EXPERIMENTAL RESULTS FOR SOUND PATH CONTRIBUTION USING AN ALTERNATIVE TPA APPROACH

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Abstract. *Transfer path analysis (TPA) is a methodology used to quantify the partial acoustic contribution associated with different propagation paths through complex systems. The accurate determination of transfer functions among sources and receivers is the main step when applying TPA methodology. This paper presents the results obtained from a controlled experiment in a semi-anechoic room comparing traditional TPA and a new methodology for sound path quantification known as operational TPA. An aluminum plate and two electrodynamic shakers were used, representing respectively the main radiator surface and the vibration energy injected through both considered transfer paths. The operational TPA aims for a practical and robust way of finding the partial acoustic pressure contribution for each path. In order to achieve that, accelerometers were used to measure the signal at the same input points of the forces. They require less space and are much lighter than force transducers and, as a consequence, they can be more easily placed at the input point of a sound path. However, to use the acceleration data, principal components analysis needs to be applied to calculate a transfer coefficient matrix. This experiment also measured the result when each shaker/plate set-up worked independently to obtain a sound pressure level curve that was used as a reference for both methodologies (traditional and operational) and to confirm their accuracy. The results have successfully illustrated the proposed approach for evaluating each sound path contribution of an aluminum plate using the alternative TPA formulation.*

Keywords: *Transfer path analysis, Operational TPA, Sound path contribution*

1. INTRODUCTION

This paper presents the results obtained from a controlled experiment in a semi-anechoic room. It makes a comparison between the traditional TPA and a new methodology for sound path quantification known as operational TPA. However, in order to understand the experimental results appropriately, it is firstly necessary to present an overview of the theory beyond the methodologies applied. The subitems 1.1 and 1.2 are dedicated to this.

1.1 Traditional TPA

Iniatially, a brief explanation to the traditional and largely used methodology for vibroacoustic investigation is given, especially for the automotive and aeronautic industries, called Transfer Path Analysis (TPA), which is also known as Sound Path Contribution (SPC).

It is a procedure used to evaluate the partial acoustic contribution associated with different propagation paths of vibroacoustic energy that exist between one or more sources and one or more receivers, linked one to another by a number m of airborne or structure-borne connections.

The traditional TPA has been used in many projects as a vibroacoustic energy quantification during the last decades to achieve a better noise comfort for the respective application. The traditional methodology, described by LMS International (1995), can be summarized by the following steps:

1. Measure the Transfer Functions associated with the transfer path j under laboratory (experimental) conditions, between the input point of vibroacoustic energy and the point where the global vibroacoustic response is measured. The sound source is normally disconnected from the structure to proceed this step;

2. Measure the Operational Forces directly or even calculate them in an indirect way using analytical methods of matrix inversion. These operational forces represent the input forces related to a specific transfer path during its steady-state;

3. Multiply the Transfer Functions mentioned in item (1), by the Operational Forces obtained after executing item (2). This procedure results in a Partial Pressure Response related to the path j. As a final result the Total Sound Pressure Response will be obtained by the sum of all partial responses. The analytical formulation in a frequency domain is represented by the Eq. (1).

$$r_i(\omega) = \sum_{j=1}^{nrpaths} \left(\frac{R_i(\omega)}{S_j(\omega)}\right)^{(exp)} S_j(\omega)^{(op)}, \quad (1)$$

where:

- $r_i(\omega)$ is the Total Noise Response at an output point i;
- $\left(\frac{R_i(\omega)}{S_j(\omega)}\right)^{(exp)} = H_{ij}(\omega)^{(exp)}$ is the Frequency Response Function (FRF) between the output point i (pressure response or acceleration) and the input point j (force or volume velocity) under laboratory conditions;
- $S_j(\omega)^{(op)}$ is the force or volume velocity at the transfer path j under operational conditions.

It is important to point out that the vibroacoustic contributions of each path (degrees of freedom) are represented by complex numbers in a frequency domain, which means the sound wave related to it have magnitude and phase. Hence, in order to obtain the total noise response, this information must be considered once destructive interferences may occur.

1.2 Operational TPA

In recent years, a new methodology was proposed and has been applied in industry, which was called operational TPA. This methodology has the advantage of not requiring measurements of transfer functions and also the operational loads, as described by Kousuke and Junji (2006) and Lohrmann (2008).

As already explained, the traditional method uses an artificial force, that excites the connection (input) points between the sound source to the radiator structure and simultaneously measure the respective acoustic or vibrational response (output) in order to calculate the Tranfer Functions.

Normally, there is no space available to do that and this step becomes hard to implement. As a consequence, it can lead to unacceptable errors. Additionally, even for situations where the space is available in laboratory conditions, as soon as the system is placed *in situ*, the forces related to each path can not be measured anymore once the former space is not available.

This new methodology aims for a practical and robust way of processing data. In order to achieve that, acceleration transducers are used to measure the vibroacoustic signals at the input points. These transducers require less space than force transducers and can be easily placed where it is necessary while the system is under operational condition. The operational TPA, illustrated in Fig. 1, can be summarized by the following three steps:

1. Measure n times, m input (acceleration $\ddot{x}_{ij}$) and the output (global sound pressure p_i) values *in situ*. It is recommended that n be the largest possible.
2. Apply the Principal Component Regression method in order to obtain the transfer coefficient vector $\{H\}_{m1}$;
3. Choose one set of m acceleration data from the execution of item (1) and multiply it by the transfer coefficients h_j obtained from the item (2).

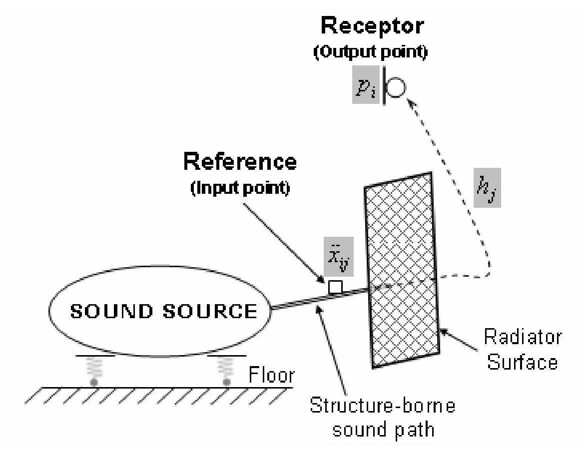

Figure 1. Illustration of the operational TPA methodology considering one source and one structure-borne sound path

The execution of item (1) generates n linear equations which produce an acceleration matrix $[\ddot{X}]_{nm}$ associated with each measurement and transfer path. The global sound pressure matrix $\{P\}_{n1}$ is measured simultaneously on an arbitrary point and the resulting system of equations (2), which have the transfer coefficient vector $\{H\}_{m1}$ as the unknown, is presented below:

$$\{P\}_{n1} = [\ddot{X}]_{nm}\,\{H\}_{m1}\,. \tag{2}$$

If the number of measurements n is the same of transfer paths m, the acceleration matrix $[\ddot{X}]_{nm}$ will be square ($n = m$) and inversible, making it possible to obtain the solution for the transfer coefficient vector $\{H\}_{m1}$, as shown by the system of equations (3):

$$\{H\}_{m1} = [\ddot{X}]_{nn}^{-1}\,\{P\}_{n1}\,. \tag{3}$$

However, the precision of this system of equations becomes very fragile when there is noise in the acceleration and pressure signals, or even when there is a high correlation among the input signals (acceleration $\ddot{x}_{ij}$). In order to minimize this problem, a statistical methodology is applied known as Principal Component Analysis (PCA).

This methodology consists firstly in applying a procedure of matrix decomposition called Singular Value Decomposition (SVD) on the acceleration matrix $[\ddot{X}]_{nm}$. This procedure can be interpreted in different ways, as shown by Jackson (1991):

1. It transforms correlated variables into a set of uncorrelated ones that better expose the various relationships among the original data items;
2. It finds the best approximation of the original data using fewer dimensions.
3. It identifies and sorts the dimensions along which data exhibit the greatest variation;

As stated by Klema and Laub (1980), SVD is based on a Linear Algebra Theorem which establishes that a rectangular matrix can be splitted up into a product of three matrices, as shown by the Eq. (4):

$$[A]_{nm} = [U]_{nn}\,[S]_{nm}\,[V]_{mm}^{T}\,, \tag{4}$$

where $[U]^T[U] = [I]$ and $[V]^T[V] = [I]$; the columns of $[U]$ are orthonormal autovectors of $[A][A]^T$; the columns of $[V]$ are orthonormal autovectors of $[A]^T[A]$ and $[S]$ is a diagonal matrix that contains the square root of the eingenvalues of $[U]$ and $[V]$ sorted in a decreasing way. Note that the matrices dimensions will be omitted for clarity from this point of the text.

Hence, the principal component matrix $[T]$ is obtained from the acceleration data matrix $[\ddot{X}]$, as described by the following algebraic manipulation:

$[\ddot{X}] = [U][S][V]^{-1}$, multiply both sides by $[V]$, obtaining:

$[\ddot{X}][V] = [U][S][V]^{-1}[V]$ and

$$[\ddot{T}] = [\ddot{X}][V] = [U][S]. \tag{5}$$

The principal components which are lower than the other ones are considered noise and because of that are discarded by the algorithm. The singular values reduced matrix $[S_r]$, obtained as soon as the lower values are discarded, and the correspondent unitary matrices $[U_r]$ and $[V_r]$, also reduced matrices, are used to reassemble the original matrix. The principal components reduced matrix, obtained from the reduced unitary matrices is shown by the Eq. (6):

$$[T_r] = [\ddot{X}][V_r] = [U_r][S_r]. \tag{6}$$

The coeficient matrix $[C]$ of $[T_r]$ for the pressure signal matrix is obtained using:

$$[P] = [T_r][C]. \tag{7}$$

Hence, the relations from the equations (6) and (7) allows the formulation of the reduced coeficient matrix $[V_r]$, which transforms the input points acceleration signal matrix into the principal components matrix $[T_r]$ where the less important components were removed, and the coefficient matrix $[C]$ transforms $[T_r]$ on the pressure response matrix $[P]$.

The reduced matrices obtained from the decomposition procedure can be used to obtain the transfer function between the measured acceleration at the input point j and the sound pressure output response, as described by the following steps:

If $[H] = [\ddot{X}]^{-1}[P]$ and $[P] = [T_r][C]$, hence

$$[\ddot{H}] = [\ddot{X}]^{-1}[T_r][C]. \tag{8}$$

If $[T_r] = [\ddot{X}][V_r]$ and $[V_r] = [\ddot{X}]^{-1}[T_r]$, hence

$$[H] = [V_r][C]. \tag{9}$$

If $[C] = ([T_r]^T[T_r]^{-1})[T_r]^T[P]$, hence

$$[H] = [V_r]([T_r]^T[T_r]^{-1})[T_r]^T[P]. \tag{10}$$

Considering that $[T_r] = [U_r][S_r]$, then premultiply it by $[T_r]^T$, obtaining:

$$[T_r]^T[T_r] = [T_r]^T[U_r][S_r]. \tag{11}$$

Premultiply equation (11) by $([T_r]^T[T_r])^{-1}$, obtaining:

$$[S_r]^{-1}[U_r]^T = ([T_r]^T[T_r])^{-1}[T_r]^T, \tag{12}$$

finally resulting:

$$[H] = [V_r][S_r]^{-1}[U_r]^T[P]. \tag{13}$$

The Eq. (13) is used to obtain the transfer coefficients h_j. The main advantages observed when using this new methodology for transfer path quantification can be summarized by the following items:

1. It reduces the number of transfer functions (transfer coefficients) to be obtained and, as a consequence, it reduces the negative effect produced by sources of error associated with the great amount of measurements;
2. The data can be measured in operational conditions, giving the possibility to obtain the real characteristic of vibration;
3. Adaptability, once this technique can be applied using only transducers for output and input signals, which are easily applicable considering the equipment characteristics to be analyzed.

2. EXPERIMENTAL SET-UP FOR THE OPERATIONAL TPA

2.1 Presentation

The basic idea of this experiment consists of reproducing a vibroacoustic system with the transfer paths between the vibration source and the radiator surface well established. An aluminum plate (0.5x0.5x0.005m) and an easel were used during the experiment. Two electrodynamic shakers, two force transducers and two accelerometers from Bruel&Kjaer, as well as a signal analyzer from LMS International were also used.

It was possible to measure acceleration, force and pressure signals simultaneously and compare both traditional and operational TPA with the results obtained from direct measurements of sound pressure. In order to get the reference signals, the electrodynamic shakers were applied on the aluminum plate separately to obtain the reference partial response and together for the reference total response.

The white noise was used independently for both electrodynamic shakers as an input signal in the range of 1.56 Hz - 12.8 kHz and the frequency step was 1.56 Hz. After getting the data in a narrow band and post-processing by the TPA algorithm, the results were converted to 1/3 octave band from 0.2 kHz to 10 kHz.

Two output channels at the signal analyzer were used for the electrodynamic shakers and either the signals as the gain were different between each other. The right shaker had twice the gain of the left shaker. Two set-ups were tested as described below:

- Set-up (A): Accelerometers and force transducers were placed aligned to the electrodynamic shaker probes. The first one at the opposite side of the excitation point on the plate. The second one at the same side of the excitation point on the plate.
- Set-up (B): It is very similar to the set-up (A), but differs from the fact that the accelerometers were placed on the periferic region of the aluminum plate.

Both set-ups can be visualized in a general way by Fig. 2 and in detail by Fig. 3. Fig. 2 shows an aluminum plate hung in an easel, placed inside a semi-anechoic chamber. Two electrodynamic shakers, accelerometers and force transducers were connected to it at arbitrary points.

A $1/2$ inch microphone was placed in a pedestal 1 m distant and 0.7 m height from the aluminum plate and the floor respectively. The microphone position establishes the output point. This configuration made it possible to control separately the vibratory energy injected on the structure, simulating the vibration input points of a real system. It was also possible to measure separately the noise produced by the electrodynamic shakers connected to the aluminum plate.

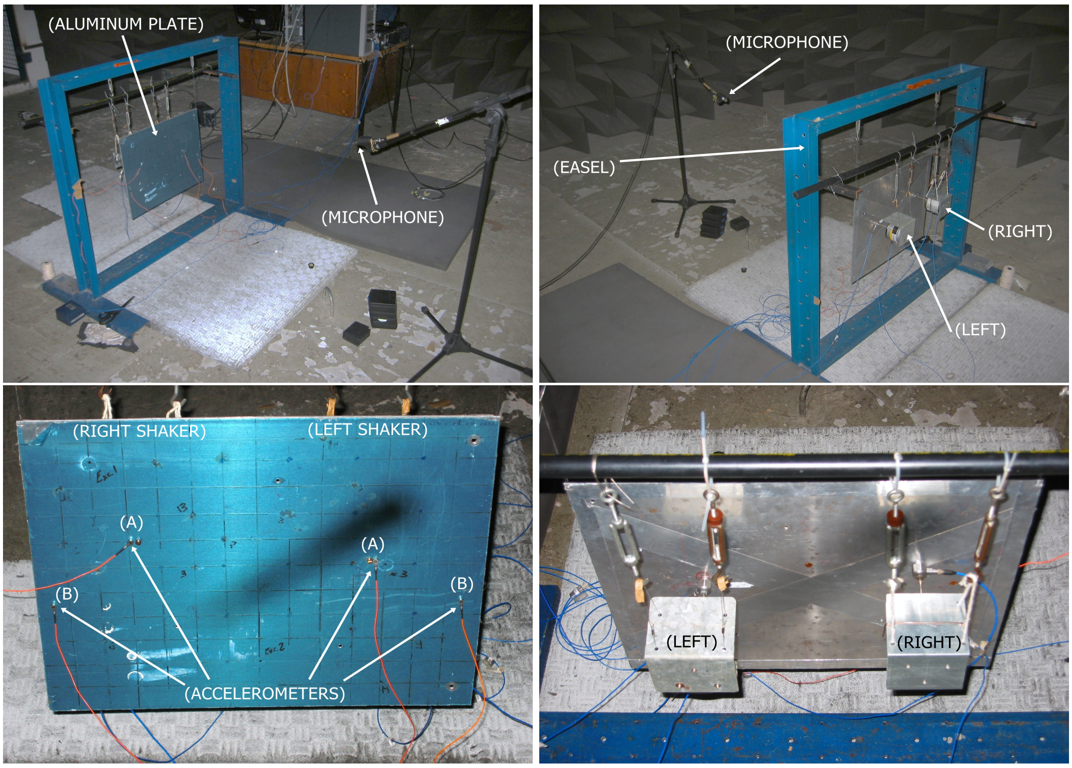

Figure 2. Set-up (A) and (B) for the TPA validation using a semi-anechoic chamber

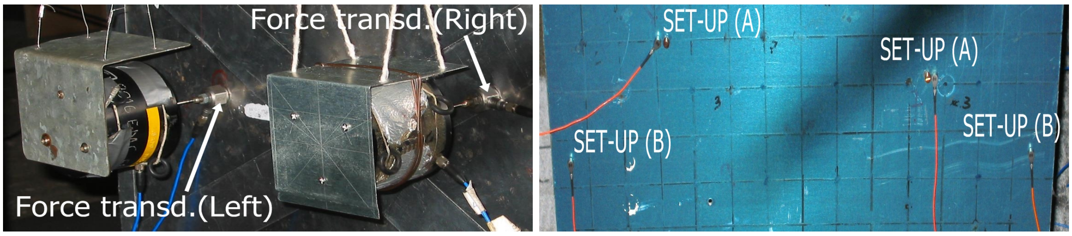

Figure 3. Force transducer and accelerometer positioning in detail for set-up (A) and (B)

This algorithm uses the accelerometer signals as the data associated with that transfer path, providing the transfer coefficient vector and also the path loads in operational condition. The microphone is responsible for capturing the global signal for both the operational and experimental conditions and, in this way, it is also used to build the transfer coefficient vector. The data provided by these transducers was used as an input for a computer algorithm implemented for the operational TPA.

2.2 Results

The result in terms of sound pressure level (SPL) for the set-up (A) is shown by Fig. 4 to Fig. 6. Basically, three types of curve were plotted for both set-ups (A) and (B):

1. The one obtained from direct measurement of the sound pressure;
2. The one which is obtained from the Operational TPA;
3. The one which is obtained from the Traditional TPA.

It is important to consider that the reference curves shown in Fig. 4 and Fig. 5 provide information about the noise path, however, they should not be considered as absolute reference curves since they were obtained by direct measurement of the sound pressure related to each eletrodynamic shaker working independently. They do not consider the constructive and destructive wave interference when the two power sources (two drivers) excite the surface of the plate simultaneously.

Figure 4 shows a good agreement among the curves, despite some discrepancies for the traditional TPA in the bands of 0.2 kHz; 1.25 kHz; 1.6 kHz and 2.5 kHz.

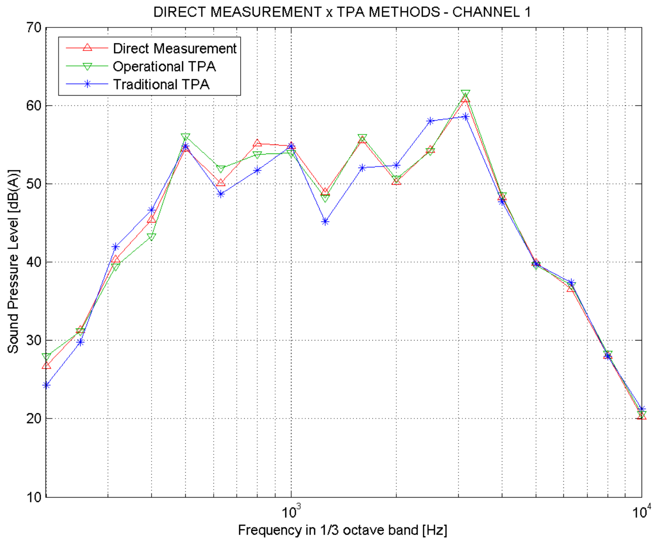

Figure 4. Partial Result - Right Shaker

Figure 5 shows a lower agreement among the curves when compared with the right shaker, which can be justified by the lower power injected by the left shaker.

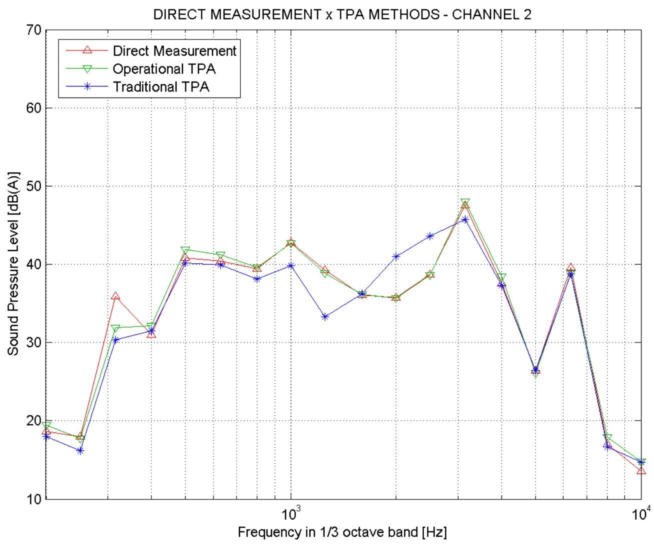

Figure 5. Partial Result - Left Shaker

Figure 6 shows the overall results, i.e. the curve highlighted as Operational TPA was obtained by a sum of the complex sound pressures for each path calculated by the operational TPA algorithm. The curve highlighted as Traditional TPA was obtained by a sum of the complex sound pressures for each path calculated by the traditional TPA algorithm. The curve highlighted as Direct Measurement is the sound pressure level (SPL) measured with both electrodynamic shakers in operation.

A better agreement can be seen than those presented in Fig. 4 and Fig. 5, with differences up to 4 dB (0.2 kHz; 1.25 kHz and 2.5 kHz) at a maximum. The differences revolve to 2 dB or less for most frequency bands. The values obtained by traditional TPA and the algorithm of operational TPA are almost identical for the bands above 4 kHz.

The better quality of this result can be justified by the sum of the partial result taking into account the sound wave phase obtained for each path. The sound partial vectors were added up to build the total sound pressure vector for each frequency analyzed.

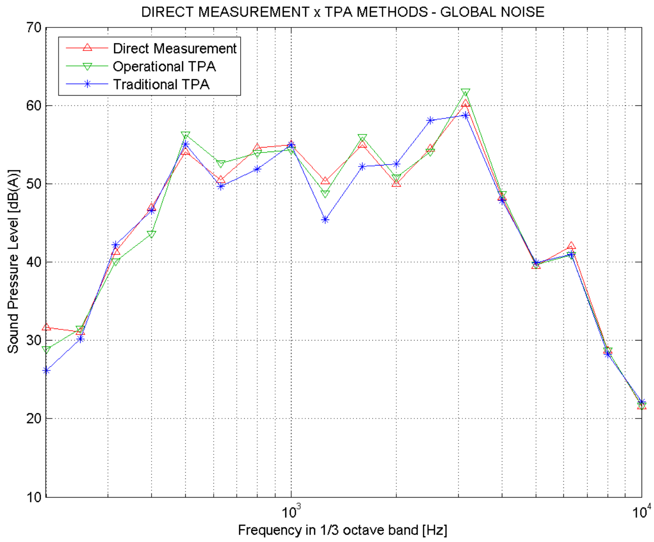

Figure 6. Total value - both electrodynamic shakers

The results in terms of sound pressure level (SPL) of the set-up (B) are represented by Fig. 7 to Fig. 9. All results obtained using the TPA algorithm are compared with reference values obtained through direct measurements.

These results show greater discrepancies among the curves than those presented by the set-up (A). However, these differences can be justified by the following evidence:

1. The accelerometers were placed on the periferic region of the aluminum plate and consequently a lower signal-to-noise ratio is established between the force and acceleration signals;

2. The accelerometers are more subjected to noise interference.

The curves shown by Fig. 7 present a medium level of agreement, except for the band of 0.63 kHz which have a discrepancy of 9 dB. The other frequency bands show differences up to 6 dB or lower.

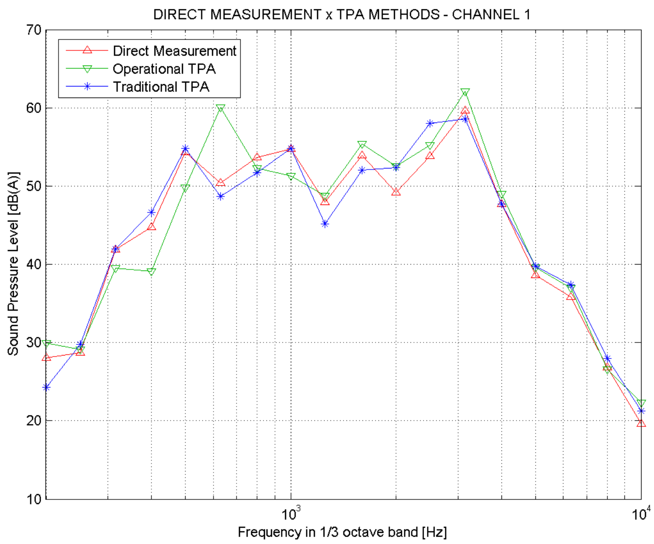

Figure 7. Partial Result - Right Shaker

In the case of the left shaker result, shown in Fig. 8, the agreement among the curves is lower than the right shaker, with maximum difference of 8 dB in the bandwidth of 0.4 kHz. It is important to remember that the power injected by the left shaker was much lower than that injected by the right shaker.

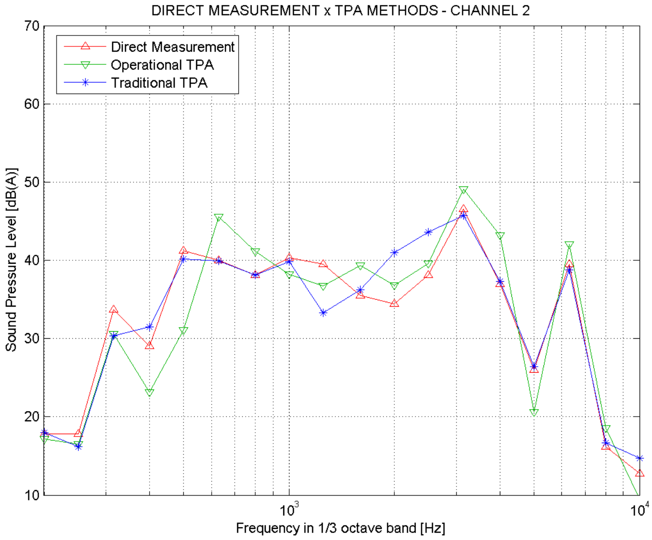

Figure 8. Partial Result - Left Shaker

A better quality is achieved when both electrodynamic shakers are injecting force. This can be justified by the sum of the partial result taking into account the sound wave phase obtained for each path and then adding up the sound pressure vector of the frequencies analyzed.

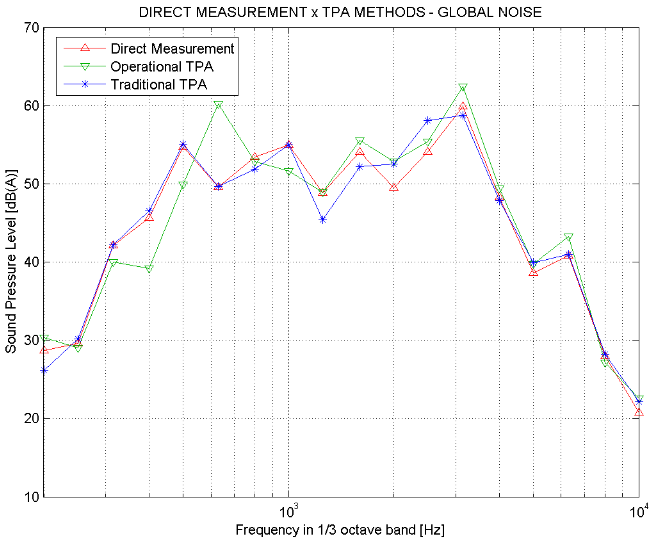

Figure 9. Total value - both electrodynamic shakers

3. CONCLUSION

The operational TPA is a new methododology which has been used in the last years. It makes possible to quantify the partial acoustic contribution associated with different propagation paths through complex structures in a practical way. The accurate determination of transfer coefficients among sources and receivers, which means to apply the Principal Component Analysis, is the main step when applying TPA methodology.

It can be concluded from this work that there are some advantages when comparing the operational and traditional TPA:

1. It becomes easier to place the transducers at the excitation points;
2. The number of transfer functions (coefficients) to be measured decrease;
3. There is a minimum system impedance changing when using accelerometers;
4. Experimental and operational set-up can be the same;

It is important to point out that a normalization procedure could be necessary to adequate the data to be processed by the SVD algorithm, if the measured input signals have different units.

Finally, the accelerometer placement shall be as near as possible to the place where the force is being applied for the analyzed transfer path.

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