# NUMERICAL COMPUTATION OF THE EFFECTIVENESS-NUMBER OF TRANSFER UNITS FOR SEVERAL CROSS-FLOW HEAT EXCHANGERS WITH DIFFERENT FLOW ARRANGEMENTS

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***Abstract.*** *In this paper is used a recently developed numerical solution methodology for calculation of thermal parameters of cross-flow heat exchangers with complex flow arrangements (Navarro and Cabezas-Gómez, 2005). According to this technique the heat exchanger is discretized into small control volume elements following the tube-fluid circuiting. Each elements works as a one pass mixed unmixed cross-flow heat exchanger, where are the energy conservation equations for both streams using local averaged values of temperature and assuming constant physical properties and heat transfer coefficients. The main aim of the paper is the graphical representation of the effectiveness-number of transfer units for several cross-flow heat exchangers, considering either the standard and complex geometrical configurations. The results are validated comparing simulation results with values obtained form available analytical relations for the ε-NTU data of known heat exchangers configurations. Afterwards, are presented results for more complex configurations, for which are not disposable analytical relation in the open literature. The results show very acceptable values of the ε-NTU for all the considered configurations. The present works allows analyzing newly proposed cross-flow heat exchangers configurations using one of the most useful parameter used in heat exchangers design, namely, the well known ε-NTU parameters.*

***Keywords:*** *Effectiveness-number of transfer units; ε-NTU; Cross-flow heat exchangers; Numerical simulation*

## 1. Introduction

Due to wide range of design possibilities, simple manufacturing, less maintenance and low cost cross-flow heat exchangers are extensively used in industries e.g. petroleum, petrochemical, air conditioning, food storage, and others. Such heat exchangers are especially well suited for gas cooling and heating. The extensive use of these apparatus has generated the need for calculation method that accurately predicts their performance (Bes, 1996).

A comprehensive review of solution methods for determining effectiveness ($\varepsilon$ or $P$) - number of transfer units (NTU) relationships for two-fluid heat exchangers with simple and complex flow arrangements is presented by Sekulic *et. al.* (Sekulic, Shah and Pignotti, 1999). The methods were categorized by the authors as: analytical methods for obtaining exact solutions, approximate methods, curve-fit to the results from the exact solutions, numerical methods, matrix formalism, and methods based on exchanger configuration properties, as the use of flow reversal symmetry of exchanger configurations. In conformity to the authors continuing efforts to design more efficient systems, more compact exchangers, or specific operating conditions may require effectiveness-NTU formulae for a new heat exchanger, not reported in the literature. Using some of these methods Pignotti and Shah (Pignotti and Shah, 1992) obtained eighteen effectiveness-NTU explicit formulas for new arrangements.

The main aim of this article is to provide a new numerical methodology for thermal performance calculation of cross-flow heat exchangers. The proposed methodology is based on physical concepts and it is characterized by the division of the heat exchanger in a number of small and simple one-pass mixed-unmixed cross-flow heat exchangers. The present approach allows obtaining effectiveness data for new configurations.

At present, solutions can be getting only for configurations where the external fluid is unmixed (considered here as air flowing over finned tube bundles) and the tube fluid is well mixed in each tube cross section and unmixed between passes. Then, heat exchangers with one to several tubes can be analyzed, including different tube fluid circuiting configurations. Next, in section 2, it is presented the proposed numerical methodology for thermal performance parameters calculation. In section 3 simulation results are presented and compared with available solutions from literature, considering cross-flow heat exchangers with simple and complex flow arrangements. Finally, the conclusions of the paper are presented in section 4.

## 2. Methodology Development

### 2.1 Governing Equations for one pass cross-flow Heat Exchanger

The governing equations presented in this section are those developed for a cross-flow heat exchangers with one fluid mixed and another unmixed following (Kays and London, 1998). These are the basic equations applied in the proposed numerical solution methodology. Figure 1 shows the temperature conditions for one pass cross-flow mixed-unmixed heat exchanger having one row. It is considered that tube side fluid is perfectly mixed in the tube cross section and external fluid is perfectly unmixed, i.e., there are fins in the airside. It is also assumed for convenience that the mixed fluid is hot and the unmixed is cold. The equations are also valid when the unmixed fluid is hot and the mixed is cold. For this condition the subscripts hot and cold must be interchanged.

Figure 1(a) illustrates the hot and cold temperature variations along a tube of length L whereas Fig. 1(b) shows both temperature variations along the cold fluid flow length in the differential length (dx). In this infinitesimal section, the cold mass flow rate is small and therefore the hot fluid temperature is constant. An energy balance in the differential length, dx, for the hot and cold fluids can be written as:

$$\delta q = -C_h \cdot d T_h \tag{1}$$

$$\delta q = dC_c \cdot \Delta T_c \tag{2}$$

where $\Delta T_c = (T_{c,o} - T_{c,i})$ is the mean temperature variation of the cold fluid in the differential length, *dx, q* stands for the heat transfer rate, and $C_h$ and $C_c$ represent the heat capacity rate of the hot and cold fluids, respectively.

Using the fact that in the differential section *dx*, the cold mass flow rate is small in comparison with the hot mass flow rate, must be assumed that the hot capacity rate, $C_h$, is constant in the section and a differential heat capacity rate ratio, $C^*$, must be expressed as:

$$dC^* = \frac{dC_c}{C_h} \rightarrow 0 \tag{3}$$

Considering Eq. (3) and the temperature conditions (Fig. 1b), a condenser type of effectiveness expression is applicable. Then, using the effectiveness definition, a parameter Γ that expresses the 'local effectiveness' in the differential length *dx* can be written as (Kays and London, 1998)

$$\Gamma = \frac{\Delta T_c}{T_h - T_{c,i}} = 1 - e^{\frac{-UdA}{dC_c}} \tag{4}$$

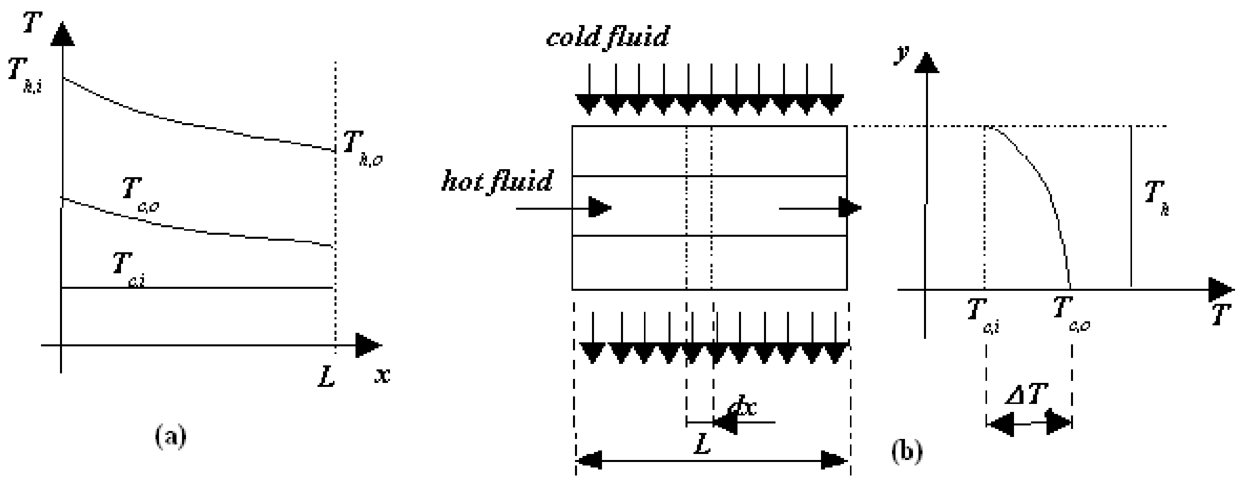



Figure 1. (a) Air and fluid temperature variations in the longitudinal direction with respect to the fluid flow; (b) cross-flow air temperature variation in a differential volume element of the heat exchanger.

Assuming that both the cold flow and heat transfer area *A* distributions are uniform, the following relations are valid

$$\frac{dC_c}{dA_{fr}} = \frac{C_c}{A_{fr}} = const \tag{5}$$

$$\frac{dC_c}{dA}=\frac{C_c}{A} \tag{6}$$

In Eq. (5) $A_{fr}$ represents the exchanger total frontal area. Thus, along the tube length $L$

$$\Gamma=1-e^{-\frac{UA}{C_c}}=const \tag{7}$$

where $U$ is the overall heat transfer coefficient. Combination of the equations (1), (2) and (4) and separation of variables lead to

$$\frac{dT_h}{T_h-T_{c,i}}=-\Gamma\, dC^*=-\Gamma\frac{C_c}{C_h}\frac{dA_{fr}}{A_{fr}} \tag{8}$$

In Eq. (8) it should be noted that $C_c$, $C_h$ and $A_{fr}$ are the total magnitudes and are not variables. As mentioned, the developed governing equations are valid for one pass cross-flow heat exchanger, one fluid mixed and another unmixed. For this kind of heat exchanger, the integration of Eq. (8) can easily be done analytically obtaining the $\varepsilon$-$NTU$ relations (see Eq. 15 latter. In these relations $\varepsilon$ represents the conventional heat exchanger effectiveness and $NTU$ stands for the number of transfer units). Nevertheless, cross-flow heat exchangers for engineering applications, commonly, have a complex flow arrangement with several circuits and rows. For these exchangers the solution of this system of equations (Eqs. 1 to 8) is not trivial. In these cases the performance of derivation and integration founded in Eq. (8) is difficult due to two reasons: due to a non-validity of Eqs. (6 and 7) for the overall heat exchanger area, and due to a variation of temperature distribution of the cold (unmixed) fluid, $T_{c,i}$, in each row of the heat exchanger. This leads to an application of numerical procedure to obtain a desired solution.

**2.2 Numerical solution methodology**

In this paper is developed a new numerical solution methodology for cross-flow heat exchangers thermal performance calculation based on application of the Eqs (1-8). The methodology consists in the following mean steps:

First step - the heat exchanger is divided into a set of three dimensional control volumes called elements identified by the triplet $(i,j,k)$. The indices $1 \le i \le N_e$; $1 \le j \le N_t$; and $1 \le k \le N_r$ represent the element position along a particular tube, the tube in each row, and the row, respectively. It should be noted that each element is modeled as a mixed-unmixed heat exchanger schematized in Fig. 2. The variables $N_e$, $N_t$, and $N_r$ represent the number of elements per tube, the of tubes per row and the number of rows in the heat exchanger, respectively.

Second step - the system of governing equations of section 2.1 are integrated and applied in each element separately. This leads to a system of algebraic equations for each element and consequently for a whole heat exchanger.

Third step - the above system of equations for a whole heat exchanger is solved iteratively. It is done following tube fluid circuits along the heat transfer surface through indices $i,j,k$ management.

As the mathematical model employed (see section 2.1) is valid for a one pass mixed-unmixed cross-flow heat exchanger, the size of all elements must be sufficiently small to ensure this condition. This means that should be used a large enough number of elements. Therefore, each element work as an independently heat exchanger connected to others by the tube fluid circuits. Solving iteratively the integrated algebraic equations in each of these small heat exchangers (i.e., elements) it is obtained the temperature distribution in the whole heat exchanger. The thermal performance parameters are founded through application of its definitions.

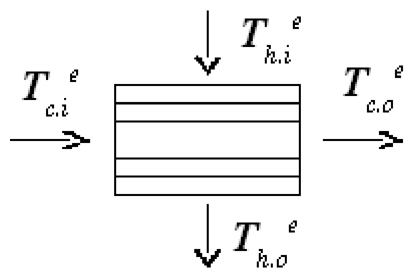



Figure 2. Scheme of a mixed-unmixed heat exchanger.

**2.3 Algebraic equation system for an element**

Next it is presented the algebraic equations for each element that can be obtained from the integration of section 2.1 equations.

Firstly due to the small element size and validity of Eq. (3), it is assume that the hot fluid (mixed) temperature has a linear variation along a control volume, whereas the cold (unmixed) has an exponential variation.

This way, the hot average element fluid temperature is expressed as:

$$T_h^e = 0.5(T_{h,i}^e + T_{h,o}^e) \tag{9}$$

where the superscript $e$ is associated with a specific element ($i,j,k$).
Now integration of the Eq. (1) in the element leads to

$$q^e = -C_h^e(T_{h,o}^e - T_{h,i}^e) \tag{10}$$

In obtaining the heat balance for the cold fluid, thought the integration of Eq. (2), and using the Eq. (5), the following expression can be written

$$q^e = \Delta T_c^e \int_e dC_c = \Delta T_c^e \int_e \frac{C_c}{A_{fr}} dA_{fr} = \Delta T_c^e C_c^e \tag{11}$$

where $C_c$ and $A_{fr}$ are the total magnitudes and are not variables and $\Delta T_c^e = T_{c,o}^e - T_{c,i}^e$ represents the mean cold fluid temperature difference in the element.
To close the algebraic equation system integration of Eq. (4) in the element results in:

$$\Gamma^e = \frac{\Delta T_c^e}{(T_h^e - T_{c,i}^e)} = 1 - e^{-\frac{(UA)^e}{C_c^e}} \tag{12}$$

The last term of Eq. (12) is equal to that of Eq. (7). This is obtained because it is assumed that each element is equal to a one pass mixed/unmixed cross-flow heat exchanger. Therefore, the equations of section 2.1 are valid.
The relations (9) to (12) represent a closed system of equations to be solved for each element for five unknowns. There are five equations for five unknowns, namely: $q^e$, $\Gamma^e$, $T_h^e$, $T_{h,o}^e$ and $\Delta T_c^e$, knowing $T_{h,i}^e$, $T_{c,i}^e$, $(UA)^e$, $C_c^e$ and $C_h^e$. To solve these equations for the whole heat exchanger, i.e. for all the elements interconnected, is needed an iterative procedure. In the following section the computational procedure are proposed.
Before that, it is shown that the above system of five equations must be rearranging to obtain the following two equations for temperature calculation in each element:

$$T_{c,o}^e = \frac{A + 2(1-\Gamma^e)}{2+A} T_{c,i}^e + \frac{2\Gamma^e}{2+A} T_{h,i}^e \tag{13}$$

$$T_{h,o}^e = \frac{2-A}{2+A} T_{h,i}^e + \frac{2A}{2+A} T_{c,i}^e \tag{14}$$

where $A = C_c^e\Gamma^e/C_h^e$. The Eqs. (13 and 14) are used in the procedure described below.

During derivation of the governing equations (section 2.1) and its discretization (present section) the following hypothesis were assumed in agreement with literature work (Shah and Sekulic, 1998). The heat exchanger operates under steady state conditions. The heat losses to the surroundings are negligible (i.e. the heat exchanger is adiabatic). There are no thermal energy sources and sinks in the heat exchanger walls or fluids. The tube fluid is perfectly mixed in each cross sectional area varying linearly through each element; and the external fluid (unmixed) is uniformly distributed at the inlet and outlet in each element, being their temperatures mean values in these regions. There are no phase changes in the fluid streams. The physical properties and heat transfer coefficients are constant for the heat exchanger surface. Considering these hypotheses, it is proposed a methodology that allows determining theoretical efficiency data. However, these data are very useful for heat exchanger design and rating procedures (e.g., (Shah and Pignotti, 1993)).

### 2.4 Procedure for thermal parameters calculation of heat exchanger

The proposed model can be used to simulate simple geometries like parallel linear cross-flow configurations and complex geometries involving multi-pass, counter cross-flow with several circuits arrangement configurations of cross-flow heat exchangers, allowing the computation of various parameters like *ε-NTU* relations, mean-temperature difference curves, cold and hot fluid temperature distributions, heat transfer area, and friction and heat transfer coefficients. This work will be related mainly with the obtainment of *ε-NTU* graphics for several cross-flow heat exchangers circuiting. During this derivation it is assumed that the heat exchanger and the tube curves are adiabatic; the mixed fluid inlet conditions are homogenous for each element; and the unmixed fluid is uniformly distributed.
To obtain the *ε-NTU* relations, computational algorithms, shown in Tab. 1, have been developed. The present algorithms consider that the hot fluid is mixed and cold is unmixed. Nevertheless, it is also valid when the hot fluid is

unmixed and cold is mixed, as assumed before. In this case the subscripts (hot and cold) are interchanged in all algorithms' referenced equations.

Table 1 shows the algorithm for calculation of the effectiveness, $\varepsilon$. The first step consists to load geometric data for a specific heat exchanger. This data file contains the necessary information like number of rows, tubes, circuits and the flow arrangement for the mixed fluid. The algorithm inputs (steps 1.2 and 1.3) are $NTU$, $C^*$, and $C_{min}$=($C_c$ or $C_h$). The effectiveness $\varepsilon$ is the algorithm output and it is computed in step 1.8. It is noted that the $T_{c,i}$, $T_{h,i}$ and $UA$ values, introduced in 1.4 (Tab. 1), are arbitrarily chosen for simulation purposes, since $\varepsilon$ depends only of $NTU$, $C^*$, and flow arrangement. The $(UA)^e$, $C_c^e$ and $C_h^e$ values are calculated in 1.5 and 1.6 and they are used to calculate the parameter $\Gamma^e$ for each element, according to Eq. (13). It should be noted that the procedure presented here (step 1.6) is valid only when the unmixed heat fluid capacity is minimum in the element, i.e., $C^e_{unmixed}/C^e_{mixed} \leq 1$. The number of element in the discretization process, $N_e$, is calculated based on this relation. This means that for the minimum heat capacity rate of heat exchanger $C_{min} = C_{unmixed}$, there is no restriction for the $N_e$ value, but when $C_{min} = C_{mixed}$, the value $N_e$ should be large enough to guarantee the validity of the above relation. In step 1.7, the heat exchanger temperature distribution is calculated iteratively and the convergence criteria is related to cold fluid outlet temperature ($T_{c,o}$). Nevertheless the same convergence criteria can be related to heat transferred rate $q$ (step 1.8). This criterion is also checked in the computational program developed.

Table 1 Effectiveness $\varepsilon$ algorithm.

1.1 Read a heat exchanger geometry from file
1.2 Read $NTU$, $C^*$
1.3 Choose $C_{min} = (C_c$ or $C_h)$
1.4 Given $T_{c,i}$, $T_{h,i}$ and $UA$
1.5 Compute $(UA)^e$ according to

$$(UA)^e = \frac{UA}{N_e\, N_t\, N_r}$$

1.6 Compute $C_c^e$ and $C_h^e$
If $C_{min}=C_c$

$$C_c^e = \frac{UA}{NTU \cdot N_e N_t} \quad \text{and} \quad C_h^e = \frac{UA}{NTU \cdot C^* N_c}$$

If $C_{min}=C_h$

$$C_c^e = \frac{UA}{NTU \cdot C^* N_e N_t} \quad \text{and} \quad C_h^e = \frac{UA}{NTU \cdot N_c}$$

1.7 Compute temperature distribution iteratively
Compute initial temperature distribution according to Tab. 2
Do
Compute the sum $S = \sum_{i,j,N_r} T^e_{c,o}$
Compute temperature distribution (Tab. 2).
Compute the new value of the sum $S_{new} = \sum_{i,j,N_r} T^e_{c,o}$
While $\frac{|S_{new} - S|}{S} < Tolerance$
1.8 Compute effectiveness of heat exchanger

$$T_{c,o} = \frac{1}{N_t N_e} \sum_{i,j,N_r} T^e_{c,o}$$

$$T_{h,o} = \frac{1}{N_c} \sum_{\text{each last circuit element}} T^e_{h,o}$$

$$q = \sum_{i,j,k} q^e \quad \text{or} \quad q = -C_h (T_{h,o} - T_{h,i}) \quad \text{or} \quad q = C_c (T_{c,o} - T_{c,i})$$

Compute $\varepsilon$ using Eq. (1)

Table 2 presents the algorithm for the temperature distribution calculation. It should be noted that it is not computed the real temperature distribution and for these reason the parameter $\Gamma^e$ is constant for all elements of the heat exchanger being calculated in the item 2.1. In case to be consider the computation of the real temperature distribution, the parameters $\Gamma^e$ as well as $(UA)^e$, $C_h^e$ and $C_c^e$ must be calculated for each element (item 2.2), using local properties and heat transfer coefficients. For both cases, the cold and hot temperatures in each element are computed using the model proposed Eqs. (10 trough 13) and these values are used to update the inlet temperatures for the next element of the circuiting. This update procedure depends of the geometry and is done for each circuit involving all its elements. It is the procedure that permits to simulate several types of cross-flow heat exchangers, i.e., with one fluid mixed and another unmixed, both fluids mixed or unmixed.

Table 2 Temperature distribution algorithm.

```
2.1 Compute Γe using Eq. (13)
2.2 Compute temperatures following circuiting
For 1 to Nc
        Update temperatures for cold fluid side
        While not(end of a circuit)
                Compute Tc,oe, Th,oe, qe using equations Eqs. (10)-(13).
                Go to next element
                Update temperatures for cold and hot fluids sides
        End while
End for
```

### 3. Results

In this section are discussed results obtained with the developed methodology. Firstly, simulation data are presented for one pass cross-flow exchangers with *n* rows. These results are compared with the available analytical relations showing a very good agreement. Next, more complex heat exchanger configurations are tested through comparison with literature proposed solutions obtaining a very good agreement. Finally, for these configurations new effectiveness data are provided. To identify the heat exchanger geometries analyzed in the paper it is used the notation $G_{Np,Nrp}$ from (Shah and Pignotti, 1993). The two subscripts represent the true number of passes (i.e. the over-and-under passes) and the number of rows per pass, respectively.

#### 3.1 Cross-flow heat exchanger with one pass and *n* rows

In this subsection the proposed methodology is validated and analyzed through comparison of the simulated results with $\varepsilon$-NTU analytical relations for one pass and n rows cross-flow heat exchanger. The six analytical relations considered are included in Tab. 3 (ESDU 86018,1991, Stevens, Fernandez and Woolf, 1957, Baclic and Hegg, 1985). For an unmixed/unmixed flow arrangement two relations were considered. One, Eq. (19), is the infinite series solution obtained by Mason and used in (Stevens, Fernandez and Woolf, 1957) and (Baclic and Heggs, 1985). Another, Eq. (20), is extensively used in literature and is taken from (ESDU 86018, 1991). An explanation of the origin of some terms of this equation can be found in (DiGiovanni and Webb, 1989).

Table 3. $\varepsilon$-*NTU* relationships for one pass cross-flow configurations with one or more rows (Eqs. 15-18 from (ESDU 86018, 1991) and (Stevens, Fernandez and Woolf, 1957); Eq. 19 from (Stevens, Fernandez and Woolf, 1957) and (Baclic and Heggs, 1985); Eq. 20 from (ESDU 86018, 1991).

| $N_r$ | Side of $C_{min}$ | Relation | Equation |
|---|---|---|---|
| 1 | A | $\varepsilon_A = 1 - e^{-(1-e^{NTU_A \cdot C_A^*})/C_A^*}$ | (15) |
| 2 | A | $\varepsilon_A = 1 - e^{-2K/C^*}\left(1+\frac{K^2}{C^*}\right)$, $K = 1 - e^{-NTU \cdot C^*/2}$ | (16) |
| 3 | A | $\varepsilon_A = 1 - e^{-3K/C^*}\left(1+\frac{K^2(3-K)}{C^*}+\frac{3K^4}{2(C^*)^2}\right)$, $K = 1 - e^{-NTU \cdot C^*/3}$ | (17) |
| 4 | A | $\varepsilon_A = 1 - e^{-4K/C^*}\left(1+\frac{K^2(6-4K+K^2)}{C^*}+\frac{4K^4(2-K)}{(C^*)^2}+\frac{8K^6}{3(C^*)^3}\right)$, $K = 1 - e^{-NTU \cdot C^*/4}$ | (18) |
| ∞ | both fluids unmixed | $\varepsilon = \frac{1}{C^* NTU}\sum_{n=0}^{\infty}\left\{\left[1-e^{-NTU}\sum_{m=0}^{n}\frac{(NTU)^m}{m!}\right]\left[1-e^{-C^*NTU}\sum_{m=0}^{n}\frac{(C^* NTU)^m}{m!}\right]\right\}$ | (19) |
| ∞ | both fluids unmixed | $\varepsilon = 1 - e^{\left[NTU^{0.22}\left(e^{-C^* NTU^{0.78}}-1\right)/C^*\right]}$ | (20) |

Fluid A mixed, Fluid B unmixed, $C_A^*=1/C_B^*$, $\varepsilon_B = \varepsilon_A C_A^*$, $NTU_B = NTU_A C_A^*$, $C_A^*=C_A/C_B$

We compare the maximum relative error between analytical ($\varepsilon_t$), Eqs. 15-18 (Tab. 3), and simulated ($\varepsilon_s$) effectiveness values, $G_{1,1}$, $G_{1,2}$, $G_{1,3}$, and $G_{1,4}$, respectively. This comparison is performed considering the available analytical expressions up to four rows. A maximum relative error is obtained from 1111 effectiveness values calculated in the following intervals $0 \le C_i^* \le 1$ and $0 \le NTU_i \le 10$ with 0.1 increment, respectively. Results shows that the maximum relative error is very small for all cases, i.e., relative error of the order of $10^{-6}$ %, indicating a perfect agreement between analytical and simulated values and the rigorousness of the present methodology. It should be emphasized that the simulation results obtained by the program are very accurate for any number of tube rows as we can see next.

Table 4 shows the convergence history of the simulation results to the infinite analytical solution, Eq. (19), for an unmixed-unmixed arrangement. Three main points can be emphasized from this comparison. First, it is observed that the maximum relative error decreases with the increase of the number of tube rows, being equal to 0.0082 % for $N_r$ = 100. This expected behavior shows the high accuracy of the developed methodology and indicates that if it is desired a smaller error a number of tube rows should be increased. This leads to the second main point of this comparison related to the unmixed fluid concept. From Tab. 4 it is seen that for a cross-flow heat exchanger have a completely unmixed-unmixed flow distribution it should have a higher enough number of tube rows to guarantee a small relative error in effectiveness.

Table 4. Comparison between model prediction and infinite series solution, Eq. (19).

| | | **Average relative error (%)* and maximum relative error (%)** | | | |
|---|---|---|---|---|---|
| $N_r$ | **Geometry** | $C_{min}=C_{air}$ | | $C_{min}=C_t$ | |
| 5 | $G_{1,5}$ | 0.63 | 2.88 | 0.45 | 2.89 |
| 6 | $G_{1,6}$ | 0.44 | 2.10 | 0.32 | 2.10 |
| 7 | $G_{1,7}$ | 0.33 | 1.56 | 0.24 | 1.56 |
| 8 | $G_{1,8}$ | 0.25 | 1.22 | 0.18 | 1.22 |
| 9 | $G_{1,9}$ | 0.20 | 0.97 | 0.14 | 0.97 |
| 10 | $G_{1,10}$ | 0.16 | 0.79 | 0.12 | 0.79 |
| 20 | $G_{1,20}$ | 0.04 | 0.20 | 0.03 | 0.20 |
| 50 | $G_{1,50}$ | 0.006 | 0.033 | 0.005 | 0.033 |
| 100 | $G_{1,100}$ | 0.0016 | 0.0082 | 0.0012 | 0.0082 |

*Average relative error = $\frac{1}{N}\sum_{1}^{N}\frac{|\varepsilon_s-\varepsilon_t|}{\varepsilon_t}100$

The third main point is related to the fact that the analytical solution is valid rigorously only for an infinite number of tube rows and there are not easily available analytical formulae for more than four rows. So, the developed methodology permits to obtain very accurate data for geometry configurations for what the effectiveness values information is scarce or approximate formulae are used (see Eq. 20). To make this point clearer the heat exchanger with one tube row is analyzed. Taking the point $C^*$ = 0.5 and $NTU$ = 5 it can be shown that an error of the order of 0.1 % in the effectiveness provokes an error on the $NTU$ of the order of 4.0 % using then $\varepsilon$–$NTU$ analytical relation. It points to the fact that even a small relative error on effectiveness can lead to a great error on NTU determination. Thus, caution should be taken when it is computed NTU from the effectiveness-NTU relations. One application where this is very important is the effectiveness-NTU reduction method, commonly used for experimental determination of the airside convective heat transfer coefficient. In Tab. 4 it is seen that even for ten tube rows the average relative error on effectiveness is of the order of 0.1 %. When the number of rows is augmented the error decreases as expected.

As it is difficult to derive analytical or polynomial relations for $N_r$=5 to $\infty$, the approximate empirical correlation (Eq. 20) is extensively used in industry and research laboratories. According to (DiGiovanni and Webb, 1989) this correlation yields to unphysical results for NTU < 1, leading to a maximum error of 3.7 % when compared with the numerical solution for an unmixed-unmixed case. Then, the proposed methodology supply more accurate results in these cases and can be used when an analytical relation is not available.

**3.2 Analysis of some cross-flow heat exchanger's complex configurations**

In the previous section were showed results produced by the model single pass crossflow arrangements. For these kinds of heat exchangers exist various theoretical correlations and an approximate correlation for an infinite number of tube rows that can be used for rating and performance prediction purposes. Nevertheless the majority of heat exchanger has other more complex configurations with many different flow arrangements. In these cases the availability of $\varepsilon$-$NTU$ data is very useful for predicting heat exchanger performance. In this section will be presented an analysis concerning to the elaboration of $\varepsilon$-$NTU$ graphics obtained with the present simulation program for different heat exchangers.

Recently, several works studying and determining overall compact heat exchanger thermal performance have been developed (for example Wang et al., 1999; Jang et al., 1998 among others). In these works the $\varepsilon$-$NTU$ theoretical relations showed in Tab. 3 for single pass crossflow heat exchangers, with one or more rows are frequently used for experimental data reduction. Nevertheless, the appropriate $\varepsilon$-$NTU$ equation to be used depends in the number of tube rows and the fluid side circuiting (Wang et al., 2000). When different flow arrangements are used as Z, DX, and others multipass counter cross-flow heat exchanger configurations, errors will be done in data reducing using relations from Tab. 3. Next it is shown the errors involved in reducing the experimental data considering only the relations from Tab. 3 for effectiveness computation of other kinds of flow arrangements.

The tube circuiting of the analyzed cross-flow heat exchangers are showed in Fig. 3. First, Fig. 3.a, is a Z-shape cross-flow - $N_t$=12 - (geometry 1), second, Fig. 3.b, is a staggered two-row and two circuit arrangement - $N_t$=10 - (geometry 2), third, Fig. 3.c, is a staggered three-row and two-circuit arrangement- $N_t$=10 - (geometry 3), fourth, Fig. 3.d, is a staggered six-row and five-circuit arrangement - $N_t$=10 - (geometry 4). These four types were chosen to show the differences in $\varepsilon$-*NTU* values obtained with the proposed model in relation with theoretical ones. The geometries 2 to 4 were taking from (Rich, 1973).

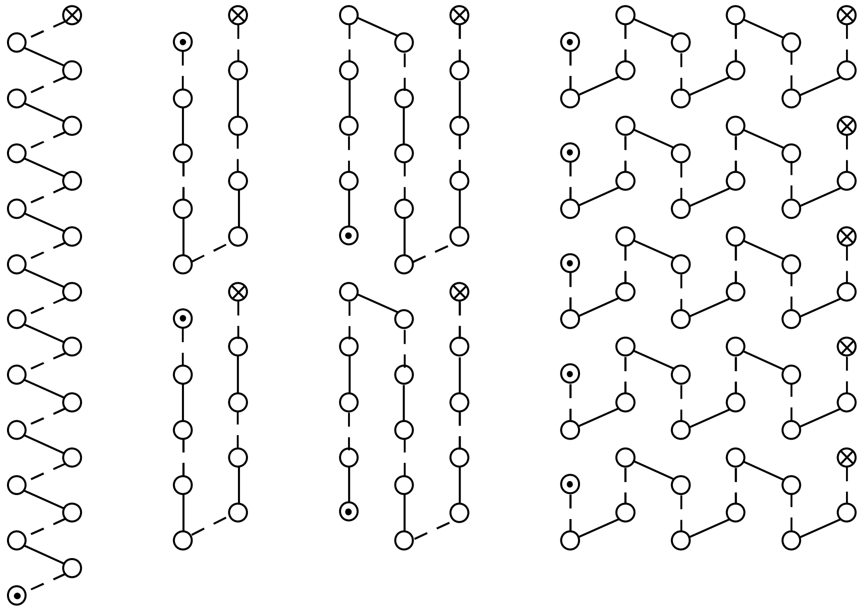


Figure 3. Schematic of the circuit arrangement for the cross-flow heat exchangers considered in present study: (a) Z-shape arrangement; (b) staggered two rows and two circuits arrangement; (c) staggered three rows and two circuits arrangement; (d) staggered six rows and five circuits arrangement.

The effectiveness of heat exchangers showed in Fig. 3 (a-d) are illustrated in Figs. 4-7. The heat capacity ratio varies from 0 to 1, with 0.25 increments. For $C^*$=0, the equation $\varepsilon=1-\exp(-NTU)$ is used. These graphics were obtained with the computational program and there is no similar in literature for these geometries (see straight lines in Figs. 4-7). Observing Figs 4 and 5, it is noted some differences due to flow arrangement influence for two rows heat exchangers. The two rows heat exchanger, geometry 2 flow arrangement, seems to have better performance that the geometry 1 configuration. This effect it is caught only by the developed program. With the theoretical correlation for two rows (see Tab. 3), which is used in these two cases, the above flow arrangement effect on $\varepsilon$-*NTU* behavior cannot be noted. This shows two important facts: first the flow arrangement exerts a big influence over the $\varepsilon$-*NTU* values, and second the use of a certain theoretical correlation can introduce errors in the experimental results.

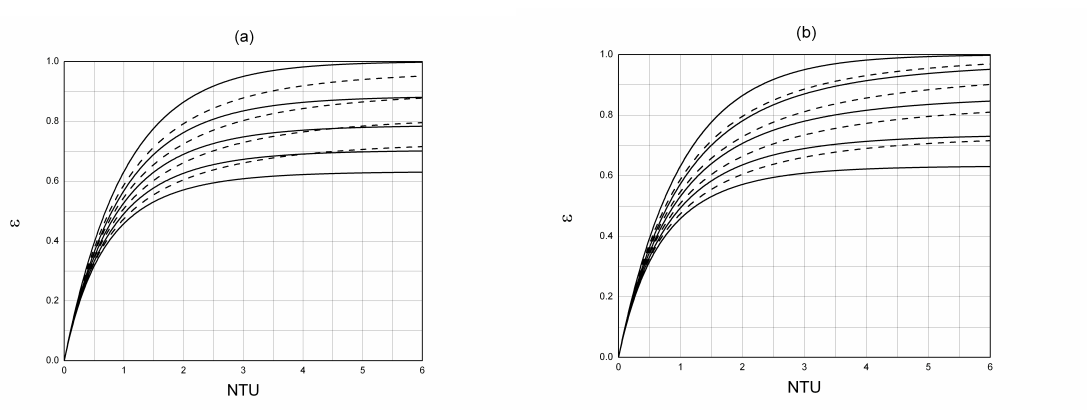
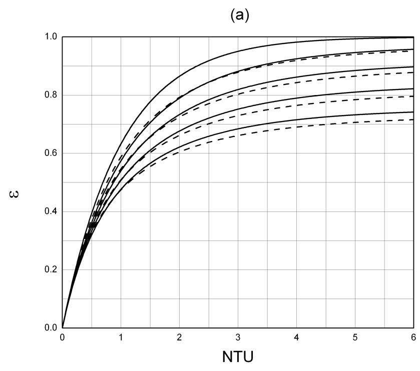



Figure 4. Effectiveness of the theoretical relation (dash line) and by the model prediction (straight line) of a Z-shape cross-flow heat exchanger (geometry 1): (a) $C_{max}$ (mixed), $C_{min}$(unmixed); (b) $C_{min}$ (mixed), $C_{max}$ (unmixed).



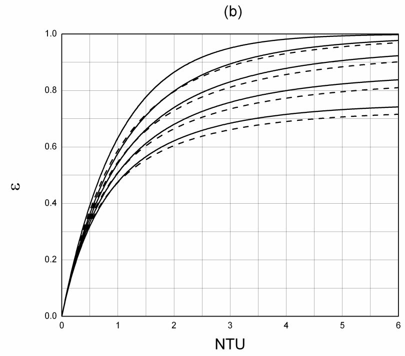
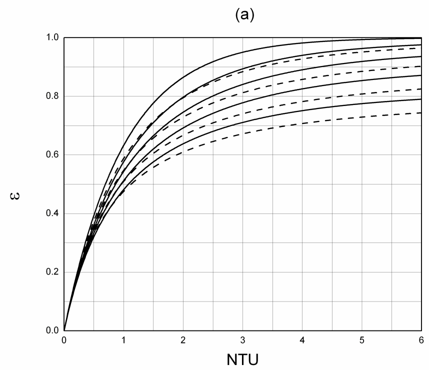




Figure 5. Effectiveness of the theoretical relation (dash line) and by the model prediction (straight line) of a staggered two rows and two circuits cross-flow heat exchanger (geometry 2): (a) $C_{max}$ (mixed), $C_{min}$(unmixed); (b) $C_{min}$ (mixed), $C_{max}$ (unmixed).



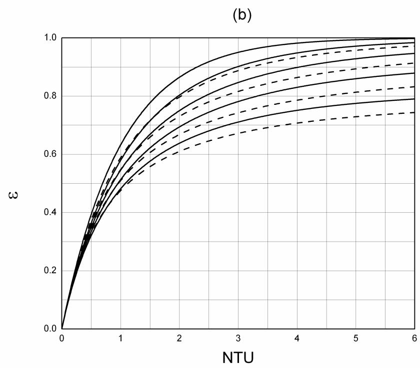




Figure 6. Effectiveness of the theoretical relation (dash line) and by the model prediction (straight line) of a staggered three rows and two circuits cross-flow heat exchanger (geometry 3): (a) $C_{max}$ (mixed), $C_{min}$(unmixed); (b) $C_{min}$ (mixed), $C_{max}$ (unmixed).

When there is not available a relation for effectiveness for a heat exchanger with complex flow arrangement, an approximation using a theoretical relation for the same number of row is used. In these cases, expressive differences between real and computed values of the theoretical effectiveness could occur. Figs 4 to 6 show a comparison between effectiveness values obtained both from theoretical relations (Eqs. 15,16, and 17) computational program's results for geometries 1-3 (Fig 3a-c).

## 4. Conclusions

This work presents a new methodology for cross-flow heat exchanger thermal performance computation. For this purpose a computational program *HETE (Heat Exchanger Thermal Efficiency)* is implemented. It is an extension of papers published in (Navarro and Cabezas-Gomez, 2005, Cabezas-Gomez, Navarro, Saiz-Jabardo, 2007). The proposed methodology is validated through comparison with well-established analytical and approximate theoretical results from research works, obtaining very small errors. New effectiveness data are obtained for some complex flow arrangements. A simulation element-by-element model discretizes the whole heat exchanger in small ones along the tube fluid path based on a local effectiveness concept. Computing the temperature distribution of both fluids the overall heat exchanger effectiveness is calculated. The model has been validated through comparison with theoretical relations from literature relations, obtaining very small relative errors in all cases. The comparisons reveal that the model is accurate and suitable for predicting the theoretical performance of coils. Effectiveness values were estimated for cross-flow heat exchangers with complex flow arrangements. For the analyzed cases, a maximum difference of about 10 % was noted between effectiveness model prediction and those based on theoretical formulae. This should be considered in experimental works.

## 5. Acknowledgments

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