

THE MONTE CARLO METHOD APPLIED TO THE RADIATIVE TRANSFER IN AN ANISOTROPICALLY SCATTERING

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Abstract. *This work presents an analysis of the Radiative Transfer Equation solution using the Monte Carlo Method. A cold media and one-dimensional slab geometry with a normal collimated beam incidence onto the media is considered. Anisotropic scattering is specially analyzed and an algorithm is improved for this purpose. Optical thickness, Albedo, number of bundles and number of control volumes are investigated. A comparative analysis with the Discrete Ordinate Method is also performed.*

Keywords*: Monte Carlo Method, Radiative Heat Transfer, Anisotropic Media*

1. INTRODUCTION

In recent years, various works had been carried on radiation heat transfer to diverse applications such as: combustion engines, boilers, furnaces, rocket engines and many other examples that involve high temperatures. Few analytical solutions are available in literature for these problems and the use of numerical methods is a powerful tool to study radiative transfer phenomena. One of these numerical methods is the model based on the Monte Carlo Method (MCM). Since 1963, the MCM had been used to solve problems on radiation heat transfer (Yang *et al*., 1995). In despite of an important number of works of the MCM in radiative heat transfer problems founded in literature (Yang *et al*., 1995) some analyses miss to be performed.

Until few years ago the MCM applied to solve radiative heat transfer problems had not been considered the anisotropy and the internal reflections. Prahl *et al*. (1989) considered these effects to analyze the laser incidence on the skin tissues. Ambirajan (1996) applied the MCM in multiple scattering of a narrow light beam with normal incidence onto a narrow parallel slab. In this work it was observed that the MCM is efficient until the second order scattering. Henson and Malalasekera (1997) realize a comparison of discrete transfer and MCM in three-dimensional nonhomogeneous isotropically scattering media. They founded good agreement with benchmark results. The average deviation between the two methods was less than 1.2%. Wang (1998) developed a modeling of diffuse reflectance of light in turbid slabs. He used a hybrid method which combines the MCM and the diffusion theory, using the advantages of high precision to MCM with the speed of diffusion theory.

The advantage of MCM is that even the most complicated problem (multidimensional or gas) can be solved with relative ease, in opposite of the finite difference or volume and finite element techniques that increase much more rapidly the complexity of formulation (Modest, 2003). MCM may easily be applied for multidimensional geometries, not homogeneous, with time dependent boundary conditions, where others techniques are almost impossible to be implemented (Wong and Mengüç, 2002). Ruan, *et al*. (2002) developed a MCM applied to a medium with nongray absorbing-emitting-anisotropic scattering particles and gray approximation. Modest (2003) presented the MCM comparisons with the reference solutions. He shows that the errors (difference between MCM and reference solution) increase strongly with the optical thickness.

Lataillade *et al*. (2002) and Eymet *et al*. (2005) presented a boundary-based net-exchange MCM applied to solve a scattering media with optically thick absorption (and/or for quasi-isothermal configurations). They presented the number of statistical realizations needed in order to get 1% standard deviation over the slab emission value, as a function of slab total optical thickness. They showed the number of statistical realizations increase strongly with the optical thickness. In case of very short photon mean free paths, most bundles are absorbed in the vicinity of their emission positions which means that only very few bundles effectively participate to distant radiative transfers. The consequence is that MCM based on bundle transport formulations require very large numbers of statistical realizations for sufficiently accurate radiative transfer estimations. The computational costs impose constraints to geometrical grid sizes that cannot be reduced sufficiently for the optically thin assumption to be valid. They presented the exchange formulation in order to reduce the number of statistical realizations to the acceptable values.

Tess *et al*. (2004) applied the MMC to solve the radiative transfer in a turbulent sooty flame. The MCM shows very convenient for these radiative calculations, because it allows them to easily treat the turbulence–radiation interaction, as well as to include sophisticated models of gas properties, without computation time increase.

Petrasch *et al*. (2006) used the MCM to a reticulate porous ceramic to calculate the extinction coefficient and scattering phase for diffusely and specularly reflecting surfaces. The adopted assumptions are: the medium is statistically homogeneous and isotropic, diffraction is neglected, geometrical optic equations are valid, solids are opaque, and the gas phase is non-participating (transparent). A large number of statistical realizations is used (10^7).

Xavier Filho and Moura (2007) used a MCM to solve the Radiative Transfer Equation to one-dimensional slab condition. An isotropic scatter media with a collimated normal incident beam onto to the sample was analyzed. A comparative analysis with Discrete Ordinate Method (DOM) was performed to show the influence of optical thickness, albedo, bundles number, the number of control volumes and processing time.

The Discrete Ordinates Method (DOM) was proposed by Schuster (1905) and Schwarzschild (1906) to study radiative transfer in stellar atmospheres (Siegel and Howell, 2002). Until now days many works were proposed to improve this method and a review it can be found in Fiveland's works (1985 and 1987). The DOM is a multi-flux method extension and it has been used to solve multi-dimensional radiative transfer problems.

In this work, a Monte Carlo formulation is employed to solve the Radiative Transfer Equation to an one-dimensional slab condition. An anisotropic scatter media with a collimated normal incident beam onto to the sample is analyzed. The self-emission is not considered. An uncertainty analysis is performed to show the influence of optical thickness, albedo, bundles number and the number of control volumes. A comparative analysis with Discrete Ordinate Method is also performed. A processing time analysis is presented for both methods.

2. RADIATIVE TRANSFER EQUATION

The Radiative Transfer Equation (RTE), which describes the variation of the spectral radiation intensity, *I*, (in a solid angle Ω, function of optical depth τ) in an absorbing-emitting-scattering medium, can be written as:

$$\mu\frac{\partial I(\tau,\mu)}{\partial\tau}+I(\tau,\mu)=(1-\omega)I_o(T)+\frac{\omega}{2}\int_{-1}^{1}I(\tau,\mu')p(\mu',\mu)d\mu' \qquad (1)$$

where ω is the albedo, *p* is the phase function, and I_o is Planck's blackbody function (in order to simplify the notations, the spectral subscript λ is not considered in the text). These properties are those of a pseudo-continuum medium equivalent, in terms of radiative transport, to the real dispersed material. The boundary conditions assumed a normally incident collimated beam onto the sample with bidirectional transmittance and reflectance measurements. The boundary conditions can be expressed like:

$$\begin{cases} I(\tau=0,\mu)=I_C\,; & \mu_o<\mu<1 \\ I(\tau=0,\mu)=0\,; & 0<\mu<\mu_o \\ I(\tau=\tau_o,\mu)=0\,; & -1<\mu<0 \end{cases} \qquad (2)$$

where I_C is the radiative intensity of the incident beam with a divergence angle, θ_o ($\mu_o = cos\,\theta_o$).

The RTE in a scattering media has been studied, analytically and numerically in astrophysics, atmospheric, heat transfer and, more recently, in medical applications. However, analytical solutions are not always possible, and the numerical solutions must be employed. Some assumptions can be adopted to facilitate the solution of these problems, for example, homogeneous media, isotropic scattering, one-dimensional geometry, constant radiative properties, etc.

Numerical methods such as the DOM (Chandrasekhar, 1960) have been used in radiative transfer problems when the analytical solutions are not available. Statistical techniques like Monte Carlo method supply good approaches, but they present a high computational time (Churmakov *et al.*, 2003).

The phase function has been approximated by a combination of two Henyey-Greenstein (HG) functions coupled with an isotropic component (Nicolau, 1994):

$$p_{HG,g}(\theta_o,\mathrm{g})=\frac{1-\mathrm{g}^2}{\left(1+g^2-2\mathrm{g}cos\theta_o\right)^{3/2}}$$

$$\mathrm{p}(\theta_o)=\mathrm{f_1 f_2 p_{HG,g_1}}(\theta_o)+(1-\mathrm{f_1})\mathrm{f_2 p_{HG,g_2}}(\theta_o)+(1-\mathrm{f_2}) \qquad (3)$$

where the parameters g_1 and g_2 govern the shape of HG functions (p_{HG},g_1 and $p_{HG,g2}$) in the forward and backward directions. f_1 is the weighting factor between forward and backward anisotropy in the phase function, f_2 is the weighting factor between anisotropic and isotropic scattering.

Figure 1 shows a randomly generation of the phase function to $g_1 = 0.86$; $g_2 = 0.8$; $f_1 = 0.96$; $f_2 = 0.96$ to 100,000 statistical realizations.

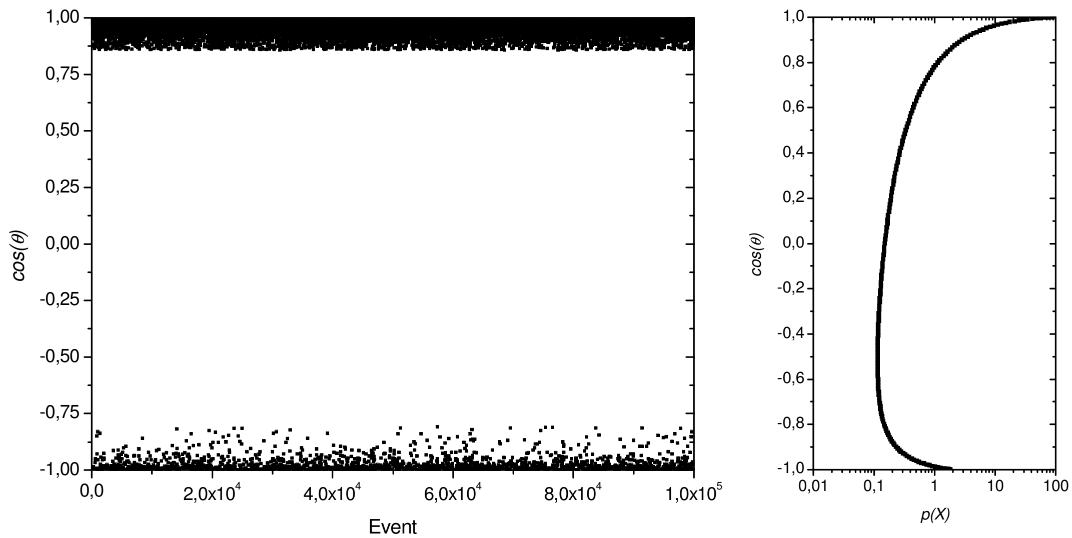

Figure 1. Anisotropic phase function randomly generated to 100,000 statistical realizations. $g_1 = 0.86$; $g_2 = 0.8$; $f_1 = 0.96$; $f_2 = 0.96$.

2.1. Discrete Ordinate Method (DOM)

The DOM was initially used by Schuster (1905) and Schwarzschild (1906) for studying radiative transfer in stellar atmospheres (Siegel and Howell, 2002), and after, Chandrasekhar (1960) extended the formulation to astrophysics problems. Carlson and Lathrop in 1968 had developed a solution to the neutrons transport equation.

The majority of works use the RTE formularization presented by Chandrasekhar (1968) and Özisik (1973). These techniques of solution of the RTE can be found in Moura *et al.* (1997 and 1998). The RTE solution by DOM is constituted of two stages: i) an angular discretization, where the integral term of RTE is substituted by a radiative intensities weighted sum of the angular directions. In this way, the integro-differential equation is transform on a set of first-class ordinary partial equations; ii) a space discretization, considering control volume, for solution of partial equations. To a "cold" media (no self-emission media), Eq. (1) can be re-write as:

$$I_{i+1/2,j} = \frac{1}{\left(1 + f\alpha_j\right)}\left[f\alpha_j \frac{\omega}{2\beta}\left[\sum_{n=1}^{N} w_n \left(p_{nj} I_{i+1/2,n}\right)\right] + I_{i,j}\right] \tag{4}$$

where *i+1/2* represent the control volume center coordinate, *f* is the interpolation function that can be: upwind (*f*=1), linear (*f*=1/2), integral (*f* is function of the α_j calculated from integration of RTE) or exponential (*f* is function of the α_j calculated from the solution of RTE) (Moura *et al*., 1998), *w* is the weight and α_j is:

$$\alpha_j = \frac{\Delta\tau_{i+1/2}}{\mu_j} \tag{5}$$

where $\Delta\tau$ is the optical thickness of the control volume.

2.2. Monte Carlo Method (MCM)

Radiative heat transfer by Monte Carlo Method (MCM) is based on probability concepts applied to the physical phenomena, such as: emission, reflection, and absorption of the photon.

Solving RTE by MCM, radiative energy is not treated as a continuous energy flux, but is considered a pack of photons, each with a fixed amount of energy. To quantify the radiative energy attenuation in a monochromatic source is considered the Beer's law. The Beer's law expresses the attenuation of radiant energy inside a volume of a thickness, S. In the MCM the Beer's law can be modify to express the radiative energy extinction probability emitted from a point in the media (or surface) and travel over the distance, S (Yang *et al*., 1995):

$$Rs = 1 - e^{KS} \tag{6}$$

where Rs is the random number, K is the extinction coefficient and S is trajectory distance by the photon until be absorbed or scattered. If the value of S is bigger than the distance takes from the photon point emission the photon was absorbed or scattered. Whether each photon is absorbed or scattered it is determined by the albedo, ω, and the uniform random number for scattering albedo, R_ω. If R_ω is bigger than albedo the photon is absorbed, otherwise the photon is scattered (Brewster, 1992). The Monte Carlo algorithm for an anisotropic media with a collimated beam incident beam onto a slab surface has this flow (Xavier Filho and Moura, 2007):

1. Determine the direction of propagation by the photon inside the solid angle of the incident beam onto the face of the slab.
2. Determine by Eq. (6) if it was absorbed, scattered or remaining in its trajectory.
3. If it was absorbed or kept the media, it is initiate a new photon analysis.
4. Verify the absorption or scattering criteria. If it was scattered, to choose a new direction (random choice by the phase function - Eq. (3) and to repeat step 2, considering the current position of the photon.

Repeat these steps for a number of photons to assure the accuracy.

3. RESULTS

A comparison between the MCM and DOM is presented in this section considering anisotropic scattering and a normal incident beam onto a slab. Hemispherical transmittance and reflectance are used to compare the results. The hemispherical transmittance and reflectance to DOM are defined as:

$$T_{eh} = \frac{\sum_{\mu>0} w_n \mu_n I_{i,n}}{I_o \mathrm{d}\omega_o} \tag{7}$$

$$R_{eh} = \frac{\sum_{\mu<0} w_n \mu_n I_{i,n}}{I_o \mathrm{d}\omega_o} \tag{8}$$

where $I_o \mathrm{d}\omega_o$ is the incident radiative onto the slab with a slid angle, $\mathrm{d}\omega_o$.

The hemispherical transmittance and reflectance to MCM are defined as.

$$T_{eh} = \frac{Ejected\ photons\ number\ by\ face\ 1}{Total\ \ photons\ considered} \tag{9}$$

$$R_{eh} = \frac{Ejected\ pthotons\ number\ by\ face\ 0}{Total\ photons\ considered} \tag{10}$$

where *face 0* has an incident beam onto the face and *face 1* is the other face without incidence.

Although the DOM is a numerical method, consequently, an approached solution, it will be used as reference for the comparative analysis with MCM. For these specific proposed cases, analytical solutions are not available.

For all cases, the DOM was solved using linear interpolation and the Eq. (4) was used to not allow negative radiative intensities. For this purpose, the minimum control volumes number, function of the optic thickness, are presented in Tab. 1 (Xavier Filho and Moura, 2007). The MCM use the same number of control volumes to respect the same conditions of analysis.

The DOM quadrature has 24 directions (Nicolau, 1994). This quadrature is particularly appropriate to considerer the divergence angle of the incident beam.

Firstly, an isotropic and conservative (unitary albedo, $\omega = 1$) case is used to validate the algorithm. The solution of MCM was implemented from Xavier Filho and Moura (2006). Figure 2 presents the results to this case. A P1 solution presented by Modest (2003) is used like reference. The analytical solution is closed to the P1 solution and its implementation is very hard due to the use of Helmholtz equation (Modest, 2003). In such a way, the P1 solution is compared with MOD and MCM solutions function of the optic thickness. Two different divergence angles of the incident beam, θ_o, two different number of the spatial discretization control volumes, dx, and two different number of packages, np, are used to investigate a probably influence of these parameters on the MCM. It can be observed that they don't present a significantly influence in the nondimensional flux. Like presented by Modest (2003), Lataillade *et al.* (2002) and Eymet *et al.* (2005) the MCM errors (difference between MCM and reference solution) increase strongly with the optical thickness. It can be also observed that the DOM presents very closed agreement with the reference solution.

Table 1. Number of control volumes used to different optic thickness. (Xavier Filho and Moura, 2007)

Optical thickness, τ_0	**Number of control volumes**
0.1	3
1.0	19
5.0	91
10.0	180
20.0	359
30.0	538
50.0	895
75.0	1342
100.0	1789

Now an anisotropic scatter media with a collimated normal incident beam onto to the sample is analyzed. The self-emission is not considered and it is used the Nicolau phase function, described by Eq. (3) to $g_1 = 0.86$; $g_2 = 0.8$; $f_1 = 0.96$; $f_2 = 0.96$ parameters. For all test cases were used dx=100, np=10^7 and θ_o=2.5º.

Figure 3 presents the results to a media with albedo, ω=1.0 and Nicolau phase function. The hemispherical transmittance and reflectance are presented. It can be observed the same effect to the MCM deviation with the increase of the optical thickness. Like this case is conservative the hemispherical transmittance and reflectance values for the same optical thickness is the unity. The MCM deviation to the reference solution has a monotonic increase with optical thickness. To a very short photon mean free paths, most bundles are absorbed in the vicinity of their emission positions which means that only very few bundles effectively participate to distant radiative transfers. The consequence is that MCM based on bundle transport formulations require very large numbers of statistical realizations for sufficiently accurate radiative exchange estimations (Lataillade, 2002). The other numerical difficulty is related to spatial discretization for the optically thin assumption. Computational costs impose constraints to geometrical grid sizes that cannot be reduced sufficiently.

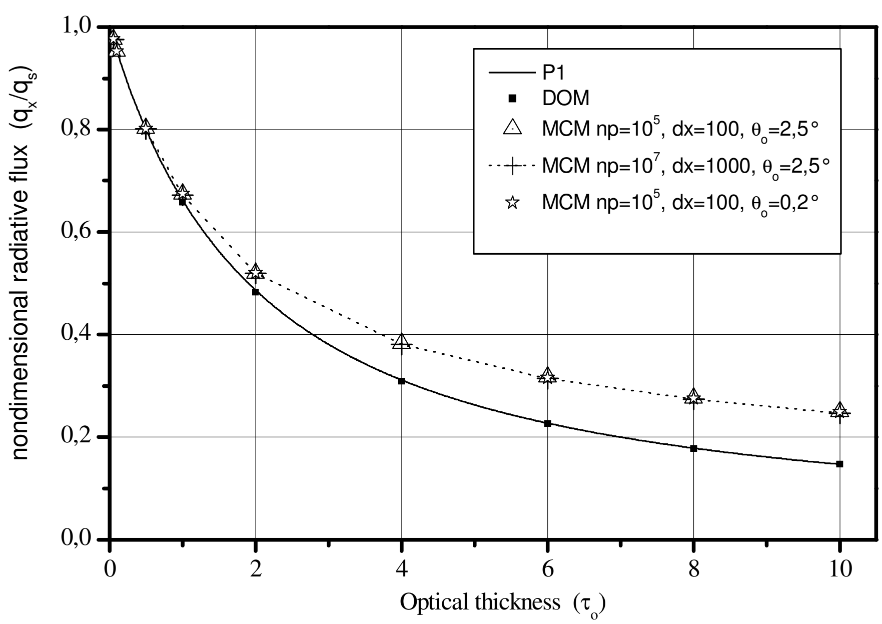

Figure 2. Comparison between DOM and MCM to a nondimensional radiative heat flux in a purely scattering layer with a normal collimated irradiation and isotropic scatter.

Figures 4 and 5 show the same analysis presented on Fig. 3 changing the albedo to ω=0.5 and ω=0.1, respectively. The MCM deviation on hemispherical transmittance is less important notwithstanding the MCM hemispherical reflectance is much bigger than DOM values. Another point is the divergence radiative flux is different for the two methods because the sum of the hemispherical transmittance and hemispherical reflectance are different for these two methods.

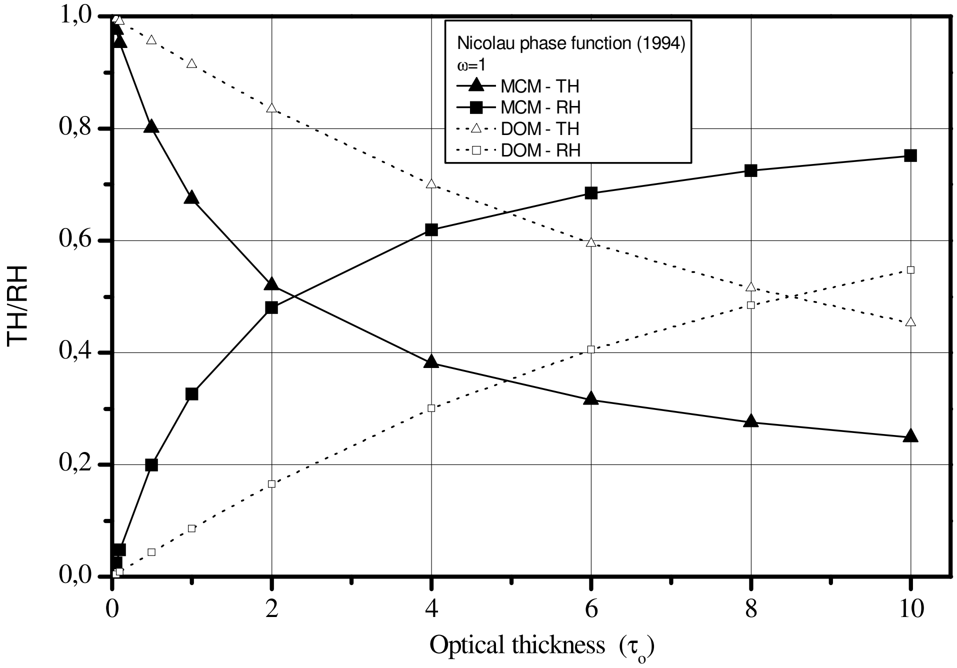

Figure 3. Hemispherical transmittance and reflectance to MCM and DOM to an unitary albedo, ω=1.0, anisotropically scattering and Nicolau phase function.

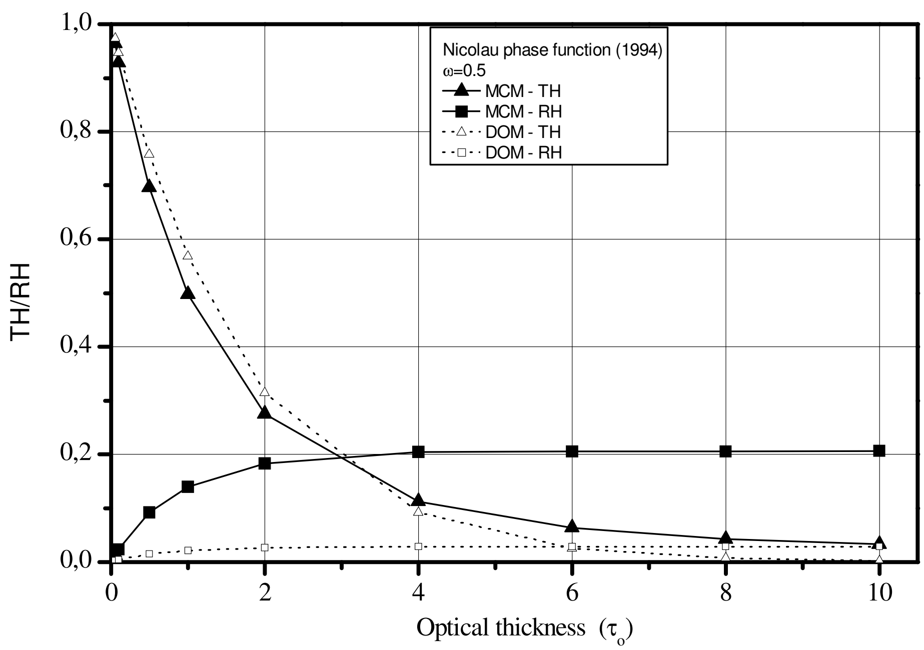

Figure 4. Hemispherical transmittance and reflectance to MCM and DOM to a unitary albedo, ω=0.5, anisotropically scattering and Nicolau phase function.

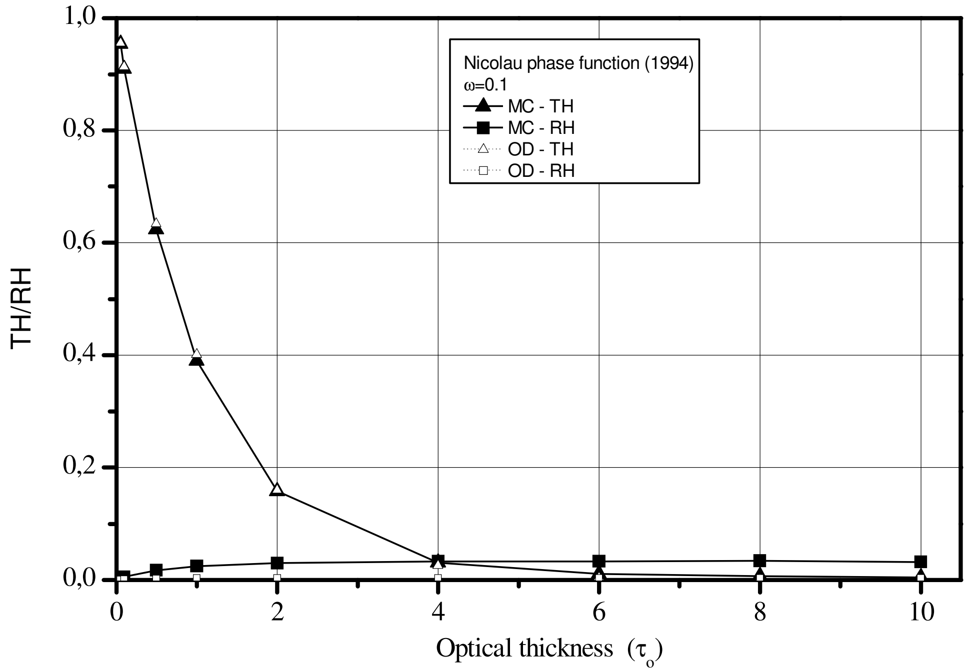

Figure 5. Hemispherical transmittance and reflectance to MCM and DOM to an unitary albedo, ω=0.1, anisotropically scattering and Nicolau phase function.

Figures 6 and 7 presented the computational time to ω=0.5 and ω=0.1 to different optical thicknesses, using a 3.0GHz PC computer. The two results are similar. The processing time of the DOM increase with the optical thickness function of the increase of the control numbers to avoid negative radiative intensities, Tab. 1. The number of the control volumes to the MCM is kept constant. If the MCM control volume is increase to improve the accuracy of this method the processing time must be considerable increased.

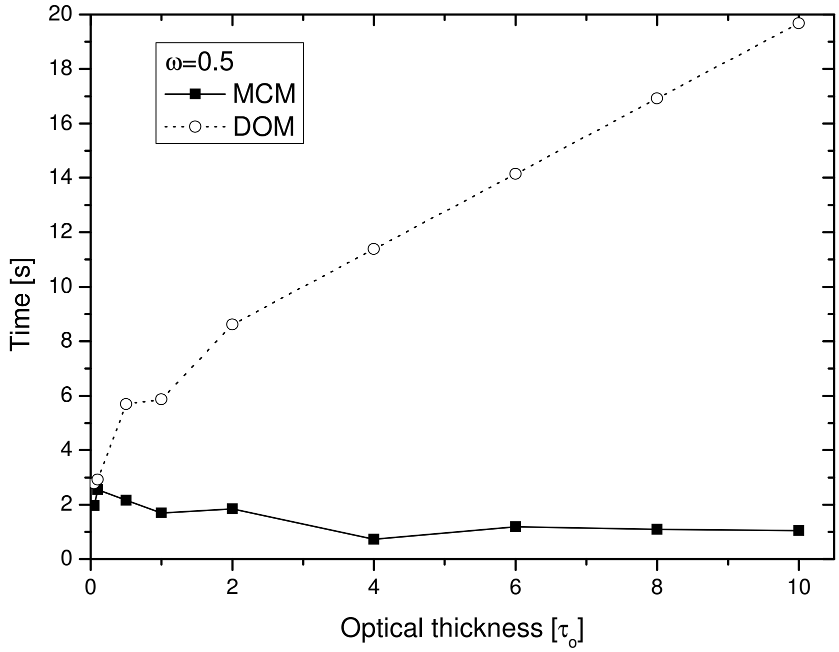

Figura 6. Computational time to MCM and DOM to albedo $\omega = 0.5$.

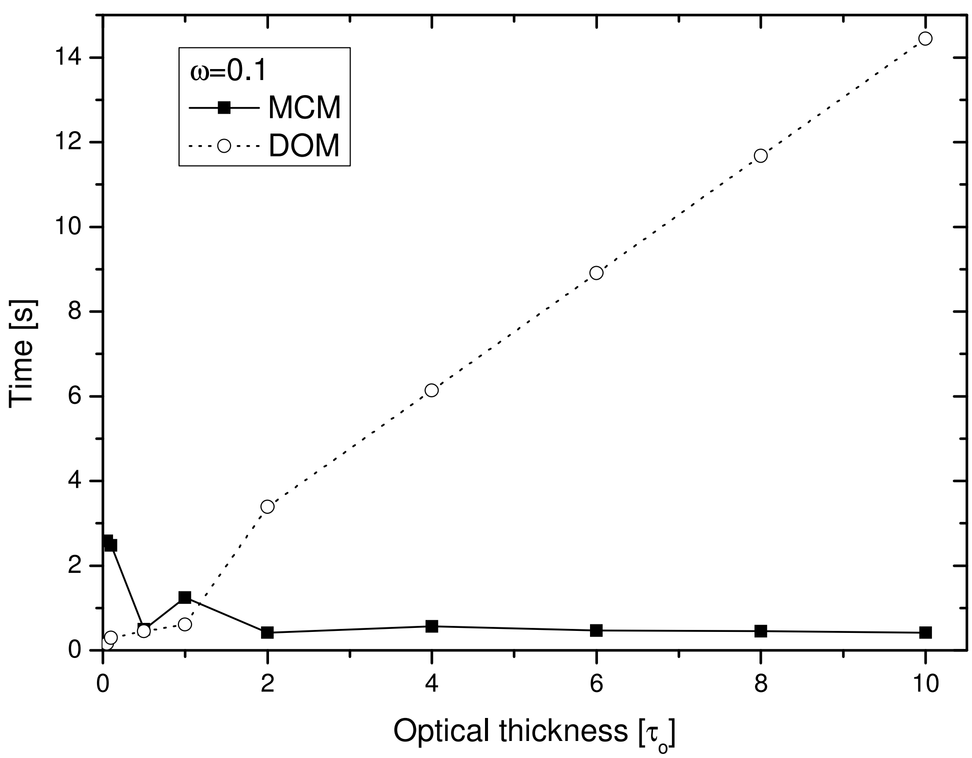

Figure 7. Computational time to MCM and DOM to albedo, $\omega = 0.1$.

3. ACKNOWLEDGEMENTS

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4. CONCLUSION

In this work a comparative study of the Monte Carlo and Discrete Ordinate methods to the radiative transfer problem in anisotropic one-dimensional geometry was presented. Considering the Ordinate Discrete method like a benchmark, tests are performed, changing the optical thickness and the albedo. Hemispherical transmittance and reflectance are used to compare the results. Monte Carlo method presents better results when it works with a significant number of packages and optically thin media.

MCM are the difficulties related to numerical behavior in the optically thick limit. It can be observed to the MCM a monotonic increase of statistical error with optical thickness. Due to a very short photon mean free paths, most bundles are absorbed in the vicinity of their emission positions which means that only very few bundles effectively participate to distant radiative transfers. The consequence is that MCM based on bundle transport formulations require very large numbers of statistical realizations for sufficiently accurate radiative exchange estimations (Lataillade, 2002). The other numerical difficulty is related to spatial discretization to an optically thick media. Computational costs impose constraints to geometrical grid sizes that cannot be reduced sufficiently for the optically thin assumption to be valid (Lataillade, 2002).

To improve the accuracy of the MCM the boundary-based net-exchange Monte Carlo algorithm (Lataillade *et al.*, 2002 and Eymet *et al.*, 2005) will be implemented like a new step of this work.

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