ANT COLONY OPTIMIZATION APPLIED TO LAMINATED COMPOSITE PLATES

Rubem Matimoto Koide, rubemkoide@hotmail.com
Marco Antonio Luersen, luersen@utfpr.edu.br
Programa de Pós-Graduação em Engenharia Mecânica e de Materiais (PPGEM)
Universidade Tecnológica Federal do Paraná (UTFPR)
Avenida Sete de Setembro, 3165, Curitiba – PR - Brazil

Abstract. *Laminated composite materials consist of stacks of layers, each layer usually composed by a matrix of polymeric material and fibers oriented in a specific direction. Laminated composite materials have excellent mechanical properties due to their good characteristics, specially the ratio stiffness-weight. These materials offer the possibility to create an unlimited set of different combinations depending on the structural design needs, being the number of layers, orientation of the plies and type of material of each layer, usually the design variables. In order to achieve superior designs, optimization techniques have been developed. The more successful ones to solve this class of problems are those based on stochastic procedures (like genetic algorithms) because their capability of avoiding getting stuck in local optima. Another technique, which is still in progress and has not been much used to solve laminated composite problems, is the Ant Colony Optimization (ACO). Thus, this work aims at analyzing the ACO applied to weight minimization and buckling load maximization of laminated composite plates under static loading. ACO is a new class of algorithms initially proposed to solve combinatorial optimization problems. It takes inspiration from the studies of real ant colonies foraging behavior. The ants deposit on the ground a chemical substance called pheromone. The pheromone concentration influences the choice of subsequent ants probabilistically. Called stigmergy, this kind of behavior is the mechanism that controls ant activity and connects them to take the shortest paths. The ACO is based in the information obtained in the pheromone matrix. Artificial ants are used to construct solutions for laminated composite plates, by choosing probabilistically the orientation of the stacks of the laminated. The solutions are built on the past search experience based on the level of pheromone, and the heuristic matrix, which brings the information about the laminated problem. The ACO procedure starts with the random selection of fiber orientation for a laminated layup. After the artificial ants have finished building the first laminated configurations, the pheromone is released with evaporation and deposited the new amount of pheromone based on the best solutions found at local and global iterations. The algorithm stops when the maximum number of iterations or function evaluations is reached. The following optimization constraints are considered in this work: first ply failure criterion, buckling and maximum number of contiguous plies with the same orientation. Also, the plate is considered symmetric and balanced, and the allowable ply orientations are (0/0/+ 45/- 45/90/90). To obtain the structural response, the plates are analyzed using the classical lamination theory. The results of this study are compared with those obtained from the literature, showing that ACO has a good performance in solving laminated optimization problems.*

Keywords*: laminated composite material, ant colony optimization, optimization*

1. INTRODUCTION

Laminated composite materials consist of stacks of layers, each layer usually composed by a matrix of polymeric material and fibers oriented in a specific direction. Due to their excellent mechanical properties, specially stiffness and weight it can be used to develop innovative lightweight materials with specific properties for any applications. To deal with these materials the composite structures can be optimize. Optimization means, in this kind of materials to achieve a minimum weight or a maximum strength reaching a ply orientation and sequencing, thickness and geometry. The result is significant weight savings.

Compared to linear isotropic materials laminated composite materials have a complex structural analyses and their optimization require a further computational effort. Due to these materials offer the possibility to create an unlimited set of different material that can be tailored to structural design, which depends an optimal distributions of layers, their thickness and orientations. This optimum problem has been examined by different researchers since Haftka and co-workers (Le Riche and Haftka, 1993; Le Riche and Haftka, 1995; Todoroki and Haftka, 1998; Liu *et al*., 2000) used genetic algorithm (GA) as heuristic search techniques. After them many others researchers also tested different genetic algorithms (Deka *et al.*, 2005; Girard, 2006; Lopez *et al.*, 2008) and other metaheuristic+ such as fractal branch and bound method (Terada *et al.*, 2001), and recently the simulated annealing by Akbulut and Sonmez (2008).

A new class of algorithms, Ant Colony Optimization (ACO), was developed to solve combinatorial optimization problems. Ant colony optimization was inspired by the observation of the behavior of real ants (Dorigo and Socha, 2007). Regarding the ant colony optimization applied to laminated composite in Aymerich and Serra (2008) is presented the maximization of buckling load using ACO and in Abachizadeh and Tahani (2009) a multi-objective optimal design for maximization of fundamental frequency and minimization of cost.

Based on the real ants communications, stigmergy or indirect stimuli, the simulation of artificial ants in ACO was developed. This kind of communication can be observed in colonies of ants. When the ants walking from-and-to a food source, deposit in the ground a chemical substance called pheromone. The quantity of pheromone on the grounds forms a pheromone trails. And a concentration of it influences the choice of their path. Artificial ants may simulate pheromone laying by modifying appropriate pheromone variables associated with problem states they visit while building solutions to the optimization problem (Dorigo and Socha, 2007). Based on this behavior above described, Marco Dorigo introduced the Ant System (AS) algorithm in his Ph.D. thesis. He and coworkers developed many variants of ACO algorithms improving the Ant System. One of them is Ant Colony System (ACS). This metaheuristic has a good performance to solve combinatorial optimization problems and has been successfully applied in many complex discrete problems.

This study investigated the minimum weight of laminates searching the optimal stacking sequence and the minimum cost with buckling load factor and maximum weight. The constraints of the optimization are first ply failure criterion, maximum number of contiguous plies with the same orientation, symmetry and balance of the laminate and a set of possible orientation choices ($0_2, \pm 45, 90_2$) for each of the two ply stacks.

This paper aims at analyzing the stacking sequence optimizations combinatorial problem for three cases obtained in the literature. The first problem is applied the genetic algorithm to minimize weight of the laminate plate regarding Tsai-Wu failure criteria as a constraint, the second case the maximum buckling load solved by ant colony optimization, the third is to find the minimum cost of the laminate with minimum buckling load and maximum weight limit 85 N as constraints.

In both cases the analysis of laminated composites is based on concepts developed in static and mechanics of materials. Establishing the stress-strain relationships, failure criteria for orthotropic lamina, classical lamination theory of laminated plate, symmetric and balanced laminated, maximum numbers of contiguous plies with the same fiber angle. The heuristic method optimization employed for all cases above is Ant Colony Optimization.

An application and performance of ACO are further investigated. The results of ACO metaheuristic procedure are compared with the same problem adopting other optimization techniques.

2. LAMINATED COMPOSITES

Composite materials are constructed of two or more materials, commonly referred to as constituents, and have characteristics derived from the individual constituents (Gürdal *et al*., 1999). Composite laminated consist of stacks of layers or many unidirectional lamina stacked and tailored in a specific manner (see Fig. 1).

Lamina is a single layer (ply) of unidirectional (or woven) composite material (Staab, 1999). The laminae thicknesses in this study are considered the same. The standard sequence determines the fiber orientations of the different layers, starting from the top of the laminate to the bottom.

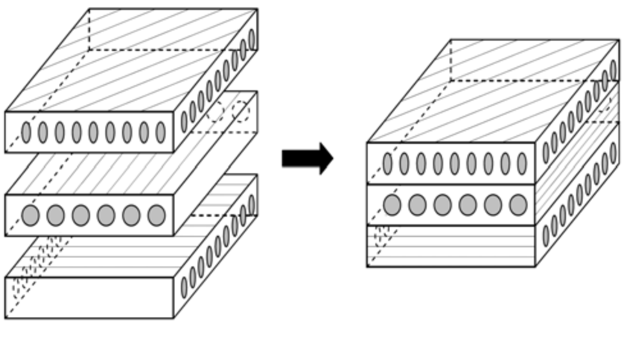

Figure 1. Laminated composite plates

In the present study the laminated plates are considered balanced and symmetric. Balanced implies a laminate with all laminae oriented at an arbitrary angle of $+\theta$ are balanced by an equal number of laminae oriented at $-\theta$. Symmetric means a laminate that has both material and geometric symmetry with respect to the geometric central plane (mid-plane) of the laminate (Staab, 1999). Another consideration is that the materials are orthotropic, in other words, they have three mutually perpendicular planes of symmetry.

Laminate strength prediction is carried out by evaluating the stress state within each layer based on the classical lamination theory (Gürdal *et al*., 1999).

In terms of structural design of composite materials, Haftka and Gürdal (1992) considered that finding an efficient composite structural design that meets the requirements of a certain application can be achieved not only by sizing the cross-sectional areas and member thickness, but also by global or local tailoring of material properties through selective use of orientation, number, and stacking sequence of lamina that make up the composite laminate. This means that this problem requires discrete programming techniques.

3. ANT COLONY OPTIMIZATION (ACO)

Ant colony optimization is a metaheuristic in which a colony of artificial ants cooperates in finding good solutions. This technique is applied to difficult discrete optimization problems (Dorigo and Stülzle, 2004). The first ACO algorithm was Ant System (AS), which was developed by Marco Dorigo for solving the traveling salesman problem (TSP).

The formal representation that the artificial ants use in a problem formulation for ACO is as follow

$$\textit{Minimize: Problem } (S,f,\Omega) \tag{1}$$

where S is the set of candidate solutions, f is the objective function, f(s) is the objective functions to each candidate s $\in$ S and Ω is a set of constraints.

Searching an optimal solution s*, in a set of feasible candidate solutions with minimum cost available. The problem in a instance (S,f,Ω) is mapped. It is characterized as:

- define the components in finite set $C=\{c_1,c_2,\ldots,c_{nc}\}$
- define the all possible sequences $x=<c_i,c_j,\ldots,c_h,\ldots>$
- identify a subset S of X ($S \subseteq X$)
- define $x \in \overline{X}$, where $\overline{X}$ is a set of feasible states that satisfy the constraints Ω
- the optimal solution S* is considered the non-empty where $S^* \subseteq \overline{X}$ and $S^* \subseteq S$
- finally estimate of a cost $J(x)$ available from feasible candidate solutions.

After the characterization of the problem with the information above, the graph Gc, the construction graph is performed by artificial ants. It is defined by Gc=(C,L), where C are the components and the all connects of components C is the set L. Dorigo and Stülzle (2004) define the graph Gc as construction graph and the elements of L are called connections. The ACO algorithm adopted the graph Gc concepts where the artificial ants build solutions in stochastic constructive procedures until complete the connected graph Gc=(C, L).

In a construction graph procedure there are two elements associated in this algorithm steps. The first is the pheromone trail τ, associated with the components $c_i \in C$ and the connections $l_{ij} \in L$. It influences in artificial ants search process, and the pheromone update by ants. The second is defined by heuristic value η or heuristic information and is related to the problem information. Both values influence the probabilistic decisions when the ants use heuristic rule in a constructive solutions procedure.

As describe Dorigo and Stülzle (2004), the main procedure of the ACO metaheuristic manages the scheduling of the three components of ACO algorithms via the *ScheduleActivities* construct: (1) management of the ant's activity, (2) pheromone updating, and (3) daemon actions.

The pseudo-code of ACO metaheuristic is describe by Dorigo and Stülzle (2004) as

```
Procedure ACOmetaheuristic
    ScheduleActivities
        ConstructAntsSolutions
        UpdatePheromones
        DaemonActions      % optional
    End-ScheduleActivities
End-procedure
```

The *ConstructionAntsSolutions* is a procedure where the solution of the optimization problem is build. In this way the artificial ants apply a stochastic decision rule in a construction graph problem defined by Gc. While the solution is being built, based on the pheromone trails and heuristic information, ants evaluate the solutions for searching the optimal feasible candidate.

UpdatePheromone procedure based on the solution available, influence the quantity of pheromone. It can be increase or decrease. The deposit of new amount of pheromone increases the probability of a good solution in the next decision. The decrease process is related to pheromone evaporation. It influences in a choice of candidate due to the reduction of pheromone trail for the solution available and in this way the bad candidate is not selected. This process actives the convergence of the algorithm.

The *DaemonActions* procedure can be used as optional process to optimize the algorithm. The local or global information can be used to decide or influence in a search optimization procedure.

The ACO algorithm is based in a three procedures as explained above. But there are many extensions of Ant System (AS). One of them is Ant Colony System (ACS) that achieves a good performance with some improvements in the original AS. The additional mechanisms in the ACS is that it exploits the search experience accumulated by the ants more strongly than AS does through the use of a aggressive action choice rule, as explain Dorigo and Stülzle (2004). Another characteristic is that the pheromone evaporation and pheromone deposit are update only in a best-so-far connection.

ACS algorithm has a framework based on three rules that manage the optimization problem. In this variant the first procedure is the Tour Construction or pseudorandom proportional rule defined by

$$j = \begin{cases} \arg\max_{l \in N_i^k} \left\{ \tau_{il} [\eta_{il}]^{\beta} \right\}, q \le q_0 \\ p_{ij}^k = \dfrac{[\tau_{il}]^{\alpha} [\eta_{il}]^{\beta}}{\Sigma l \in N_i^k [\tau_{il}]^{\alpha} [\eta_{il}]^{\beta}}, q > q_0 \end{cases} \tag{2}$$

where q is a random variable uniformly distributed in [0,1], q_0 is a parameter for the best possible move ($0 \le q_0 \le 1$), k is a ant, α is a parameter which determine the relative influence of the pheromone trail, β is a parameter which determine the relative influence of the heuristic information, η is the heuristic information value, *i,j* are the initial and the next choice or candidate, l is a candidate solution, N_i^k is the feasible neighborhood of ant k, p_{ij}^k is a probability which ant k choose the next solution if $q > q_0$. If $q \le q_0$ it means that the ant is exploiting the learned knowledge based in the pheromone trails and the heuristic information. If $q > q_0$ the ant explores other tours or search around the best-so-far solution.

The second rule is Global Pheromone Trail Update. In the following the formulation for this update is

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\Delta\tau_{ij}^{bs}, \quad \forall (i,j) \in T^{bs} \tag{3}$$

where ρ is the global pheromone evaporation rate ($0<\rho\le 1$), $\Delta\tau_{ij}^{bs}$ is the amount of pheromone that ant k deposits on each best-so-far solution, T^{bs} is a set of best connections. When this rule is applied both the evaporation and new pheromone deposit are update only to the best-so-far ant.

Local Pheromone Trail Update, the last rule, is applied during the tour construction. The pheromone evaporation and a new pheromone deposit are updated when an ant is exploiting or exploring the connection according to the pseudorandom proportional rule. It is formulated by

$$\tau_{ij} \leftarrow (1-\xi)\tau_{ij} + \xi\tau_0 \tag{4}$$

where ξ is the local pheromone evaporation rate $0 < \xi < 1$, τ_0 is the initial pheromone trails value.

The effect of the local updating rule is that each time an ant uses an arc(i,j) or connection its pheromone trail τ_{ij} is reduced, so that the arc becomes less desirable for the following ants as explained Dorigo and Stülzle (2004).

3.1. Ant Colony System (ACS) applied to laminated composite

The laminated composite stacking sequence is a combinatorial optimization problem as exposed in a previous section. To solve this optimization problem, the laminated composite stacking sequence optimization by ACO, it follows the formal representation as defined in Section 3. Table 1 shows this formulation.

Table 1. Formal representation to the artificial ants use in a laminate composite.

Characteristics Representation	Problem mapped	Laminate stacking sequence optimization
Set components C	$C=\{c_1,c_2,\ldots,c_{nc}\}$	$C=\{0_2,\pm 45,90_2\}$, fiber angle orientation
Set candidate solutions X	$x=<c_i,c_j,\ldots,c_h,\ldots>$	All candidates solutions for n plies
Subset s of x	$s \subseteq \chi$	Candidates solutions for n plies
Constraint Ω	Ω	Symmetric and balanced; maximum number of contiguous plies=4; first ply failure constraint; critical buckling load factor
Set the feasible solutions $\overline{X}$	$x \in \overline{X}$	Feasible stacking sequence
Optimal solutions s*	$s^* \in \overline{X}$	Optimal stacking sequence
Estimate of objective function	J(χ)	Minimize weight; maximize buckling; minimize cost

The ACS algorithm implementation is based on the three rules as described in Section 3. First of all the set parameters are defined in algorithm initialization, followed by the initial pheromone trail.

Dorigo and Stülzle (2004) suggested the parameters for different kind of extensions of ACO. In this work the parameters are set following their recommendations. The Table 2 expresses the value for each parameter.

Table 2. Ant Colony System parameters.

m	α	β	q_0	ξ	ρ
5	1	2	0.9	0.1	0.1

The parameter m is the ant quantity used in a tour construction. The others parameters from Tab. 2 are derived from Eq.(2), Eq. (3) and Eq.(4) as informed above.

As mentioned, the ACO is based on a construction graph Gc=(C, L) that the ants construct the solutions following the state rule and the Rolette wheel. Starting with random selection of the initial stacking sequence and initialization the pseudorandom proportional rule defined by Eq. (2). Aymerich and Serra (2008) explain that a candidate solution is then constructed step-by-step by choosing probabilistically the orientations of the buildings stacks of the laminate. This tour construction is repeated applying the two sub-rules from Eq. (2) until the end of constructing the solution. To simulate the evaporation of pheromone the local updating rule is actives during the *ConstructAntSolutions* procedure.

The reduction of amount of pheromone minimizes the risk of stagnation of stacking sequence in local optima and the new feasible combinatorial lay-up candidates can be explored.

The Global Pheromone Trail Update process focuses on the best solution from the beginning of the stacking laminate sequence. For the best orientations laminate, available by the objective function, the amount of pheromone is deposited for this specific ant trail. Abachizadeh and Tahani (2009) related that this rule, which acts as positive feedback, makes the search for the real best solution more directed.

4. NUMERICAL RESULTS

In this paper the Ant Colony System (ACS) is applied to laminated composite stacking sequence optimization. The ant colony optimization is tested for three problems. The algorithm is written in Matlab and the numerical cases are described and presented below. All the parameters of ACO algorithm for this study are in according to Tab. 2.

4.1. Case 1-Weight minimization with Tsai-Wu failure criterion as constraint – Comparative ACO (presente work) x GA (Lopez e*t al.*, 2008)

The numerical results obtained using ACO are compared with the same case studied by Lopez *et al*. (2008). They employed the genetic algorithm to investigate the effect of the failure criterion on the minimum weight of a laminate plate.

The optimization problem is formulated as

$$\begin{array}{ll} \textit{Find:} & \theta_k\,,\quad \theta_k \in \{0_2,\, \pm 45,\, 90_2\}\,,\ k=1 \text{ to } n \\ \textit{Minimize:} & \textit{Weight} \\ \textit{Subject to:} & \text{- First ply failure constraint} \\ & \text{- Symmetric and balanced laminate} \end{array} \tag{5}$$

where θ_k is the orientation of each stack of the laminate and n is is the total number of stacks.

Table 3 presents the elastic material properties and the lamina's strength properties of the carbon-epoxy.

Table 3. Carbon/Epoxy lamina properties.

Elastic Materials Properties					Lamina's Strength Properties				
E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	Mass density (kg/m^3)	X_T MPa	X_C MPa	Y_T MPa	Y_C MPa	S_{21} MPa
127.59	13.03	6.41	0.3	1605	2062	1701	70	240	105

The optimization problem described in Eq. (5) considers a carbon-epoxy square laminated plate, subjected to in-plane loads N_x, N_y and N_{xy}. The problem with different shearing load is solved in according to the number of plies (see Table 4). The lighter structure is pursued for this case, adopting the Tsai-Wu failure criterion, symmetry and balanced laminate as constraints. Each layer is 0.1 mm thick and the length and width of the plate are 1.0 m.

The characteristics about the geometry of lamina are found in Tab. 4. Lopez *et al*. (2008) investigated the optimum weight for biaxial load condition for different shear loads considering only one material stacking. The conditions loads for laminated plate are detailed in Tab. 4.

Table 4. Geometry characteristics and loading of laminated plate.

Geometry Characteristics				Loading		
Number of plies	Thickness	Length	Width			
n	t (mm)	a (mm)	b (mm)	Nx (N/mm)	Ny (N/mm)	Nxy (N/mm)
32	0.1	1000	1000	-3000	-3000	0
32	0.1	1000	1000	-3000	-3000	100
32	0.1	1000	1000	-3000	-3000	250
36	0.1	1000	1000	-3000	-3000	500
40	0.1	1000	1000	-3000	-3000	1000

Table 5 shows the results for comparison between the genetic algorithm and the ant colony optimization for biaxial compressive load and different shear load values. Note that this problem is independent of the bending stiffness and as a consequence, it is independent of the stacking sequence. Thus, if the plies shown in Table 5 are arranged in any order, the laminate extensional stiffness remains the same as long as the number of plies with the same orientation angle is kept constant. The results are very quite similar. There are a little difference in the stacking sequence for the first situation, Nxy=0.

Table 5. Comparison ACO x Genetic Algorithm (Lopez *et al*., 2008).

Loading (N/mm)			Failure Criteria	References					
Nx	Ny	Nxy		GA (Lopez et. al., 2008)			ACO (present work)		
				Stacking Sequence			Stacking Sequence		
				0_2	±45	90_2	0_2	±45	90_2
-3000	-3000	0	Tsai-Wu	4		4	3	1	4
-3000	-3000	100	Tsai-Wu		8			8	
-3000	-3000	250	Tsai-Wu		8			8	
-3000	-3000	500	Tsai-Wu		9			9	
-3000	-3000	1000	Tsai-Wu		10			10	

4.2. Case 2-Buckling load maximization– Comparison ACO x ACO (Aymerich and Serra, 2008)

This problem was investigated by Aymerich and Serra (2008). They applied the ant colony optimization metaheuristic to the lay-up design of laminated plates for maximization of buckling load.

The optimization problem can be defined as

Find: θ_k , $\theta_k \in \{0_2, \pm 45, 90_2\}$, k=1 to n

Maximize: $\lambda c = \min(\lambda cb, \lambda cf)$ (6)

Subject to: *- Maximum number of contiguous plies=4*

- Symmetric and balanced laminated

where λc is the maximum buckling load, λcb is the critical buckling load factor , λcf is the critical failure factor , θ_k is the orientation of each stack and n is the total number of stacks.

The materials properties are presented in Tab. 6. It is complemented with the allowable strains ε_1^u , ε_2^u , γ_{12}^u . Those ultimate strains' values are used to calculate the critical failure load factor λcf using a safety factor of 1.5.

Table 6. Graphite/Epoxy Properties.

Elastic Materials Properties				Allowable strains		
E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	ε_1^u	ε_2^u	γ_{12}^u
127.59	13.03	6.41	0.3	0.008	0.029	0.015

The characteristics of n plies employed in this case, composed only by graphite/epoxy material are exposed in Tab. 7. The Ny loading corresponds to 25% of Nx loading.

In Table 8 are included the numerical data obtained for case 2. The staking sequence does not the same in spite of it the ACO buckling load factor for λcb and λcf present a good results.

Table 7. Geometry characteristics and loading of laminated plate.

Geometry Characteristics				Loading	
Number of ply	Thickness	Length	Width		
n	t (mm)	a (mm)	b (mm)	Nx	Ny
48	0.127	508	127	175 N/m	Nx/4 N/m

Table 8. Comparison ACO x ACO (Aymerich and Serra (2008)*.

Stacking Sequence	Reference	Load Factor	
		Buckling (λcb)	Failure (λcf)
$[\pm45_2/90_2/\pm45_3/0_2/\pm45/0_4/\pm45/0_2]_s$	Aymerich and Serra (2008)	12743.45	12678.78
$[\pm45/90_2/\pm45_4/(0_2/\pm45/0_2)_2]_s$	Aymerich and Serra (2008)	12725.26	12678.78
$[90_2/\pm45_5/(0_2/\pm45/0_2)_2]_s$	Aymerich and Serra (2008)	12674.85	12.678.78
$[\pm45_3/90_4/\pm45_2/0_2/\pm45/0_4]_s$	ACO	12459.75	12690.69
$[90_2/\pm45_4/(0_2/\pm45)_3/0_2]_s$	ACO	12418.12	12690.69
$[\pm45_2/90_2/\pm45_2/0_2/\pm45_2/0_2/\pm45/0_4]_s$	ACO	12634.43	12690.69

* problem based on Haftka and co-workers using GA search procedures.

4.3. Case 3- Cost minimization with minimum buckling load and maximum weight as constraints

The laminated composite optimization considering the minimum buckling load factor and the maximum weight limit 85 N as constraints was formulated by Girard (2006). Lopez *et al.* (2009) solved the same problem. They have applied the genetic algorithm (GA) technique to achieve the minimum cost. The problem imply in two different materials with three discrete fiber angles 0_2, ±45, 90_2.

The formulation of the problem was defined as

Find: $\{\theta_k, mat_k\}$, $\theta_k \in \{0_2, \pm 45, 90_2\}$,
mat_k={carbon/epoxy(C-E), glass/epoxy(G-E)}, k=1 to n
Minimize: *Material Cost* (7)
Subject to: - $\lambda_{fc} \geq \lambda_{\min}$
- *Maximum weight: 85 N*
- *Symmetric and balanced laminate*

where mat_k is the lamina material, $\lambda_{\min}$ is the minimum value for critical buckling load, λ_{fc} is the critical buckling factor.

In this case the constant of gravity considered by Girard (2006) to calculate the weight in Newton is g = 9.9 m/s^2. This test presents the composite with two distinct materials, carbon/epoxy (C-E) and glass/epoxy (G-E). Their properties are described in Tab.9.

Table 9. Laminae material properties.

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	Thickness (mm)	Mass density (kg/m^{3})
mat_1- (C-E)	138.0	9.0	7.1	0.30	0.127	1605
mat_2- (G-E)	43.4	8.9	4.55	0.27	0.127	1993

The constraints for this case are the maximum weight limit 85 N and the critical buckling load. To pursue the feasible solution satisfying all constraints, characterize the problem hard to find the convergence of the algorithm in reaching the global optimum.

The laminated plate investigated in this case 3 is rectangular with length *a* and width *b* and is subjected in-plane loads *Nx* and *Ny*, as shown in Fig. 2.

The three different situations are tested for cost optimization. In the first situation the number of plies is 48 and λ_{min} is limited to 150. In the second one, the laminated plate has with 52 laminae and buckling load factor is $\lambda_{min} = 250$. For the third test the composite laminate has 60 plies and $\lambda_{min} = 375$ is the limiting for the minimum buckling load factor. The biaxial compressive load (175 N/m) has the same value for *Nx* and *Ny*.

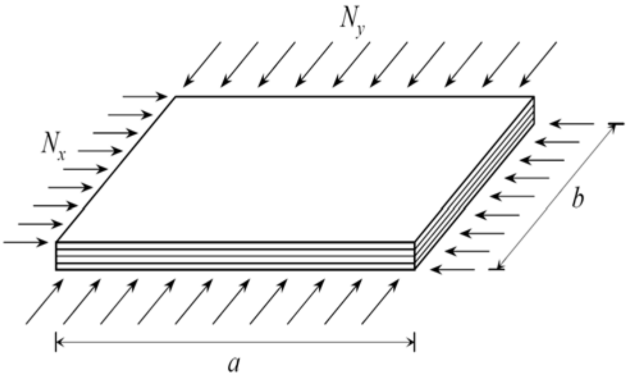

Figure 2. Laminated plate geometry and applied loading

The geometry of the laminated plate is defined as length=0.9144 m and the width=0.762 m for all tests in this case.

The λ_{min}, the minimum value for critical buckling load is given as a condition to this problem. The buckling load factor λ_{fc} represents the failure buckling load divided by the applied load. The results for the three situations evaluated in this case are reported in Table 10. In the stacking sequence show in this table, the underlined figures correspond to G-E stacks and the remaining figures to C-E stacks. Note that optimum laminate configuration is achieved with the C-E layers at the outer surface which gives a higher bending stiffness allowing satisfying the buckling constraint. Besides the slightly differences in the orientations of the plies, the material sequence and the cost obtained by ACO agree in comparison to the GA.

Table 10. Comparison ACO x GA (Girard, 2006 and Lopez *et al*., 2009).

No. of plies	λ_{min}	References	Stacking Sequence	Material (C-E/ G-E)	Cost (U)	Weight (N)	λ_{fc}	$\bar{n}_f$
48	150	Girard (2006) Lopez *et al*. (2009)	$[\pm45_3, \pm\underline{45}_9]_s$	12/36	19.99	79.73	165.56	23945
52	250	Girard (2006) Lopez *et al*. (2009)	$[\pm45_6, \pm\underline{45}_7]_s$	24/28	32.21	82.64	259.47	27345
60	375	Girard (2006) Lopez *et al*. (2009)	$[\pm45_{15}]_s$	60/0	68.19	84.39	442.79	24894
48	150	ACO	$[\pm45_3, 90_2 \pm\underline{45}_7]_s$	12/36	19.98	79.73	190.23	23744
52	250	ACO	$[90_2\pm45_60_2, 90_2\pm\underline{45}_50_2]_s$	24/28	32.21	82.63	283.09	25581
60	375	ACO	$[\pm45_{15}]_s$	60/0	68.17	84.36	443.02	25039

The $\bar{n}_f$ value is the total number of objective function evaluation to algorithm converges. The cost and weight values for the first test are the same in that comparison. The stacking sequence for test with 48 plies is $[\pm45_3, 90_2 \pm\underline{45}_9]_s$, it results the buckling load factor greater than GA value. Searching the global optimum in situation two the stacking sequence is $[90_2\pm45_60_2, 90_2 \pm\underline{45}_50_2]_s$ and λ_{fc} is 283.09. For the test with 60 plies and $\lambda_{min} = 375$ the function evaluation achieves the global convergence as GA.

Lopez *et al*. (2009) studied the convergence for this problem with and without local search. The test is based on 100 independent runs. The stop criterion is the total number of function evaluations (FE). Table 11 presents the results for this case. The mean and standard deviation (SD) values are considered only for feasible solution. The tests are applied for 500, 1500 and 2500 function evaluations.

Table 11. Convergence after 100 runs ACO x GA (Lopez *et al.*, 2009).

		λ_{min}	References	500 FE	1500 FE	2500 FE
Without local search	% of convergence	150	Lopez *et al.* (2009)	53	100	100
	λ_{min} - mean and (SD)			158.2(4.7)	166.0(1.9)	167.4(0.0)
With local search	% of convergence			100	100	100
	λ_{min} - mean and (SD)			165.3(3.7)	167.4(0.0)	167.4(0.0)
Without local search	% of convergence	150	ACO	31	100	100
	λ_{min} - mean and (SD)			171.9(9.4)	167.9(2.8)	167.9(0.0)
With local search	% of convergence			100	100	100
	λ_{min} - mean and (SD)			173.8(5.1)	170.8(9.2)	170.9(5.2)
Without local search	% of convergence	250	Lopez *et al.* (2009)	11	91	100
	λ_{min} - mean and (SD)			255.6(6.2)	261.6(1.6)	262.4(0.0)
With local search	% of convergence			44	100	100
	λ_{min} - mean and (SD)			260.0(3,6)	262.4(0.0)	262.40(0.0)
Without local search	% of convergence	250	ACO	0	43	87
	λ_{min} - mean and (SD)			0	285.3(9.5)	283.8(1.9)
With local search	% of convergence			24	99	99
	λ_{min} - mean and (SD)			281.7(0.0)	284.8(4.0)	284.2(3.1)
Without local search	% of convergence	375	Lopez *et al.* (2009)	100	100	100
	λ_{min} - mean and (SD)			433.5(13.6)	447.8(0.0)	447.8(0.0)
With local search	% of convergence			100	100	100
	λ_{min} - mean and (SD)			446.2(2.5)	447.8(0.0)	447.8(0.0)
Without local search	% of convergence	375	ACO	100	100	100
	λ_{min} - mean and (SD)			442.9(3.5)	442.7(0.0)	442.8(0.0)
With local search	% of convergence			99,1	100	100
	λ_{min} - mean and (SD)			443.0(0.6)	443.0(0.0)	443.0(0.0)

Comparing the results of the three analyses in case 3 using different values of λ_{min} with those obtained by Lopez *et al.* (2009), the ACO results are quite similar for λ_{min} =150 and λ_{min} =375 in comparison to mean and SD values and probability to converge to the global optimum considering the feasible solutions. For the situation where λ_{min} = 250, ACO algorithm required higher number of function evaluations in finding the global optimum and the GA is more effective than ACO with local search and without local search for different number of function evaluations as stop criterion.

5. CONCLUSIONS

In this paper, the Ant Colony Optimization (ACO) is applied to the optimization of laminated composite plates. ACO is a new class of algorithms to solve stochastic optimization problems. The algorithm procedures are introduced and explained. The formal representation that the artificial ants use in a problem formulation for ACO is applied to laminated composite plates. Based on this formulation the ACO is implemented to minimize weight, cost or maximize the buckling load factor considering constraints and their performance are tested, discussed and reported.

Many numerical tests were carried out and three cases are selected to evaluate the performance of metaheuristic procedures. The algorithm is compared with the genetic algorithm proposed by different researchers, and it is achieved good results in many tests run. Base on these numerical cases, the ACO approach for the optimization of laminated materials can be competitive with other techniques as genetic algorithm.

6. ACKNOWLEDGEMENTS

The authors are grateful to Mechanical Structural Lab (LaMEs)-CITEC-UTFPR for the computational infrastructure. Mr. Koide is also thankful to CAPES for his M.Sc. scholarship.

7. REFERENCES

Abachizadeh, M. and Tahani, M., 2009, "An ant colony optimization approach to multi-objective optimal design of symmetric hybrid laminates for maximum fundamental frequency and minimum cost", Structural and Multidisciplinary Optimization. Vol. 37, pp. 367-376.

Akbulut, M. and Sonmez, F.O, 2008, "Optimum design of composite laminated for minimum thickness", Composites and Structures. Article in Press.

Aymerich, F. and Serra, M., 2008, "Optimization of laminate stacking sequence for maximum buckling load using the ant colony optimization (ACO) metaheuristic", Composites: Part A, Vol. 39, pp. 262-272.

Deka, D. J., Sandeep, G., Chakraborty, D., Dutta, A., 2005, "Multiobjective optimization of laminated composites using finite element method and genetic algorithm", Journal of Reinforced Plastics and Composites, Vol. 24, No. 3, pp. 273-285.

Dorigo, M. and Socha, K., 2007, "An introduction to ant colony optimization", IRIDIA – Technical Report No. TR/IRIDIA/2006/010, Université Libre de Bruxelles.

Dorigo, M. and Stützle, T., 2004, "Ant colony optimization", MIT. Cambridge, USA.

Girard, F., 2006, "Optimisation de stratifiés en utilisant un algorithme génétique" (in French), M.Sc. Thesis, Faculté des Sciences et de Génie, Université Laval, Québec, Canadá.

Gürdal, Z., Haftka, R., Hajela, P., 1999, "Design and optimization of laminated composite materials", John Wiley & Sons, USA.

Haftka, R.and Gürdal, Z., 1992, Elements of Structural Optimization, 3rd rev. and expanded ed., Kluwer Academic Publishers, Dordrecht, The Netherlands.

Le Riche, R. and Haftka, R., 1993, "Optimization of Laminate Stacking Sequence for Buckling Load Maximization by Genetic Algorithm", AIAA Journal. Vol.31, pp. 951-956.

Le Riche, R. and Haftka, R., 1995, "Improved genetic algorithm for minimum thickness composite laminate design", Composites Engineering, Vol. 5, pp. 143-161.

Liu, B., Haftka, R., Akgun, M. and Todoroki, A., 2000, "Permutation genetic algorithm for stacking sequence design of composite laminates", Computer Methods in Applied Mechanics and Engineering, Vol. 186, pp. 357-372.

Lopez, R.H., Luersen, M.A. and Cursi, E.S., 2008, "Effect of the Failure Criterion on the Minimum Weight of Laminated Composites", in Proceedings of the Ninth International Conference on Computational Structures Technology, B.H.V. Topping, M. Papadrakakis, (Editors), Civil-Comp Press, Stirlingshire, United Kingdom, paper 10.

Lopez, R.H., Luersen, M.A. and Cursi, E.S., 2009, "Optimization of Hybrid Laminated Composites using a Genetic Algorithm", Journal of the Brazilian Society of Mechanical Sciences and Engineering (JBSMSE) (accepted to be published).

Staab, G.H., 1999, " Laminar Composites", Butterworth-Heinemann, Worbun, MA, USA.

Terada, Y., Todoroki, A., Shimamura, Y., 2001, Stacking sequence optimizations using fractal branch and bound method for laminated composites, JSME International Journal, Serie A, Vol. 44, No. 4.

Todoroki, A.and Haftka, R., 1998, "Stacking sequence optimization by a genetic algorithm with a new recessive gene like repair strategy", Composites Part B: Engineering, Vol. 29, pp. 277-285.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.