VARIATIONAL VISCOELASTIC MODELS FOR FIBER REINFORCED SOFT TISSUES

Louise Reips, louisereips@gmail.com
Jakson Manfredini Vassoler, jaksonmv@hotmail.com
Eduardo Alberto Fancello, fancello@grante.ufsc.br
Departamento de Engenharia Mecânica, Universidade Federal de Santa Catarina
Campus Universitário, Trindade, Florianópolis, SC, Brasil

Abstract. *This paper presents a constitutive framework appropriated to the simulation of fiber-reinforced materials, in particular biological tissues. Many biological soft tissues are constituted by a net of collagen and elastin fibers embedded in a compliant solid cellular matrix. Classical examples are ligaments, tendons, arteries, etc. The distribution of these fibers as well as the mechanical interaction among them and that of fibers and cellular matrix provide the anisotropic rate dependent (viscous) behavior of these materials. In this framework the behavior of standard inelastic materials is described by a free energy incremental potential whose local minimization provides the constraints for the internal variables updates at each load increment. This potential is additively decomposed in two contributions. The first one is related to the assumed isotropic behavior of the cellular matrix, while the second one takes into account the fiber induced anisotropy by means of the structural tensors given by the fiber directions. Different material models can be represented depending of the choice of suitable potential functions. In this paper different potentials are tested and compared in order to verify their ability to match the performance of biological fiber reinforced soft tissues.*

Keywords*: Biomechanics, Anisotropy, Nonlinear viscoelasticity*

1. INTRODUCTION

In this work is presented a variational constitutive model adequate to simulate the mechanical behavior of soft biological tissues, in particular connective tissues like ligaments and tendons. These biological tissues are formed by arrangements of collagen fibers imbedded in a cellular matrix where this internal structure presents an anisotropic mechanical behavior dependent on the fiber directions. Hyperelastic and visco-hyperelastic models incorporating anisotropy and composite concepts have been used to represent the behavior of these materials. Several classical references may be mentioned. In Limbert *et al.* (2004) is proposed a transversal isotropic viscoelastic law. In Pioletti and Rakotomanana (1998) is presented a formulation based in viscoelastic potentials dependent on strain invariants. In Holzapfel and Gasser (2001) an anisotropic viscoelastic law for transversal isotropic materials based on additive free energy decomposition is presented. All these models assume dependence between elastic and dissipative potentials and fiber orientation taking into account the strains history.

In Fancello *et al.* (2006) a variational framework for viscoelastic isotropic materials submitted to finite strain regime is proposed. The aim of the present work is to extend this variational formulation to include fiber reinforced viscoelastic behaviors in order to provide the desired anisotropy.

In Section 2, the mathematical background of this paper is stated. Section 3 and 4 particularize this approach for the case of isotropic and fiber reinforced viscoelastic models. Finally, Section 5 shows examples that illustrate the performance of the proposed model while Section 6 presents a closing discussion and remarks.

2. CONSTITUTIVE PROBLEM

Hyperelastic models are based on the existence of a free energy function W that depends on total strain only and whose derivative provides the stress state of that material point. The first Piola-Kirchhoff stress tensor $\mathbf{P}$ can be compute as

$$\mathbf{P} = \frac{\partial W(\mathbf{F})}{\partial \mathbf{F}} = 2\mathbf{F}\frac{\partial W(\mathbf{C})}{\partial \mathbf{C}} \tag{1}$$

where $\mathbf{F}$ is the deformation gradient and $\mathbf{C}$ the Cauchy-Green tensor.

Considering that compatibility and constitutive equations are satisfied and being K the set of admissible configurations, the equilibrium problem is related to the minimization of the total potential as follows:

$$\min_{x \in K} \Im(x) \tag{2}$$

$$\Im(x) = \int_{\Omega_0} W(\mathbf{C}(x)) d\Omega_0 - \left[\int_{\Omega_0} \mathbf{b}_0 .x d\Omega_0 + \int_{\Gamma_0} f_0 .x d\Gamma_0 \right] \tag{3}$$

In dissipative problems, like in the case of viscoelastic materials, the Eq.(1) cannot be stated, since the current stress state depends not only on total strains but also on the strain history. Despite this difficulty, Ortiz and Stainier (1999) have proposed a variational approach in which it is possible to state the problem analogously to Eq.(1) in an incremental way. In this formulation a *pseudo*-potential energy, also called Incremental Potential, is stated at each load step, providing the first Piola-Kichhoff stress at each load increment by means of:

$$\mathbf{P}_{n+1} = \frac{\partial \Psi(\mathbf{F}_{n+1};\xi_n)}{\partial \mathbf{F}} = 2\mathbf{F}_{n+1} \frac{\partial \Psi(\mathbf{C}_{n+1};\xi_n)}{\partial \mathbf{C}_{n+1}} \tag{4}$$

In this expression, $\xi = \{\mathbf{F}, \mathbf{F}^{i}, Q\}$ is the set of internal and external state variables. The elastic and inelastic deformation gradients $\mathbf{F}^{e}$ and $\mathbf{F}^{i}$ are obtained from the multiplicative decomposition of the total deformation gradient $\mathbf{F}$. The symbol Q represents a set of internal variables that includes all remaining variables related to the dissipative phenomena. In Ortiz and Stainier (1999) is presented the Incremental Potential as

$$\Psi(\mathbf{F}_{n+1};\xi_n) = \min_{\mathbf{F}^{i}_{n+1}, Q_{n+1}} \left\{ W(\xi_{n+1}) - W(\xi_n) + \Delta t \, \psi\left(\overset{\circ}{\mathbf{F}}{}^{i}, \overset{\circ}{Q}; \xi_n \right) \right\} \tag{5}$$

$$W(\xi) = \varphi(\mathbf{F}) + \varphi^{e}\left(\mathbf{F}\mathbf{F}^{i^{-1}}\right) + \varphi^{i}\left(\mathbf{F}^{i}, Q\right) \tag{6}$$

where φ is the conservative (elastic) potential dependent on $\mathbf{F}$, φ^{e} is the elastic potential dependent on $\mathbf{F}^{e}$, φ^{i} is the inelastic potential depending on ($\mathbf{F}^{i}$, Q) . Finally, ψ is a *pseudo*-potential that provides the dependence between the stress and the inelastic stretching (viscous effects). The arguments of ψ in Eq.(5) are the incremental approximations of their respective rate values. In the Eq.(5) and Eq.(6), the minimization problem identifies the optimal values of the internal variables associated to the corresponding increment $\mathbf{F}_{n+1}{}^{i}$ and Q_{n+1}. Once the minimization problem (5) is solved, the stress may be computed in the hyperelastic-like expression (4). The general framework stated in this section is adapted to represent different material behaviors depending on the particular choices and arrangements of potentials φ, φ^{e}, φ^{i} and ψ.

3. ANISOTROPIC VISCOELASTIC MODEL

The anisotropic viscoelastic model is based on an additive decomposition of the Incremental Potential in an isotropic and fiber-reinforcement contribution. In Fig.1 it is shown the rheological representation of the anisotropic viscoelastic model where it is put in evidence the isotropic and anisotropic contribution.

The inclusion of fibers is performed following concepts shown in Holzapfel (2000) by an additive decomposition of energies, in the present case, Incremental Potentials:

$$\Psi = \Psi_{iso} + \Psi_{f} \tag{7}$$

where, Ψ_{iso} is the potential related to the isotropic viscoelastic model proposed in Fancello *et al.*(2006) while Ψ_f is related to the fiber contribution.

3.1. Isotropic Viscoelastic Model

The strain quantities are decomposed into volumetric and isochoric strain by the classical multiplicative decomposition. In the Maxwell branch, the isochoric gradient of deformation is also decomposed in elastic and viscous contributions $\mathbf{F} = \mathbf{F}^{e}\, \mathbf{F}^{v}$. With these hypotheses, the free energy W is

$$W(\mathbf{F}) = U(J) + \varphi(\hat{\mathbf{C}}) + \varphi^{e}(\hat{\mathbf{C}}^{e}) \tag{8}$$

where

$$J = \det(\mathbf{F}), \quad \hat{\mathbf{C}}^{e} = \hat{\mathbf{F}}^{eT}\hat{\mathbf{F}}^{e}, \quad \hat{\mathbf{C}} = \hat{\mathbf{F}}^{T}\hat{\mathbf{F}} \tag{9}$$

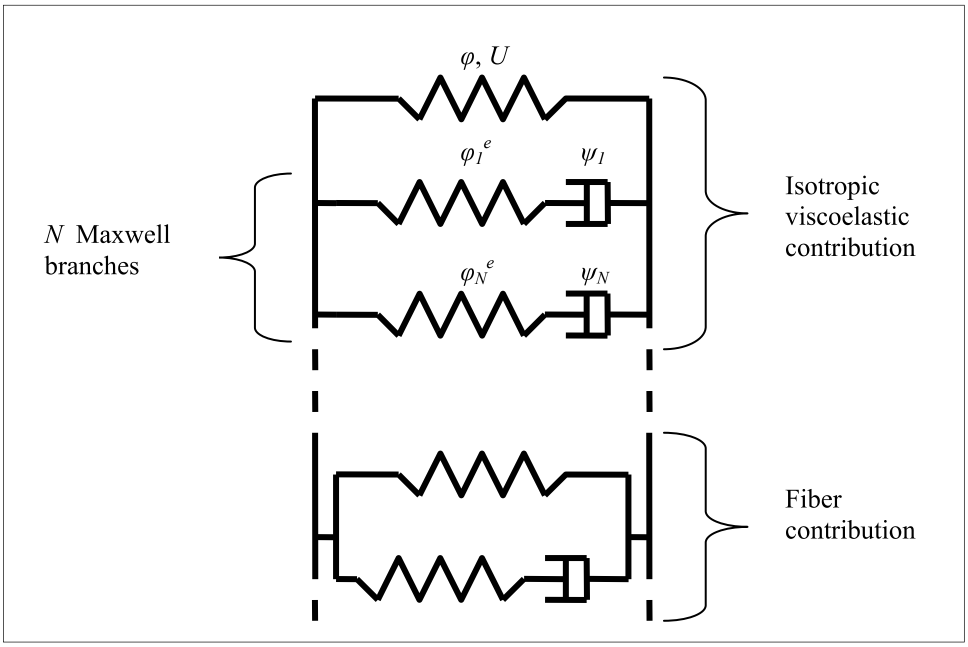

Figure 1. Rheological model

The isochoric potential φ is an isotropic function of the eigenvalues c_i of $\hat{\mathbf{C}}$**.** The (elastic) volumetric contribution comes from the potential U dependent on J:

$$\varphi\left(\hat{\mathbf{C}}\right) = \varphi\left(c_1, c_2, c_3\right),\ U\left(J\right) = \frac{k}{2}\left(\ln J\right)^2 \tag{10}$$

The viscous stretching $\mathbf{D}^v$ is defined by

$$\mathbf{D}^v = \mathrm{Sym}\left(\mathbf{L}^v\right) = \mathbf{L}^v = \dot{\mathbf{F}}^v \mathbf{F}^{v^{-1}} \tag{11}$$

Additional constraints on $\mathbf{D}^v$ define the specific characteristics of the flow rule. In Fancello *et al.*(2006) it is proposed a spectral decomposition:

$$\mathbf{D}^v = \sum_{i=1}^{3} d_j^v \mathbf{M}_j \tag{12}$$

$$d^v \in K_Q = \left\{p_j \in \mathrm{R} \Rightarrow p_1 + p_2 + p_3 = 0\right\} \tag{13}$$

$$\mathbf{M}_j \in K_M = \left\{\mathbf{N}_j \in \mathrm{Sym} \Rightarrow \mathbf{N}_i : \mathbf{N}_j = 0, \mathbf{N}_j : \mathbf{N}_j = 1, i \neq j\right\} \tag{14}$$

where the scalars d_j^v are the eigenvalues of $\mathbf{D}^v$ and $\mathbf{M}_j$ are their eigenprojections. The elastic and viscous potentials φ^e, φ^v are assumed to be isotropic functions of their respective arguments:

$$\varphi\left(\hat{\mathbf{C}}^e\right) = \varphi\left(c_1^e, c_2^e, c_3^e\right);\ \psi\left(\mathbf{D}^v\right) = \varphi\left(d_1^v, d_2^v, d_3^v\right) \tag{15}$$

From these definitions, it is shown in Fancello *et al.*(2006) that the Incremental Potential, in Eq. (4), takes the form

$$\Psi\left(\mathbf{F}_{n+1}; \xi_n\right) = \Delta\varphi\left(\hat{\mathbf{C}}_{n+1}\right) + \Delta U\left(J_{n+1}\right) + \min_{\Delta q_j^v, \mathbf{M}_j} \left\{\Delta\varphi^e\left(\hat{\mathbf{C}}_{n+1}^e\right) + \Delta t\, \psi\left(\frac{\Delta q_j^v}{\Delta t}\right)\right\} \tag{16}$$

$$\Delta\varphi\left(\hat{\mathbf{C}}_{n+1}\right)=\varphi\left(\hat{\mathbf{C}}_{n+1}\right)-\varphi\left(\hat{\mathbf{C}}_{n}\right) \tag{17}$$

$$\Delta\varphi^{e}\left(\hat{\mathbf{C}}^{e}_{n+1}\right)=\varphi^{e}\left(\hat{\mathbf{C}}^{e}_{n+1}\right)-\varphi^{e}\left(\hat{\mathbf{C}}^{e}_{n}\right) \tag{18}$$

$$\Delta U\left(J_{n+1}\right)=U\left(J_{n+1}\right)-U\left(J_{n}\right) \tag{19}$$

where the minimizing operation is constrained by Eq.(15) and Eq.(16). This minimization also shows that tensors $\hat{\mathbf{C}}^{e}$ and $\mathbf{D}^{v}$ are collinear. Moreover, the minimization with respect to Δq_j is easily solved by Newton method, which also provides the analytical material consistent matrix to be used in the global equilibrium problem. Once the minimizers of Eq.(16) are obtained, the first Piola-Kirchhoff stress tensor is computed from Eq.(4). Detailed information about these operations is found in Fancello *et al.*(2006).

3.2. Fiber contribution

The fibers introduce a strongly anisotropy related to their directions. Consequently, the Incremental Potential depends not only on Cauchy tensor **C**, but also on the structural tensor $\mathbf{A}_f$. This dependence in the present case is related to the following invariant (Holzapfel and Gasser, 2001):

$$I_f=\hat{\mathbf{C}}:\mathbf{A}_f=\hat{\mathbf{C}}:\left(\mathbf{a}_f\otimes\mathbf{a}_f\right)=\mathbf{a}_f\cdot\hat{\mathbf{C}}\cdot\mathbf{a}_f=\lambda_f^2 \tag{20}$$

where $\mathbf{a}_f$ is the unit vector defining the fiber orientation in the reference configuration. The invariant I_f has the particular physical interpretation of the quadratic stretch λ_f^2 of the fiber. Other invariants may be included, considering also coupling behaviors among fiber, but we remain with this one only for simplicity reasons. The total stretch may be also decomposed in elastic and viscous contributions:

$$\lambda_f=\lambda_f^{e}\lambda_f^{v} \tag{21}$$

This decomposition is related to Fig.1. An elastic branch depending on the total value λ_f is due to the existence of an elastic potential $\varphi_f(\lambda_f)$. Also, a Maxwell branch allows for the decomposition in Eq.(21) and for elastic potential $\varphi_f^{e}(\lambda_f^{e})$ and dissipative potential $\psi_f(d^{v})$ where d^{v} is defined as

$$d^{v}=\dot{\lambda}_f^{v}{\lambda_f^{v}}^{-1} \tag{22}$$

This equation defines the viscous flow of fibers that must be integrated in order to obtain the incremental expressions. To this aim it is used the exponential mapping proposed in Anand and Weber (1990) that allows to write

$$\lambda^{v}_{f\,n+1}=\exp\left(\Delta t d^{v}\right)\lambda^{v}_{f\,n}\,;\qquad d^{v}=\frac{1}{\Delta t}\ln\left(\frac{\lambda^{v}_{f\,n+1}}{\lambda^{v}_{f\,n}}\right) \tag{23}$$

With these definitions and following analogous arguments found in Fancello *et al.*(2006) it is defined the fiber potential

$$\Psi_f=\Delta\varphi_f\left(\lambda_{f\,n+1}\right)+\min_{\lambda^{v}_{f\,n+1}}\left\{\Delta\varphi_f^{e}\left(\lambda^{e}_{f\,n+1}\right)+\Delta t\,\psi\left(d^{v}\left(\lambda^{v}_{f\,n+1}\right)\right)\right\} \tag{24}$$

$$\Delta\varphi_f\left(\lambda_{f\,n+1}\right)=\varphi_f\left(\lambda_{f\,n+1}\right)-\varphi_f\left(\lambda_{f\,n}\right) \tag{25}$$

$$\Delta\varphi_f^{e}\left(\lambda^{e}_{f\,n+1}\right)=\varphi_f^{e}\left(\lambda^{e}_{f\,n+1}\right)-\varphi_f^{e}\left(\lambda^{e}_{f\,n}\right) \tag{26}$$

Since the fibers only contribute for positive elongations, the minimization problem (24) only makes sense for positive stretches. In this case, optimality conditions are easily solved by Newton technique..

Once the value of λ_f^v is obtained by minimization, the contribution to the first Piola-Kirchhoff stress tensor is obtained from Eq.(4).

4. MATERIAL MODELS

Different material behaviors may be represented from the adequate choice of the potential functions. Those used in the present article are shown bellow.

4.1. Ogden Model

Ogden model is usually chosen due to its flexibility of representation of polymeric materials. For the isotropic contribution the following expressions are used:

$$\varphi = \sum_{j=1}^{3} \sum_{p=1}^{3} \frac{\mu_p}{\alpha_p} \left(\left[\exp(\varepsilon_j) \right]^{\alpha_p} - 1 \right) \quad ; \quad \varphi^e = \sum_{j=1}^{3} \sum_{p=1}^{3} \frac{\mu_p^e}{\alpha_p^e} \left(\left[\exp(\varepsilon_j^e) \right]^{\alpha_p^e} - 1 \right) \tag{27}$$

$$\psi = \sum_{j=1}^{3} \sum_{p=1}^{3} \frac{\eta_p^v}{\alpha_p} \left(\left[\exp(d_j^v) \right]^{\alpha_p^e} - 1 \right) \tag{28}$$

4.2. Fiber Model

In Holzapfel and Gasser (2001) is presented a hyperelastic model in which the collagen fibers are governed by a potential of type

$$\varphi = \frac{k_1}{2k_2} \left\{ \exp\left[k_2 \left(I_f - 1 \right)^2 \right] - 1 \right\} \quad ; \quad \varphi^e = \frac{k_1}{2k_2} \left\{ \exp\left[k_2 \left(I_f^e - 1 \right)^2 \right] - 1 \right\} \tag{29}$$

where $I_f^e = (\lambda_f^e)^2$. Note again that this expression is only valid for positive values of λ_f.

5. NUMERICAL EXAMPLES

In order to verify the capability of the proposed model to represent the behavior of anisotropic materials some examples are shown in this section. The model was implemented in the GNU Octave code and in the academic finite element code METAFOR - developed by LTAS, Belgium - to perform, respectively, uniaxial and three-dimensional examples. The rheological model chosen for the examples is shown in Fig.1 with only one Maxwell branch. For the isotropic contribution a Neo-Hookean material is used. For the fibers, the potential proposed by Holzapfel and Gasser (2001) is tested. In the first test the body is submitted to an axial stretching following a sinusoidal law *f(x) = 1 + 0.5 sin(0.3x)*. The material parameters, taken from Limbert *et al.* (2004), are shown in Tab.1 and the results in Fig.2.

Figure 2 shows clearly a hysteretic behavior due to the viscous contribution as well as the hardening introduced by the fibers for positive elongations.

Table 1. Material parameters (Limbert *et al.*, 2004).

Potential	Neo-Hookean	Holzapfel
φ	$\mu = 2$ MPa	$c_1 = 1.7939$ MPa e $c_2 = 11.2055$ MPa
φ^e	$\mu^e = 2$ MPa	$c_1 = 1.7939$ MPa e $c_2 = 1.2055$ MPa
ψ	$\eta = 15.0087$ MPa.s^{-1}	$\eta = 15.0087$ MPa.s^{-1}

In the second example, the same material parameters are used in a membrane-like 3D body subject to a pressure of 1 N/mm^2 from bellow. The membrane is clamped on its borders. 4 hexahedrical 8-node elements along the thickness are used. A family of fibers oriented along *X* direction are included. The result is shown in Fig. 3.

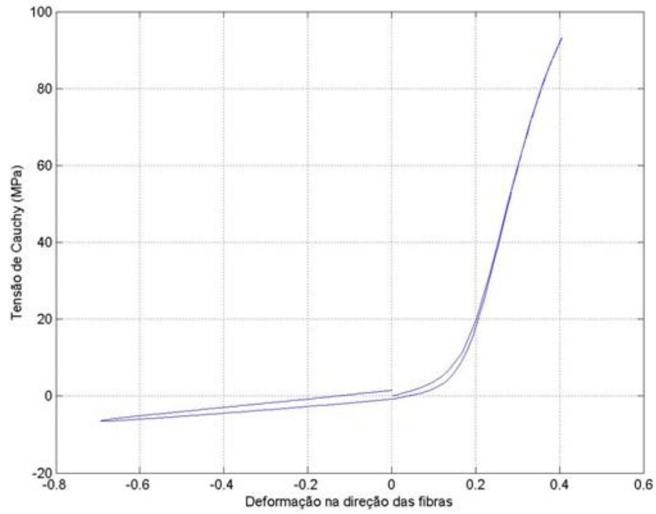

Figure 2. Stress-strain (natural) behavior.

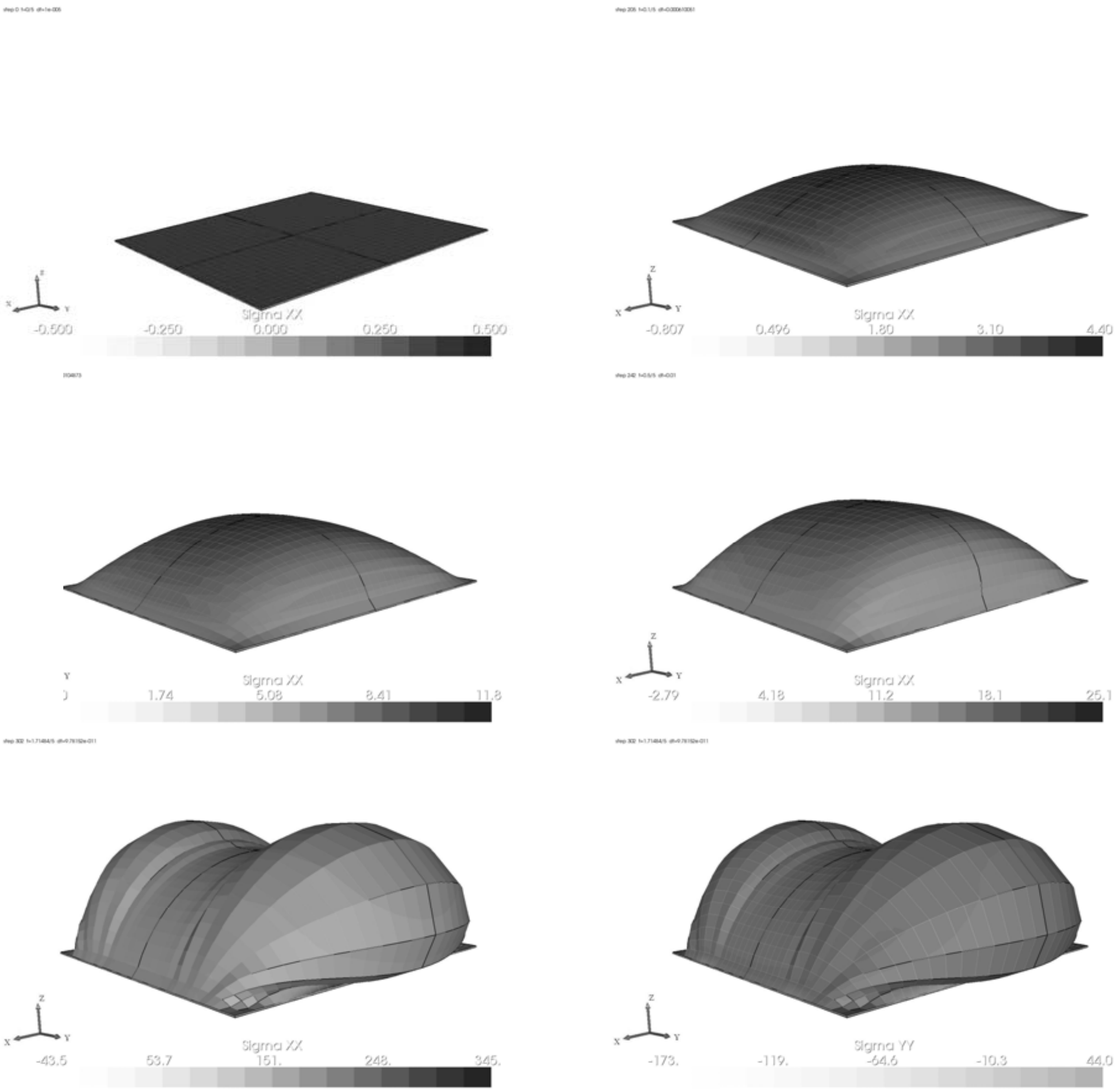

Figure 3. Sequence of configurations in the membrane.

A third example is presented to show an anisotropic behavior in a fiber reinforced composite tube, where the orientation of the fibers reinforcing at the wall are 30°. The tube is submitted to an axial linear stretching and the same elements and material parameters of the last example are used. Figure 4 shows the obtained behavior.

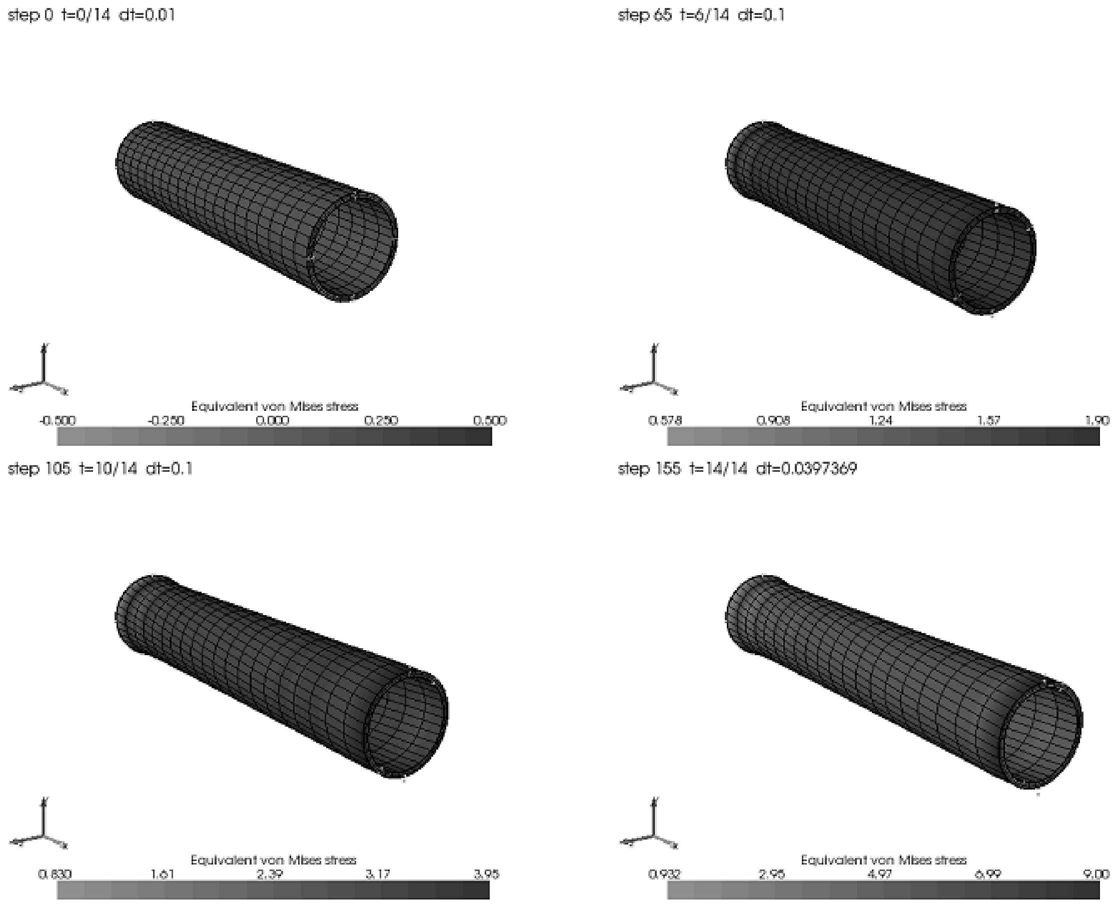

Figure 4. Sequence of configurations in the fiber reinforced composite tube.

Fig.3 clearly shows the anisotropic behavior introduced by the fibers. The stretching along X direction is somehow contained while typical membrane instability is rapidly achieved for stretches along Y direction.

Fig. 4 shows the typical behavior of reinforced tubes in which the reinforcing fibers are not aligned with the axis of the tube (X axis). In this case the reinforced tube rotates under axial stretching.

6. FINAL REMARKS

The main objective of extending the isotropic variational approach to the case of fiber-reinforced materials was achieved. In the preliminary results, the developed model showed the expected oriented behaviors including rate dependence of both isotropic and fiber contributions and the proposed numeric examples demonstrated the capability of the anisotropic model to represent different cases of application.

It is worth mentioning also that the presented examples have the goal of verifying the ability of the proposed approach to follow expected qualitative behaviors. Material identification for the proposed potentials is the objective of further works, relating this formulation to specific experimental data.

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