OPTIMIZATION OF SATELLITE ATTITUDE ON TERRESTRIAL CIRCULAR ORBIT USING THREE INTERNAL REACTION WHEELS

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Abstract. *The attitude of an artificial satellite on terrestrial orbits is analyzed through the application of external torques to the vehicle using thrusters, reaction wheels or external gravitational torque, and considering also the presence of solar wind and magnetic torques. The use of reaction wheels eliminates the need of thrusters or gas jets as external perturbation sources and they can be used also as moment wheels in the stabilization procedure. This work presents the method of attitude optimization of artificial satellites equipped with three reaction wheels. In the vehicle being studied, which is on a terrestrial circular orbit, gravitational gradient torques are applied. The angular velocities of the reaction wheels have their vectors coincident with the satellite principal axes of inertia. For the determination of optimal trajectories, numerical methods are developed with applied torques as external perturbation on the internal wheel axles. A numerical solution is presented on an approach employing performance range for minimum energy. The numerical solution is accomplished through a program implemented within the Matlab software, which uses the function bvp4c of the Matlab subroutines for optimization of a boundary value solution in two points.*

Keywords: *Satellite, Attitude, Control, Optimization, Minimization*

1. INTRODUCTION

Considering that nonlinear coupled equations would result from models based on realistic satellite architecture, many projects in the field of attitude maneuver control of a rigid space vehicle have been developed using satellite configurations with only one or two control axes**.** Concerning this, a historic description will be presented in this chapter, which will be very useful for the comprehension of the present research.

The regions of stability of a satellite equipped with one single internal wheel, along with one of the satellite principal axis of inertia, and subjected to disturbing gravitational torques, have their limits of stability determined for all its equilibrium positions (Longman, 1981). The limits of stability were obtained for all possible satellite equilibrium positions, where the variations of these regions corresponded to changes on the internal body angular momentum magnitude, or on the alignment of this moment with another body principal axis. The research in this field evolved with a work based on a satellite equipped with more than one internal wheel (Sarychev, 1982), where it was presented a devise for damping the satellite nutation oscillations, which comprised a gyroscopic system, embodying only one degree of freedom, and with a single spring-mass-dashpot system within it. More recently, an interesting research (Druzhinin, 1999) dealt with the permanent rotation motion of a satellite, developed with internal rotors, assembled over the satellite center of gravity, where it was proved that in case the total angular momentum of the that spacecraft is not null, the permanent rotations can occur only around its principal axis of inertia. Subsequently, a research studied the stabilization of the equilibrium positions of a satellite with internal rotors and with the rotors moment control employing servo-motors (El-Gohary, 2001). As a satellite can not be stabilized around a single equilibrium position, a method was conceived focusing on a control law with feedback, which would stabilizes globally and asymptotically the vehicle under a revolute motion, around a specific inertial axis (Kim, 2001). In the same research, an analytical method showed that there are no less than eight and no more than twenty four isolated cases of equilibrium on a satellite, when the angular motion is exclusively around the vehicle body Y axis (Sarychev, 2001). Considering the attitude optimization, a procedure for calculating the attitude maneuvering with minimization of time was developed using a gyroscopic device with moment control (Kranton, 1970). The problem of time minimization of space vehicles attitude was dealt with using a numerical method, with limitations of forces and torques and with the time determined through sequential reductions of the maneuvering time, until the limits would approach the bounds imposed for the problem, configuring the problem as a Bang Bang procedure (Li and Bainum, 1990). The boundary value problem in two points was derived from the application of the minimum principle of Pontryagin, solved using a quasi-linearization program. It was established analytically in a further work that a rigid space vehicle, embodying inertial symmetry, can be controlled independently on three axes; but without being simultaneously singular, in any moment, in the minimum time maneuvering procedure (Bilimoria, 1993). In a complementary work on the field of minimum time optimization control, the solution was obtained through a switching algorithm, using an open loop procedure (Meier and Bryson, 1993). The same algorithm was used to obtain approximate solutions, with switching control, for minimization of time of a satellite attitude maneuver (Byers, 1993). A solution employing the minimization of time procedure, offering many control options of the attitude maneuver problem of an inertial symmetric space vehicle, where three independent control torques are applied, each one aligned with one vehicle inertia principal axis and where the bang bang method for a finite and infinite singular control strategy was used, was published showing that for certain problems, all the control procedures for minimum cost were of an infinite order (Seywald, 1993). In a later publication, it was shown that it is possible to simulate many solutions using a variety of space vehicle configurations, through a minimum time maneuver attitude control, for rigid and flexible bodies (Scrivener, 1994). Lately, a control method for assuring satellite optimal stabilization was proposed, for a vehicle equipped with three reaction wheels. In this case, the controlling action is obtained through the rotational motion of the internal reaction wheels.

As a complement of the above-mentioned studies, the present work will analyze, in particular, the case of the optimization control strategy, based on the minimum energy approach.

2. EQUATIONS OF MOTION

The satellite inertial parameters are defined as follows (Bryson, 1994):

$$a = \frac{I_{yy} - I_{xx}}{I_{zz}} \tag{1}$$

$$b = \frac{I_{zz} - I_{yy}}{I_{xx}} \tag{2}$$

Where:

a and *b* are the inertia parameters of the vehicle;
I_{xx}, I_{yy} and I_{zz} are the principal moments of inertia of the vehicle.

The angular motion is relative to the vehicle center of mass. The kinematics equations are:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + T\phi \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + T\phi T\theta \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + T\phi T\theta T\psi \begin{bmatrix} 0 \\ -n \\ 0 \end{bmatrix} \tag{3}$$

$$\begin{aligned} \dot{\phi} &= p - n \sec\theta \ \ \sin\psi + r\cos\phi \ \ \tan\theta + q\sin\phi \ \ \tan\theta \\ \dot{\theta} &= q\cos\phi - n\cos\psi - r\sin\phi \\ \dot{\psi} &= \sec\theta\left(r\cos\phi + q\sin\phi - n\sin\theta \ \ \sin\psi\right) \end{aligned} \tag{4}$$

Where:

n is the orbit angular velocity, and is defined as:

$n = \sqrt{\frac{g}{R}}$, being *g* the earth gravity acceleration and *R* the distance from the earth center of gravity and satellite center of gravity.

p, q and *r* are the angular velocities of the vehicle, relative to the x, y and z body axes, respectively

ϕ, θ, ψ are the Euler's angles

The angular velocities are in function of *n* and of the Euler's angles for the equilibriums positions.

$$
\begin{aligned}
p &= -n\cos\theta \quad \sin\psi \\
q &= -n(\cos\phi \quad \cos\psi + sen\theta \quad \sin\phi \quad \sin\psi) \\
r &= -n(-\cos\psi \quad \sin\phi + \cos\phi \quad \sin\phi \quad \sin\psi)
\end{aligned} \tag{5}
$$

The internal reaction wheels dynamic equations of the angular momentum are (Junkins, 1986):

$$
\begin{aligned}
\dot{h}_1 &= u_1 - J_a\dot{p} \\
\dot{h}_2 &= u_2 - J_a\dot{q} \\
\dot{h}_3 &= u_3 - J_a\dot{r}
\end{aligned} \tag{6}
$$

Where $\dot{h}_1$, $\dot{h}_2$ and $\dot{h}_3$ correspond to the time rate of the angular momentum of the vehicle motion and u_1, u_2 and u_3 correspond to the torques applied on the internal reaction wheels and J_a is the polar moment of inertia of the reaction wheels.

The dynamic equations are as follows:

$$
\begin{aligned}
\dot{p} &= \frac{2((1+ab)h_3nq - (h_2n(1+ab) + (a-1)bq)r + u_1 + abu_1) + 3n^2(a-1)b\cos^2\theta\sin(2\phi)}{2(a-1+J_a+abJ_a)} \\
\dot{q} &= \frac{-(1+ab)h_3np + (h_1n(1+ab) - (a+b)p)r + u_2 + abu_2 - 3n^2(a+b)\cos\theta\cos\phi\sin\theta}{(1+ab)(J_a-1)} \\
\dot{r} &= \frac{(1+ab)h_2np - h_1nq - abh_1nq + apq + abpq + u_3 + abu_3 + 3n^2a(1+b)\cos\theta\sin\theta\sin\phi}{-1-b+J_a+abJ_a}
\end{aligned} \tag{7}
$$

In the equations (6), the angular momentum, as well as its time rate, the torque applied on the internal reaction wheel and its moment of inertia, are "parameterized" as a function of the principal moment of inertia I_{yy}:

$$
h_{1,2,3} = \frac{h_{x,y,z}}{I_{yy}}, \quad \dot{h}_{1,2,3} = \frac{\dot{h}_{x,y,z}}{I_{yy}}, \quad u_{1,2,3} = \frac{u_{x,y,z}}{I_{yy}}, \quad J_a = \frac{J_a}{I_{yy}} \tag{8}
$$

The system is described through the nonlinear Eq. (4), (6) and (7), to satisfy the requirements of the model:

$$
\dot{x}(t) = f(x,u,t) \tag{9}
$$

With:

$$
x = \{p, q, r, h_1, h_2, h_3, \phi, \theta, \psi\}; \qquad u = \{u_1, u_2, u_3\}
$$

The performance index is defined as follows (Lewis, 1986):

$$
J(t_0) = \phi(x(T),T) + \int_{t_o}^{T} L(x(t),u(t),t)dt \tag{10}
$$

The Hamiltonian function is written as:

$$
H(x,u,\lambda) = L(x,u,t) + \lambda^T(x,u,t) \tag{11}
$$

And the co-states dynamic:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x}(x, u, \lambda) \tag{12}$$

Being the optimality condition defined as:

$$0 = \frac{\partial H}{\partial u}(x, u, \lambda) \tag{13}$$

The Eq. (9) and (10) and the constraints $x(0) = x_0, (x_0 \in R^n)$, and $\psi(x(T), T) = 0, (\psi \in R^q)$, with $q \leq n$, lead to a "boundary value problem in two points", whose boundary conditions vector is:

$$R[x(0), x(1)] = \begin{bmatrix} y(0) - y_0 \\ \psi[y(1), 1] \\ \lambda(1) - \dfrac{\partial \phi^T(1)}{\partial y} - \dfrac{\partial \psi^T(1)}{\partial y} \nu \\ H(1) \end{bmatrix} = 0, R[x(0), x(1)] \in R^N \tag{14}$$

3. ATTITUDE OPTIMIZATION

In this chapter, the problem formulation will be presented with its computational implementation, and the results of the motion optimization control of artificial satellites on circular orbits, subjected to moments due to the gravitational force.

The attitude maneuver is done from a position defined by the known initial and final states, characterizing it as a boundary value problem (BVP) in two points, of the continuous systems relative to time optimal control theory. The maneuver is done having as control sources the applied torques over the three internal reaction wheels.

3.1. bvp4c – Description of the numerical method

The MatLab function bvp4c implements a method for the solution of a BVP as following (Shampine, 2000):

$$y' = f(x, y, p), \qquad a \leq x \leq b \tag{15}$$

Where:
a is the parameter initial state x
b is the parameter final state y
and *p* is a unknown parameter vector

Subjected to boundary conditions in two points, generally nonlinear:

$$g(y(a), y(b), p) = 0$$

And the above relation satisfies the differential equations at the intermediate and final points of each interval:

$$S'(x_n) = f(x_n, S(x_n)) \tag{16}$$

$$S'\left(\frac{x_n + x_{n+1}}{2}\right) = f\left[\frac{x_n + x_{n+1}}{2}, S\left(\frac{x_n + x_{n+1}}{2}\right)\right] \tag{17}$$

$$S'(x_{n+1}) = f(x_{n+1}, S(x_{n+1})) \tag{18}$$

These conditions result in a non linear system of algebraic differential equations with the S(x) coefficients. Differently from the method of shots, the solution *y(x)* is approximate over the interval [a, b] and the boundary conditions are taken into consideration at each step of the solution process. The algebraic equations are solved iteratively by linearization, as the methods considered in this study employ the MatLab tools for the solution of linear equations. The basic method utilized by the bvp4c function is known as "collocation method".

3.2. Formulation of the actuation control problem for the minimization of energy

The model is described by the nonlinear equation variables on time. The performance index is chosen considering as a constraint the activation control minimum energy approach, for accomplishing the desired maneuvering attitude, taking the system from the initial to the final position.

The applied torqueses, as control sources, are applied on the three internal wheels $(u_1(t))$, Eq. (6), determining the system function (Junkins, J. L, Kim, Y., 1993):

$$L(x(t),u(t),t) = \frac{1}{2}\sum_{i=1}^{3} u_i^2(t) \tag{19}$$

The problem is formulated with the final state free, which is a condition that includes in the equation the performance index, the final state function $\phi(x(T),T)$, where the state variables tend to assume a very close value to the desired final state variables.

Then, the performance index, if a weighing value S is included in its final state, considering the Eq. (9), is as follows:

$$J = \frac{1}{2}(x(T)-r(T))^T S(T)(x(T)-r(T)) + \frac{1}{2}\int_0^T L dt \tag{20}$$

The function L is defined as follows:

$$L = \frac{1}{2}(u_1^2 + u_2^2 + u_3^2) \tag{21}$$

The nine differential equations which define the system, being three of them the ones that determine its kinematic behavior, Eq. (1), and six that determine its parameterized dynamic behavior, Eq.(6) and Eq.(7), being inserted in Eq. (11) result in the Hamiltonian equation, defined as follows:

$$H = \frac{1}{2}(u_1^2 + u_2^2 + u_3^2) + \lambda_p \dot{p} + \lambda_q \dot{q} + \lambda_r \dot{r} + \lambda_{h_1}\dot{h}_1 + \lambda_{h_2}\dot{h}_2 + \lambda_{h_3}\dot{h}_3 + \lambda_\phi \dot{\phi} + \lambda_\theta \dot{\theta} + \lambda_\psi \dot{\psi} \tag{22}$$

The co-state differential equations are determined employing the Eq.(22), and the control equations, in conformity with Eq.(13).

By deriving the Hamiltonian equations, in relation to each state variable, we obtain the nine co-states differential equations (Bettiato, 2003).

The control inputs are obtained under the condition of "optimality" Eq.(14) being applied to the Hamiltonian equation:

$$u_1 = -\frac{\lambda_{h1}(a-1) + \lambda_p(ab+1)}{J_a + abJ_a + a - 1} \tag{23}$$

$$u_2 = -\frac{\lambda_{h2} + \lambda_q}{J_a - 1} \tag{24}$$

$$u_3 = -\frac{\lambda_{h3}(b+1) + \lambda_r(ab+1)}{J_a + abJ_a - b - 1} \tag{25}$$

3.3. Computational implementation of the minimum energy control problem

The elaboration of a computational program for solving the optimum control problem with the control actuation for the minimization of energy was done with the aid of the software Matlab 6.0, through the Matlab optimization package function "bvp4c", for the solution of a boundary value problem in two points.

The computational implementation presented in this item constitutes a generalized form, which will be applied on the other optimization program, with some distinctions.

The state and costate vectors are as follows:

$$y = [p, q, r, h_1, h_2, h_3, \phi, \theta, \psi, \lambda_p, \lambda_q, \lambda_r, \lambda_{h1}, \lambda_{h2}, \lambda_{h3}, \lambda_\phi, \lambda_\theta, \lambda_\psi] \tag{26}$$

The system dynamic is described through nine state differential Eq. (1), (6) and (7) and nine co-state differential equations (Bettiato, 2003), totalizing eighteen equations for the whole system, including the three internal wheels dynamics. The controls are determined through the Eq. (23)-(25).

For this case of the optimization problem solution for the minimization of energy of the actuation control, eighteen boundary conditions were established, being nine for the initial states and nine for the final ones.

For the final states, the vector boundary conditions components were computed as a function of the final state $\phi(x(T),T)$, from the Eq. (20), which for a quadratic index of performance with a weighted final state, is:

$$\phi(x(T),T) = \frac{1}{2}(x(T) - r(T))^T S(T)(x(T) - r(T)) \tag{27}$$

Defining the co-state problem as:

$$\lambda_X(T) = \left.\frac{\partial \phi}{\partial x}\right|_T = S_X(x(T) - r(T)) \tag{28}$$

The difference between the co-state and the right side of the Eq. (27) generates a residue for the final states which are terms of the boundary conditions vector.

The program includes the "continuation method", being this method characterized for repetitions on the program execution, where the result obtained in the former execution is utilized, with the purpose of adjusting the weighed variables (S(T)) for improving the precision of the result.

The proposed problem was developed with the purpose of determining the applied torques on the internal wheels, to produce the angular displacement of a satellite with some inertial relations, minimizing as a result the actuation control energy. The satellite is on a circular terrestrial orbit and, undergoing the effects of the moments due to the gravitational force.

The following figure illustrates the initial and final position of a satellite with the internal amount of angular displacement:

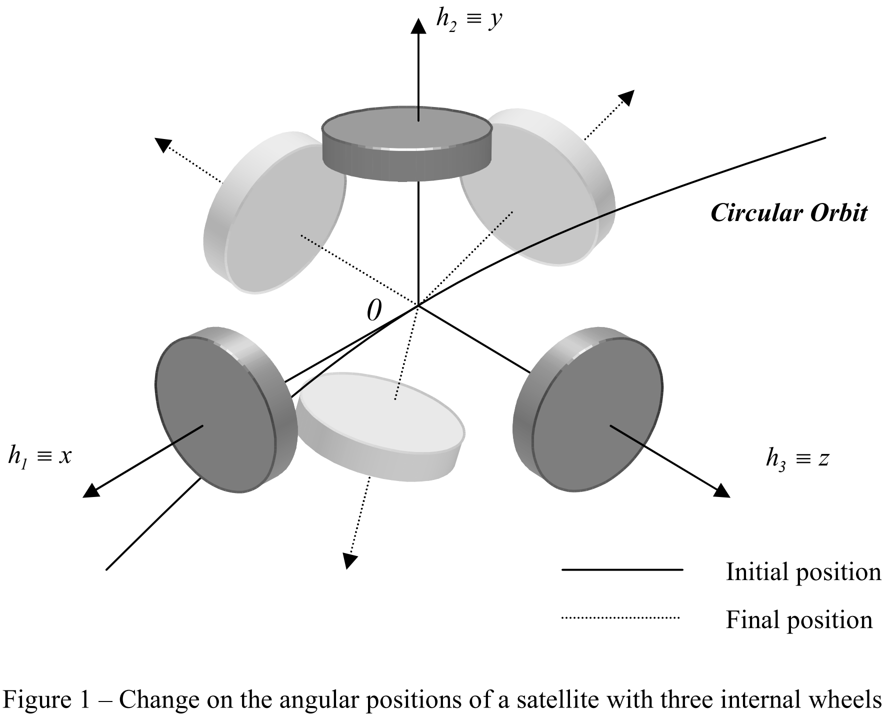

Figure 1 – Change on the angular positions of a satellite with three internal wheels

The satellite being studied is of the flat kind, with the following values of the inertial parameters ***a*** and ***b***:

a = - 0.5
b = 0.2

These parameters correspond to the following inertial relations:

I_{xx}=1.667 I_{yy}
I_{zz}=1.333 I_{yy}

This work uses time expressed in canonic units. Its advantage is to integrate the dynamic equations easily in the numerical development. In this paper, one unit of time (u.t.) in canonic units is the necessary time for the satellite to translate one radius arc in its orbit. In this case, the distance from the center of mass to the center of the inertial referential frame, is 6,978 Km. Then, one canonic unit of time is, approximately, 923 seconds (01 *u.t.* $\cong$ 923 sec).

The initial as well as the final positions constitute equilibrium positions and are functions of the internal amount of angular displacement, and in this case are:

Initials:	$\phi = 0.081 \quad rd$ $\theta = -0.014 \quad rd$ $\psi = -0.174 \quad rd$ $h_1 = 0.1 \quad rd/u.t$ $h_2 = 0.1 \quad rd/u.t$ $h_3 = 0.1 \quad rd/u.t$	Finals:	$\phi = 0.014 \quad rd$ $\theta = -0.081 \quad rd$ $\psi = -1.398 \quad rd$ $h_1 = 0.1 \quad rdc.u.t.$ $h_2 = 0.1 \quad rd/u.t$ $h_3 = 0.1 \quad rd/u.t$

The orbit angular velocity is:

n= - 1 rd/u.t.

As the angular positions are equilibrium positions, the satellite angular velocity is the orbit angular velocity itself, which is expressed in the reference system of the body through the components p, q and r, due to the Euler's angles ϕ, θ and ψ .

So, due to the satellite positions, initial and final, the components p, q and r are:

Initials: p = - 0.173 *rd/u.t* q = 0.982 *rd/u.t* r = - 0.077 *rd/u.t*	Finals: p = - 0.982 *rd/u.t* q = 0.173 *rd/u.t* r = 0.077 *rd/u.t.*

In addition to the orbit angular velocity n and to the inertial parameters a and b and to the initial and final states, the following data are also used:

Parameterized axial moment of inertia of the internal wheels:

J_a = 0.05

Number of intervals of the time vector:

Int = 10

The weighing values of the final state are:

S_1 = 100, S_2 = 10000 and S_3 = 100000

The maneuver final time:

T = 0.2 u.t.

Obs.: The same data are used in the subsequent problems

The problem results are presented in graphic form as follows:

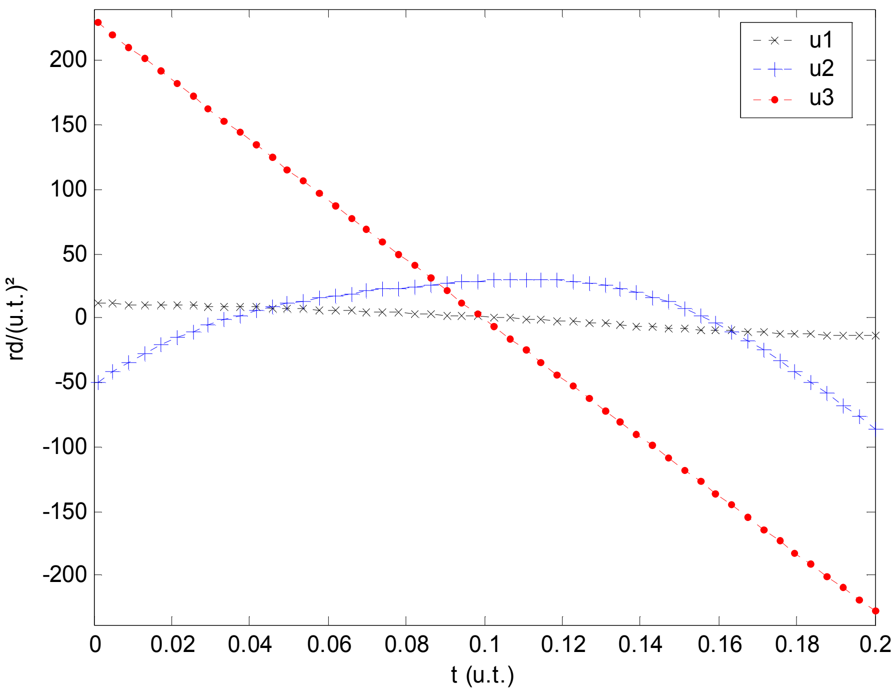

Figure 1. Controls input (u1 u2 u3)

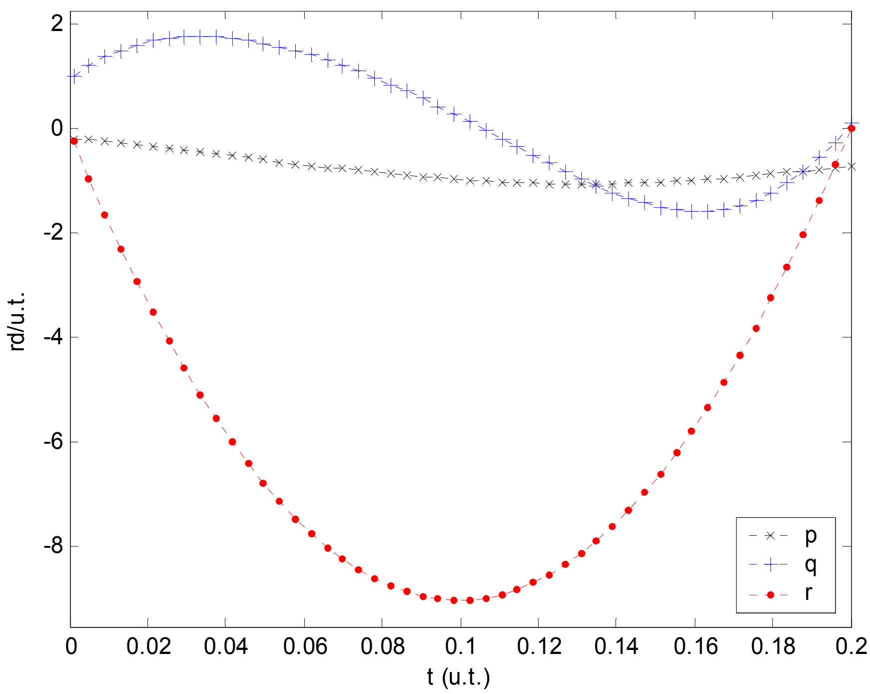

Figure 2. Angular velocities

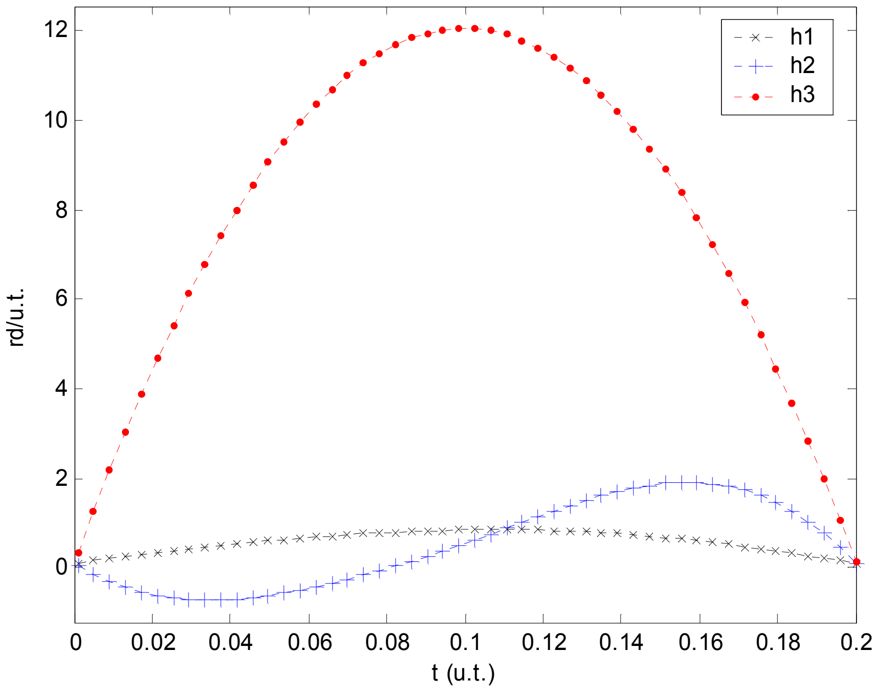

Figure 3. Parameterized angular momentum (h1 h2 h3)

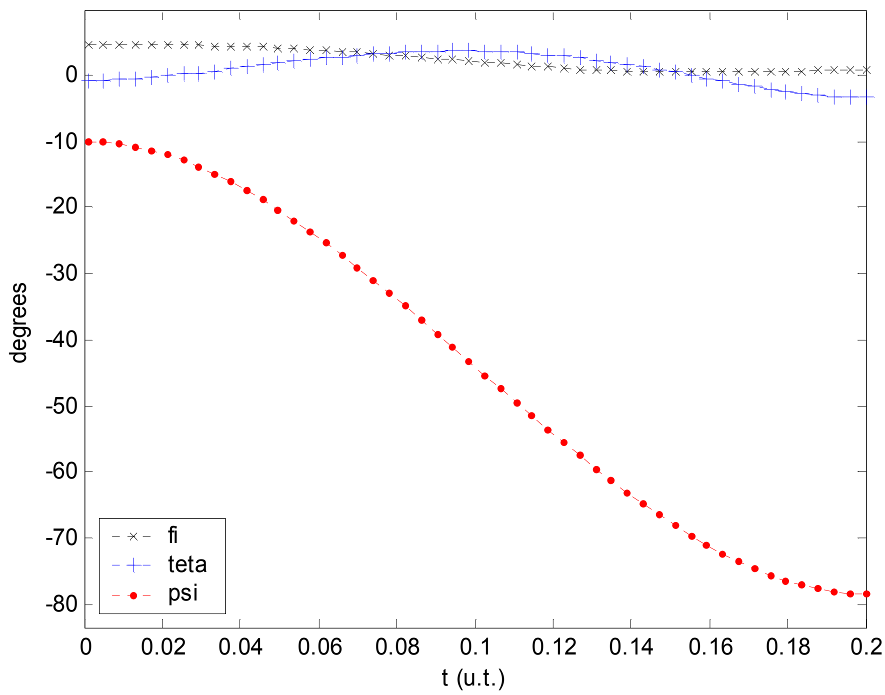

Figure 4. Euler's angles ($\phi\ \theta\ \psi$)

4. CONCLUSIONS

The intent of this work, to determine the equilibrium positions of a satellite equipped with three internal reaction wheels, through an optimization procedure using numerical methods, was satisfactorily accomplished using the Matlab function "bvp4c", for the solution of a boundary value problem in two points. The use of numerical methods was justified due to the fact that the manipulation of analytical methods for the solution of a rigid body equation of motion is extremely difficult. However, this can be the base for a future work in this field, to complement the literature on satellite control and the theoretical support on the matter of satellites equipped with three reactions wheels with internal motion. Also, the inclusion of the magnetic torques and body center of gravity offset effect could be analyzed in a future work, being this aspect important in the research of satellite optimal control.

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