periodic force [Ecker, Dohnal, Springer, 2005]. It is known that the analysis of such a system leads to equations where the time-periodic force appears in the stiffness matrix of the system. Therefore, the system is in fact parametrically excited, although this might not be obvious at first glance.

The sketch in Fig. 8 gives a brief idea, how a string running from the root to the tip of the beam can be used to create a force that mimics an axial load. By pulling at the lower end of the string, the force at the tip can be controlled. This system has been investigated theoretically [**?**], but also a test rig was built and measurements were taken. Another experimental study of a cantilever beam, employing a different type of PSE, is found in [Dohnal, 2008]. For more general experiments

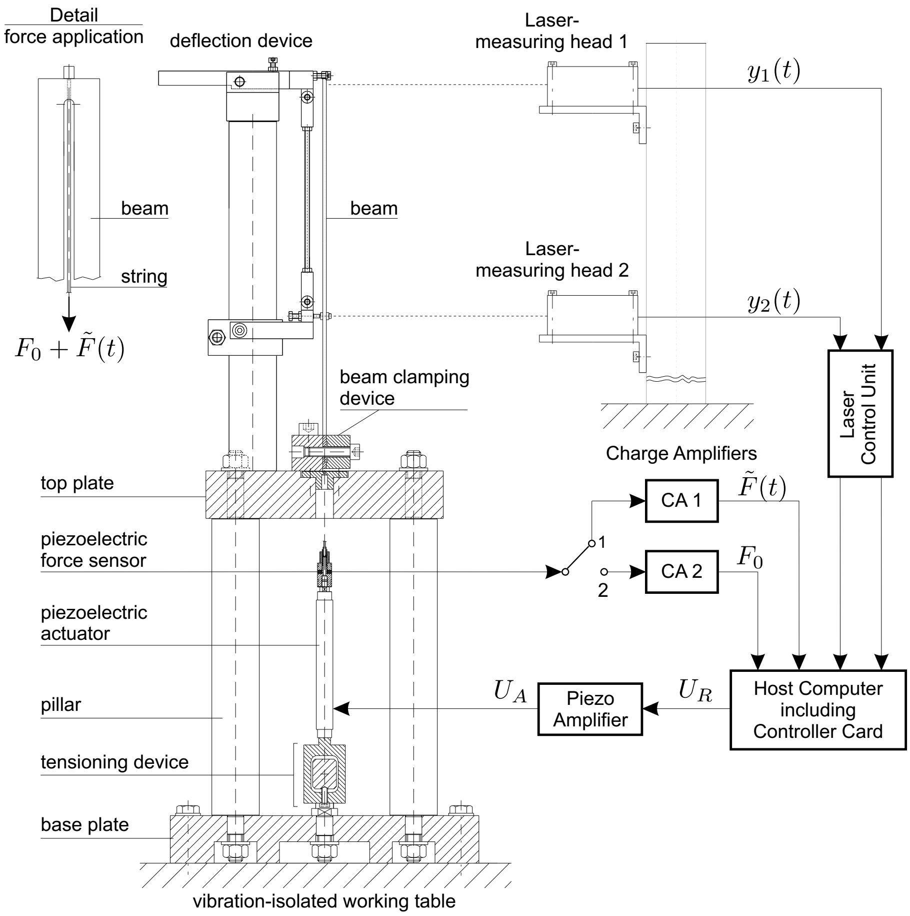


Figure 9. Experimental setup consisting of mechanical test rig on the left hand side, measurement and control equipment on the right hand side.

to proof the basic idea of vibration suppression by PSE see [Paradeiser, 2006] and [Schmidt et al. 2007].

In [Ecker, Pumhössel, 2009] one will find a detailed description of the test rig as sketched in Fig. 8 and selected results from the experiments. Two typical measured time series of first mode vibrations are shown in Fig. 8. The plot on top is a time series for the beam tip deflection $y_1(x = L, t)$ when no parametric excitation is active. One can see a rather slow decay of the vibration amplitudes, which will serve as a reference result. The diagram below shows the same signal, but with PE activated and operating near the optimal PE-frequency. Although strong beats do appear in the signal it is easy to recognize that the PE-generated signal exhibits a much faster vibration decay.

It might indeed be too optimistic to think that this idea could be installed in the future in turbo-rotors and may help to provide more damping to turbine blades. But at least this example demonstrates that it is possible to experimentally verify the theoretical results and proof the vibration suppression effect. Moreover, it shows that the basic effect can be used in many different ways and configurations. Also this example might serve as a thought-provoking impulse and trigger further ideas.

7. CONCLUSIONS

Time-periodic parameters in engineering systems, in particular stiffness parameters, can be used to faster suppress vibrations in a lower mode by energy transfer to a higher and better damped mode. This effect can be achieved if the frequency of the periodic function to vary the parameter is chosen as the difference between the respective higher and lower natural frequency of the system. The virtually increased damping parameter of the lower vibration mode is especially of avail if this mode becomes unstable. Depending on the amplitude of the parameter variation the stability threshold is raised and the rotor may run stable at higher speeds as before.

There is a number of ways to take advantage of this basic principle in a rotor system. Several of them are outlined in this paper and the possible benefit is confirmed by either numerical or experimental studies. Further application ideas will emerge from future work on this topic which may evolve into a key solution for rotor instability in certain cases.

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9. Responsibility notice