STABILITY ANALISYS OF AN AEROELASTIC TYPICAL SECTION BY THE STRUCTURED SINGULAR VALUES METHOD

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Abstract: *The main goal of this work is to validate the Structured Singular Value method to analyze the Flutter Margin, by comparing with results obtained from methods of flutter analysis traditionally employed in the aeronautical industry, that presents modal evolution or the geometric root locus, taken as a reference. In order to perform this validation, it is adopted as a model the aeroelastic typical section with three degrees of freedom subjected to aerodynamic forces and moments, derived from Theodorsen´s formulation. The reference flutter speed is calculated by the Method K. Applying the Roger rational function approximation to the aerodynamic influence matrix it is possible to create a state space approximation of the aeroelastic model and, by analyzing the state matrix eigenvalues evolution with increasing speed, the instability condition of the approximate model can be determined. Nominal model uncertainty parameters are defined by the true airspeed and the dynamic pressure, and these parameters are inserted in the model by the application of the Linear Fraction Transformation. The Flutter speed is then calculated using the Structured Singular Value and compared with its reference value.*

Keywords: *Aeroelasticity, Robust Stability, Flutter Margin, Structured Singular Values*

1. INTRODUCTION

The design requirements for commercial aircraft, related to Aeroelasticity, imposed by FAA (*Federal Aviation Administration*), are presented in the FAR (*Federal Aviation Regulation*) Section 25.629 [rgl.faa.gov]. The general requirement is that the aircraft is free from aeroelastic instability for all of the project conditions and configurations inside the aeroelastic stability envelope.

Aeroelastic instabilities are potentially destructive to the aircraft. Thus, in new aircraft designs or in new configurations to already existing aircraft, the characteristics of the aeroelastic dynamic stability should be investigated to determine if the flight envelope is free from Flutter, considering the parameters that may vary.

Flutter is an aeroelastic phenomenon characterized by a self-excited oscillation of an airfoil and its associated structure, caused by the combination of inertia forces, elasticity and aerodynamics. The structural component vibrates in its natural frequency under the effect of the aerodynamic forces. At a certain speed, called *Flutter critical speed*, the oscillation amplitude is kept at a constant value. As the speed increases, the movement amplitude grows until the point of structural failure. The mode of vibration during the Flutter is known as the *Flutter mode*.

This phenomenon is a constant concern for aircraft designers due to the large amount of possible Flutter modes and the number of tests required to determine these modes to guarantee that the aircraft is free from aeroelastic instabilities. Once the Flutter modes are known, the structural stiffness can be changed or a balance masses can be added to suppress its occurrence.

The Flutter boundary is defined as a line in the flight envelope, defined by altitude values and Mach number for which the system is at imminent instability, that is, the system's dynamic response to initial conditions that are different from equilibrium is oscillatory and non-damped, or simple harmonic oscillations. There are different ways to determine this boundary, but the use of traditional methods requires performing an analysis for variation of several parameters.

The robust aeroelastic analysis aims to find a stability margin for a system with multiple inputs and outputs, considering inherent parametric variations of the system without the need to perform the Flutter calculation for every parametric variation, reducing the time of analysis.

2. AEROELASTIC STABILITY

The aeroelastic system considered is constituted by the wing and the control surface. The physical model was idealized as constituted by two rigid bodies interconnected by springs, as illustrated in Fig. 1. The *typical section* is a model that represents the main modes of a high aspect-ratio wing and without sweep angle and, although it is a simple model, its use is justified by the possibility of validation of methods that can be used in more complex and representative models. The Eq. (1), developed for a *typical section*, refers to the structural degrees of freedom of the section. The three degrees of freedom referred to plunge, pitch and deflection of control surface are characterized by six states in the state space model corresponding to the respective displacements and speeds.

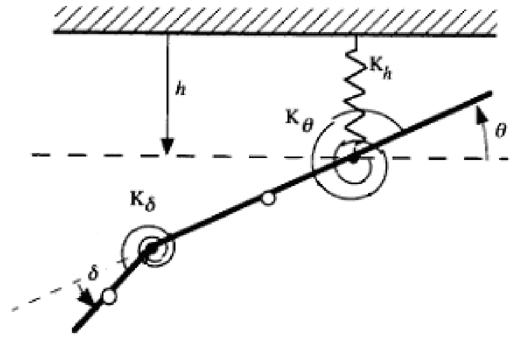

Figure 1. Aeroelastic *Typical Section* with Three Degrees of Freedom, Waszak (1998)

For the ideal system three generalized coordinates were considered: one fixed coordinate in the wing to represent the plunge (h); one fixed coordinate in the inertial system, chosen as the non-strained position of the structure, which represents the pitch (θ); and another fixed in the control surface, to represent its deflection (δ). So the application of the Lagrange equation leads to the following mathematical model for this system:

$$\begin{bmatrix} m & S_{h\theta} & S_{h\delta} \\ S_{h\theta} & I_\theta & S_{\theta\delta} \\ S_{h\delta} & S_{\theta\delta} & I_\delta \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\theta} \\ \ddot{\delta} \end{Bmatrix} + \begin{bmatrix} K_h & 0 & 0 \\ 0 & K_\theta & 0 \\ 0 & 0 & K_\delta \end{bmatrix} \begin{Bmatrix} h \\ \theta \\ \delta \end{Bmatrix} = \begin{Bmatrix} -L_b \\ M_\theta \\ M_\delta \end{Bmatrix} \tag{1}$$

The non-steady aerodynamic forces (L_b, M_θ e M_δ) are calculated based on the linearized theory of thin airfoils, by Theodorsen (1935), in which the surface is treated as a flat plate subjected to a simple harmonic movement, assuming an incompressible regime, potential flow and small oscillations. It is assumed in this situation that the airfoil oscillation happens in relation to the elastic axis. The Eq. (2) presents this calculation.

$$\begin{Bmatrix} -L_b/b \\ M_\theta/b^2 \\ M_\delta/b^2 \end{Bmatrix} = q\left[2C(k)\{R\}[S_1]\{x_S\} + \frac{2b}{V}C(k)\{R\}[S_2]\{\dot{x}_S\} + \frac{2b^2}{V^2}[M_{nc}]\{\ddot{x}_S\} + \frac{2b}{V}[B_{nc}]\{\dot{x}_S\} + 2[K_{nc}]\{x_S\}\right] \tag{2}$$

In the Eq. (2), x_s represents the structural degrees of freedom of the typical section. The parameter q corresponds to dynamic pressure and V corresponds to true airspeed; except for $C(k)$, the other parameters are functions of the section geometry. The values used in this work are the same used by Olds (1997) (listed on Tab.1).

Table 1. Model's Parameters.

Parameter	**Value**	**Unit (SI)**
b	0.914	[m]
c	1	
m	128.7165	[Kg/m]
ρ	1.225	[Kg/m^3]
S_α	23.5397	[Kg]
S_β	1.4712	[Kg]
I_α	26.90	[Kgm]
I_β	0.673	[Kgm]
K_h	m50^2	[N/m]
K_α	$I_\alpha 100^2$	[N/m]
K_β	$I_\beta 500^2$	[N/m]

The Theodorsen function, $C(k)$, characterizes (quantifies) the aerodynamic delays in the system and its magnitude relates the circulatory and non-circulatory forces [Da Silva (1994)]. The Theodorsen function can be expressed in terms of Bessel functions (J_1 e Y_1 are first order Bessel functions), as shown in the Eq. (3).

$$C(k) = \frac{-J_1 + iY_1}{-(J_1 + Y_0) + i(Y_1 - J_0)} \tag{3}$$

When the value of $C(k)$ is real and unitary (k=0) the flow is denominated quasi-steady, because the effects of aerodynamic delays are eliminated. The argument k of the Theodorsen function is called *reduced frequency* ($k = \omega \cdot b/V$).

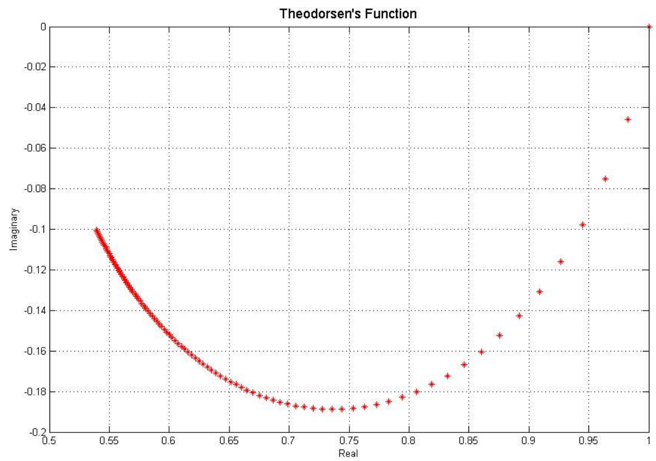

Figure 2. Theodorsen Function (C(k): $0 \leq k \leq 1$).

Defining the vector $x_s = [h \quad \theta \quad \delta]^T$, the equations that describe the model's behavior can be presented as the Eq. (4):

$$[M_S]\{\ddot{x}_S\} + [K_S]\{x_S\} = qA(s')\{x_S\} \tag{4}$$

The inertia matrix is denoted as $[M_s]$ and the stiffness matrix as $[K_s]$, weighted by chord b to correspond to force vector and aerodynamic momentum (see Eq. (2)). The matrix $A(s')$ is called aerodynamic influence matrix ($s'=i \cdot k$).

2.1. Method K to determine the *Flutter Speed*

The Method K [Karpel (1981), Nam and Kim (2006)] is the one that requires the least computational effort among the methods to solve the Flutter problem. Assuming that the system is subject to a simple harmonic movement, expressed by the equation:

$$x_S = x_{S0} e^{i\omega t} \tag{5}$$

Hodges (2002) wrote that experimental observations indicate that the energy removed by cycle during the simple harmonic oscillation is approximately proportional to the amplitude squared, but independent from frequency. This behavior can be characterized by a damping force which is proportional to the displacement time derivative. To incorporate this form of structural damping into the analysis, a structural damping g is added and $1/\omega^2$ is substituted by:

$$\lambda = {(1 + i \cdot g)}/{\omega^2} \tag{6}$$

The numeric values for damping g, which are obtained for each *reduced frequency* value, can be interpreted only as the necessary damping (at a certain way) to obtain a simple harmonic movement at a given frequency. Note that this damping is, in fact, an artificial structural damping, that does not really exist and was introduced as an artifact to produce the desired movement. Using the Laplace transform and substituting Eq. (6) into Eq. (2):

$$\frac{1+ig}{\omega^2}\{x_S\} = [\overline{K}]^{-1}[[\overline{M}] + \frac{1}{\pi\mu}(\frac{1}{k^2}C(k)\{R\}[S_1] + i\frac{1}{k}C(k)\{R\}[S_2] + \ldots$$
$$\ldots - [M_{nc}]\{x_S\} + i\frac{1}{k}[B_{nc}]\{x_S\} + \frac{1}{k^2}[K_{nc}])]\{x_S\} \tag{7}$$

Substituting Eq. (5) into Eq. (7) reveals a configuration of an “eigenvalue problem”, which can be solved calculating the respective values of λ.

Figure 3 shows the variations in the frequencies of aeroelastic modes versus flight speed. The point where the results are correct is exactly in the Flutter speed. Nevertheless, the concept of the physical phenomenon responsible for aeroelastic instability can be observed. At first, at low speed, the frequencies of the aeroelastic modes are very distinct and easy to identify. As the speed increases, the frequency of the pitch mode decreases and the frequency of the plunge mode increases to converge to the Flutter speed. This coalescence of frequencies is the result of aeroelastic coupling by the modes that are responsible for the aeroelastic instability of the studied system.

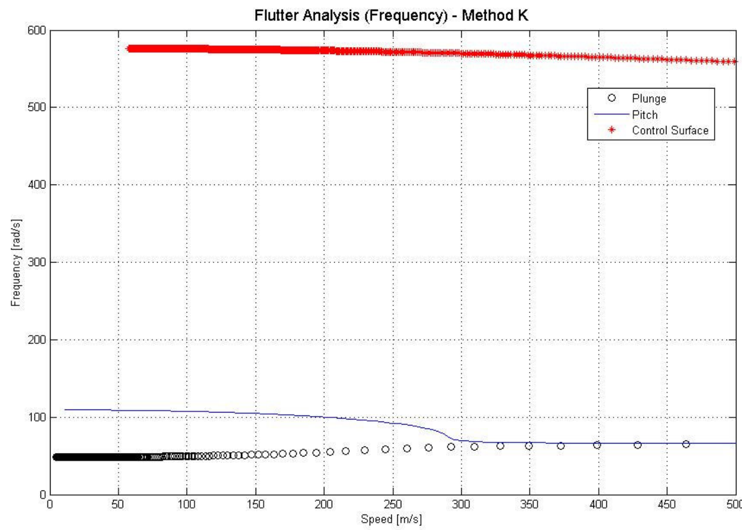

Figure 3. Modal Frequency Evolution.

Figure 4 shows the evolution of structural modes damping as a function of flight speed. The speed which, as shown in Fig. 3, the coalescence of the frequencies of the pitch and plunge modes, the damping referred to the pitch mode crosses the zero damping line (Fig. 4), presenting negative damping.

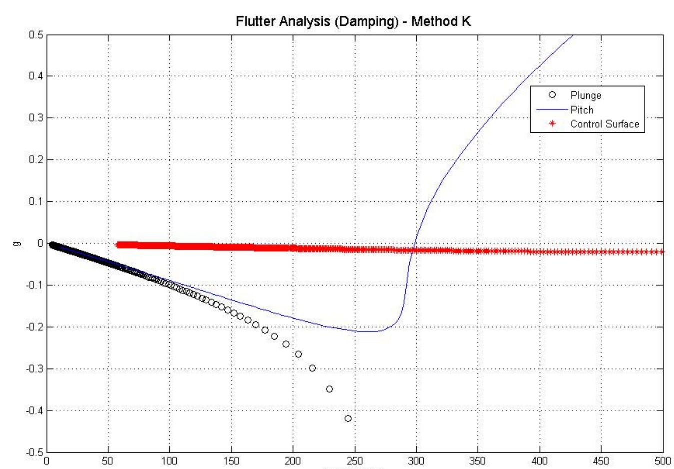

Figure 4. Damping Evolution.

Figure 3, along with Fig. 4, correspond to the so-called *V-g-f Diagram*, because it contains information on the evolution of Speed, Damping and Frequency of the aeroelastic modes of the system. The advantages of the *V-g-f Diagram* are: presenting physical information of the system as frequency, damping and speed, and without needing a state space model to the generation of its data.

2.2. State Space

The utilization of the *Structured Singular Values* method [Toivonem (1998), Lind and Brenner (1998) and Damen and Weiland (2002)], requires that the model's equations are represented in the state space form and assuming a linear time-invariant system (LTI) and finite dimension. To build this model, the equations of the non-steady aerodynamic forces are approximated in the frequency domain in terms of rational functions on the Laplace s variable.

The main approximation methods by rational functions, presented by Karpel (1981), are the Roger Method, the Padè Matrix Method and the Minimum-State Method. Applying these approximation methods, aerodynamic states are added to the system to represent the aerodynamic lags due to non-steady flow.

The Roger method approximates the aerodynamic influence matrix by the following expression, where coefficient matrices are calculated using the Least Squares Method. The matrix A_{ap}, calculated this way, represents the rational approximation of the aerodynamic influence matrix, $A(s')$.

$$\left[A_{ap}\right]=\left[P_0\right]+\left[P_1\right]s'+\left[P_2\right]s'^2+\sum_{j=3}^{N}\frac{\left\lfloor P_j\right\rfloor s'}{s'+\gamma_{j-2}} \tag{8}$$

The state vector is increased by adding the aerodynamic lags, as defined:

$$\left\{\underset{nx1}{X_{ai}}(s)\right\}=\frac{s}{s+(V/b)\gamma_{i-2}}\left\{X(s)\right\} \tag{9}$$

The values of γ_{j-2} correspond to the aerodynamic poles selected in the interval of reduced frequencies of interest. Here the values of the approximation poles used are the same as in Karpel's work (γ_1=0.2, γ_2=0.4, γ_3=0.6 and γ_4=0.8).

The approximation of the developed aerodynamic model, associated to the representative model of structural behavior, is added to associate distinct physical phenomena in a single mathematical representation, by the state space representation. The index "s" in the states refers to the structural states and the index "a" refers to aerodynamic lags.

$$\begin{Bmatrix}\dot{x}_S\\ \ddot{x}_S\\ \dot{x}_{a3}\\ ..\\ \dot{x}_{aN}\end{Bmatrix}=\begin{bmatrix}0 & I & 0 & .. & 0\\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{B} & q\overline{M}^{-1}P_3 & .. & q\overline{M}^{-1}P_N\\ 0 & I & -\left(\frac{V}{b}\right)\gamma_1 I & .. & 0\\ .. & .. & .. & .. & ..\\ 0 & I & 0 & 0 & -\left(\frac{V}{b}\right)\gamma_{N-2}I\end{bmatrix}\begin{Bmatrix}x_s\\ \dot{x}_S\\ x_{a3}\\ ..\\ x_{aN}\end{Bmatrix} \tag{10}$$

where:

$$\overline{M}=M-qP_2\left(\frac{b}{V}\right)^2 \tag{11}$$

$$\overline{B}=B-qP_1\left(\frac{b}{V}\right) \tag{12}$$

$$\overline{K}=K-qP_0 \tag{13}$$

The development of analytical expressions and rational approximations to the aerodynamic influence coefficients allows the utilization of eigenvalue extraction techniques, directly from the state matrix, as a function of speed, as the Geometric Root Locus (see Fig. 5).

Once the state space system equations are formulated, it is possible to observe that the state matrix has speed dependant parameters, therefore the eigenvalues of the matrix, which corresponds to the open loop system poles, are also speed dependant. Thus, by gradually increasing the speed, a new state matrix is calculated and its eigenvalues are extracted to find the Flutter speed where one eigenvalue crosses the imaginary axis.

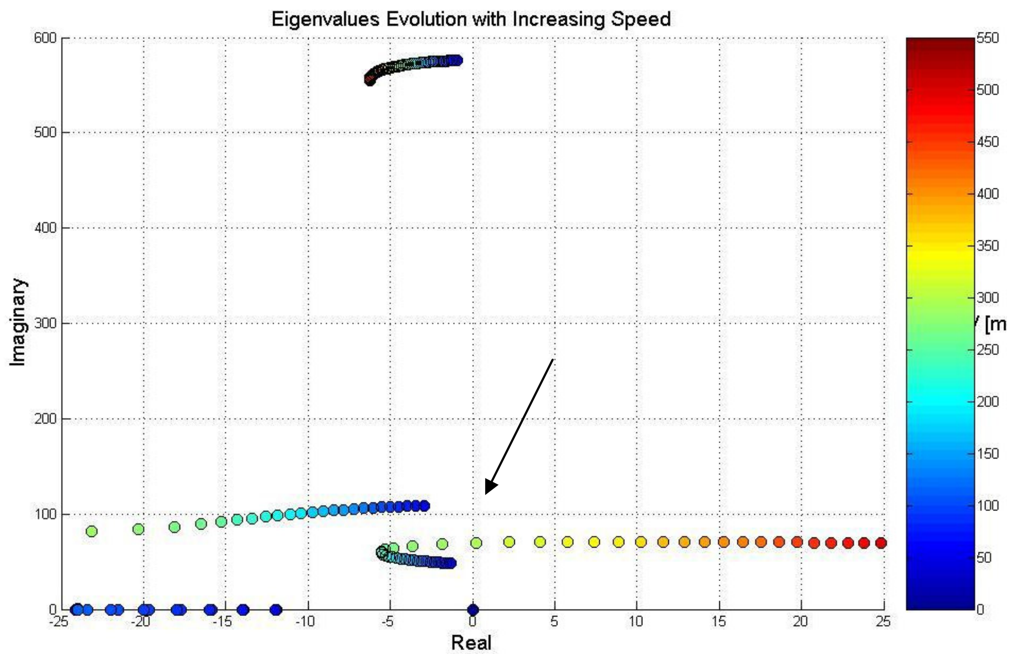

Figure 5. Nominal Plant Eigenvalues Evolution with Increasing Speed.

The system adopted for this work, with its aerodynamic model approximated by rational functions given by Eq. (8) and Eq. (9), presented a value of 299.04 [m/s] as the critical Flutter speed.

2.3. Structured Singular Values

The aeroelastic model is re-written in an appropriate form for the robust stability study, which used norm bounded operators, Δ, to describe errors and uncertainties. One measure of multivariable stability, known as *Structured Singular Value*, μ, allows the determination of a stable flight speed which is robust to model uncertainties described by Δ.

The basic idea to model a system with uncertainties is to separate what is known (nominal plant) from what is unknown (uncertainties) and create a feedback link with boundaries for the possible parametric and uncertainties variations. The *Linear Fractional Transformation* (LFT) is a powerful and flexible tool to represent uncertainties in systems and matrices. A complex operator P is considered and partitioned into four elements to describe the nominal plant:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{14}$$

The upper LFT, $F_u(P,\Delta)$, is defined as the interconnection matrix such that the P upper loop is closed with the complex operator Δ. This transformation is applied to include the effect of uncertainties into the nominal plant as Eq. (15).

$$F_u(P,\Delta) = P_{22} + P_{21}\Delta(I - P_{11}\Delta)^{-1} P_{12} \tag{15}$$

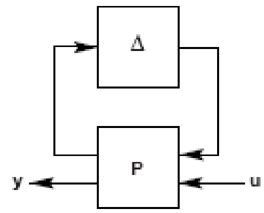

Figure 6. Upper Linear Fractional Transformation $F_u(P,\Delta)$.

The problem formulation by structured parametric uncertainties encloses the norm bounded operators set, Δ, associated with the plant P by a LFT feedback. The set of possible plants is generated by $F_u(P,\Delta)$ including every possible Δ. It is assumed that the true model belongs to this set.

The *Structured Singular Value*, μ, represents an alternative for the robust stability analysis. The measure μ is defined as in the works of Toivonem (1998), Lind and Brenner (1998) and Damen and Weiland (2002) as being equal to zero for the case without any uncertainty or parametric variation, or Eq. (16), for the other cases:

$$\mu(P) = \frac{1}{\min\limits_{\Delta \in \Delta}\{\bar{\sigma}(\Delta) : \det(I - P\Delta) = 0\}} \tag{16}$$

The *Structured Singular Value* is an exact measure of robust stability for systems with structured uncertainties. The value of μ determines the amplitude that the uncertainties can present without compromising the absolute stability of the system. It is said that P is robustly stable in respect to the operator Δ, knowing that $||\Delta||_\infty \leq \alpha$, for every Δ if, and only if, $\mu(P) < (1/\alpha)$ [Zhou, Doyle and Glover (1994), Toivonem (1998), Lind and Brenner (1998), Damen and Weiland (2002)].

The system P is, in general, internally weighted in a way that the interval within which the system's errors and perturbations variations can be described by the set of uncertainties **Δ**, bounded by the unit value ($||\Delta||_\infty$**<**1). A value of $\mu(P)<1$, for a set of uncertainties bounded by one (unit value), imply that there are no uncertainty in this set of uncertainties which is able to make the system unstable. This means that the system is stable for any uncertainty inside the described model.

The *Structured Singular Value* depends on the modeled uncertainty structure. The calculated robust stability analysis by μ will be precise only if the uncertainty model is realistic enough.

The determination of the Flutter critical condition needs the description of uncertainties on dynamic pressure or true airspeed. In the problem under study, the state space model used for analysis does not have all non-steady aerodynamic components linearly varying with dynamic pressure. The aerodynamic poles are proportional to the speed.

It can be observed, in Eq. (12), that there is a factor q/V multiplying the real coefficient matrix P_1, therefore $\overline{B}$ is also not linearly dependant with dynamic pressure. This factor is considered as q/V_0 where V_0 is the speed at which the model is generated so that this parameter depends linearly on the dynamic pressure. This approximation increases faster than the original factor, which is thought of to be conservative, and this is good when the dynamic pressure is calculated at a speed close to V_0.

Thus, the state matrix is divided, according to what was pointed out above, in two large blocks for the addition of parametric uncertainties. The first block consists in the group of parameters that are proportional to the dynamic pressure, directly influencing the states referred to the structural displacements of the system. The second block consists in the group of parameters that are proportional to the speed, originated by the inclusion of the aerodynamic lags.

The second consideration made refers to the relation of the dynamic pressure variation from the first block to the speed variation of the second block, in such a way that its variations have amplitudes that correspond to the actual values and are coherent with each other. Therefore, for a variation of 10% in the speed it is allowed a variation of 21% in the dynamic pressure ($d_q = 2d_v + d_v^2$).

The Flutter margin is dependant upon the flight conditions and μ is defined as the lowest perturbation Δ that causes instability in the system. In this way, uncertainties in the flight parameters (dynamic pressure and speed) are introduced and it reaches the least perturbation that leads to instability, which means the Flutter speed.

In this work, all analysis are made considering only sea level flight (ρ=1.225 Kg/m^3), therefore the speed is related to the dynamic pressure by its square and a constant ($q = 0.5\rho V^2$).

Perturbations in the dynamic pressure and in the speed must enter the system by a feedback represented by a LFT (*Linear Fractional Transformation*). Additive real perturbations are considered in the system: δ_q for the dynamic pressure and δ_v for the speed.

$$\begin{cases} q = q_0 + \delta_q d_q \\ V = V_0 + \delta_V d_V \end{cases} \tag{17}$$

Next, the dynamics referring to δ_q and δ_v are separated from the nominal system.

The first step is the insertion of the additive uncertainty in the dynamic pressure of the nominal aeroelastic system. Starting from Eq. (10):

$$\ddot{x} = -\overline{M}^{-1}\overline{K}x - \overline{M}^{-1}\overline{B}\,\dot{x} + q_0\overline{M}^{-1}P_3 x_{a3} + \ldots + q_0\overline{M}^{-1}P_6 x_{a6} + \delta_q z_1 \tag{18}$$

where:

$$z_1 = d_q\left[\overline{M}^{-1}P_0 x + \overline{M}^{-1}P_1\left(\frac{b}{V_0}\right)\dot{x} + \overline{M}^{-1}P_3 x_{a3} + \ldots + \overline{M}^{-1}P_6 x_{a6}\right] \tag{19}$$

The second step is the insertion of additive uncertainty in the speed, which correspond to the aerodynamic poles of the nominal aeroelastic system. This uncertainty can be modeled as speed uncertainty or as an uncertainty in the non-steady aerodynamic force model which can result from computational fluid dynamics algorithms or by the assumed hypotheses for the development of the equations.

The consideration of speed uncertainty is illustrated for the first aerodynamic pole as a model for the development of the equation in the required form because the procedure is exactly the same for all other aerodynamic pole. Thus, from the nominal equation:

$$\dot{x}_{a3} = \dot{x} - \left(\frac{V_0 - \delta_V d_V}{b}\right)\gamma_3 I x_{a3} = \dot{x} - \left(\frac{V_0}{b}\right)\gamma_3 I x_{a3} - \left(\frac{\delta_V d_V}{b}\right)\gamma_3 I x_{a3} \tag{20}$$

where:

$$z_2 = -\left(\frac{\gamma_3}{b}\right) I x_{a3} \tag{21}$$

The signals z_i and w_i (where each signal w_i corresponds to the z_i multiplied by the respective δ factor) were introduced to associate the perturbations in the dynamics to the nominal dynamics by feedback connections. Having this in mind, the perturbation information is added to the nominal aeroelastic model to result in a new model that will be used in the μ stability analysis.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{x}_{a1} \\ \ldots \\ \dot{x}_{a6} \\ z_1 \\ z_2 \\ \ldots \\ z_5 \end{bmatrix} = \left[\begin{array}{ccccc|ccc|c} 0 & I & 0 & \ldots & 0 & 0 & \ldots & 0 & \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{B} & q_0\overline{M}^{-1}P_1 & \ldots & q_0\overline{M}^{-1}P_6 & I & \ldots & 0 & \\ 0 & I & -\left(\frac{V_0}{b}\right)\gamma_1 I & \ldots & 0 & 0 & \ldots & 0 & B \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \\ 0 & I & 0 & \ldots & -\left(\frac{V_0}{b}\right)\gamma_6 I & 0 & \ldots & I & \\ \hline \overline{M}^{-1}P_0 & \overline{M}^{-1}P_1\left(\frac{b}{V_0}\right) & \overline{M}^{-1}P_1 & \ldots & \overline{M}^{-1}P_6 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \left(\frac{\gamma_1}{b}\right)I & \ldots & 0 & 0 & \ldots & 0 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 & \ldots \\ 0 & \ldots & \ldots & \ldots & \left(\frac{\gamma_6}{b}\right)I & 0 & \ldots & 0 & 0 \end{array}\right] \begin{bmatrix} x \\ \dot{x} \\ x_{a1} \\ \ldots \\ x_{a6} \\ w_1 \\ w_2 \\ \ldots \\ w_5 \end{bmatrix} \tag{22}$$

The Eq. (22) represents the augmented plant considering uncertainties in speed and in dynamic pressure. The uncertainty matrix, Δ, is diagonal and contains the dynamic pressure uncertainty parameter in the first three positions and the speed uncertainty parameter in the following twelve positions.

The evaluation of the maximum structured singular value as a function of frequency resulted in a speed of 292 [m/s] to the instability condition. It was considered a nominal speed of 260 [m/s] in the stability analysis that led to the previous value and a parametric variation of 15% was considered around this nominal value. It can be observed, in Fig.

7 and by the use of the *Structured Singular Value* stability criterion ($\mu(P) < 1$), that the maximum variation before instability is approximately (15/1.2) %, which results in the speed found.

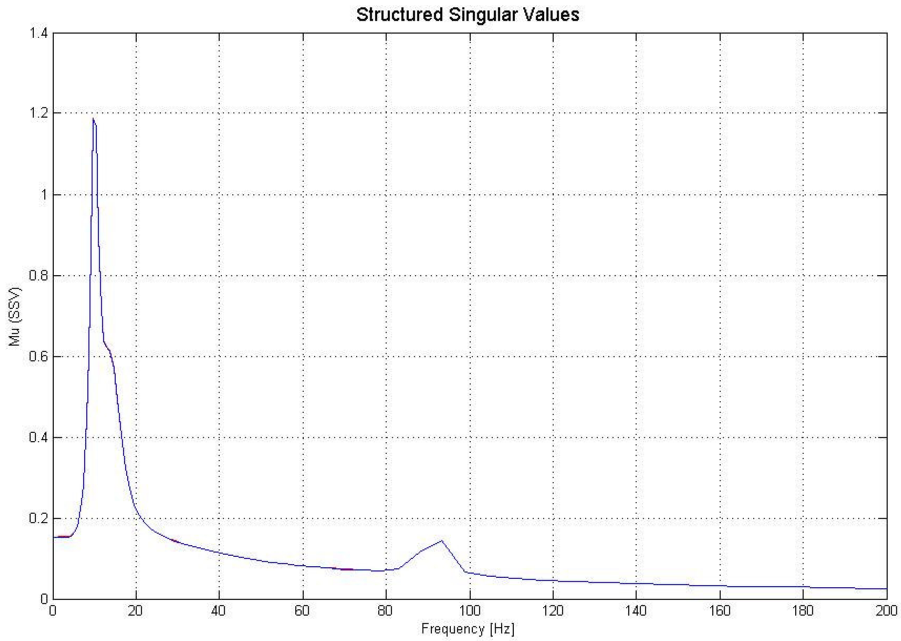

Figure 7. Aeroelastic System Structured Singular Values –260 [m/s].

This difference between the values of the *critical speed* obtained by the Method K and by the *Structured Singular Value* Method is mainly due to the simplification made to allow the linear dependence of the system to the dynamic pressure. As previously stated, the approximation is only good when it is near the nominal value from which the variation is being considered.

Therefore, taking the value of 290 [m/s] as the nominal speed, and allowing the same variation of 15% around the nominal values, the new Flutter speed found is 298.97 [m/s]. Comparing with the value of 299.04 m/s obtained by the Method K, they are practically the same but with a slightly lower value, which is conservative.

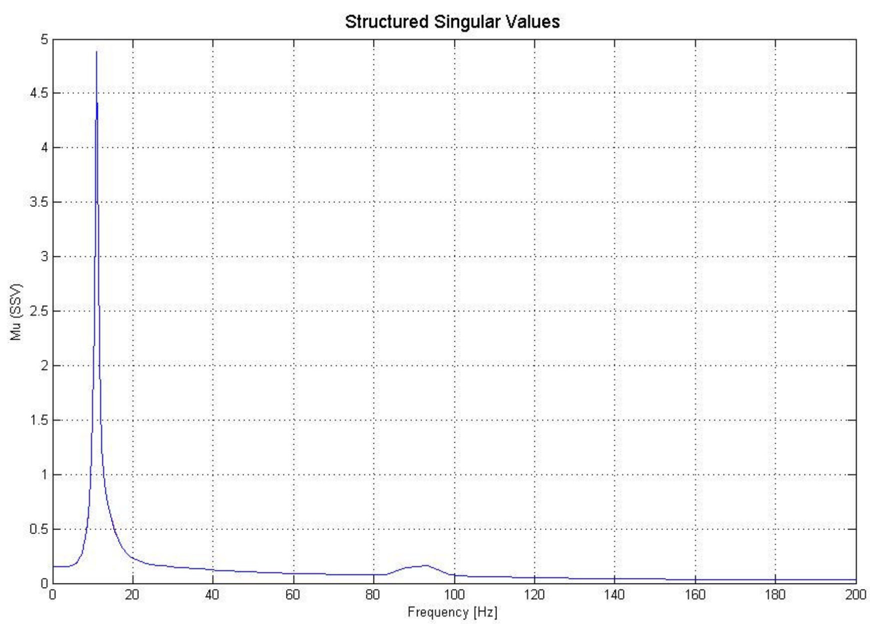

Figure 8. Aeroelastic System Structured Singular Value –290 [m/s].

It can be observed, in Fig. 8, the percentage variation of (15/4.9) % from the nominal speed under consideration. It is possible to create an iterative algorithm, with the procedure presented above, so that the μ-analysis converges to the eigenvalue analysis results.

3. CONCLUSION

The methods presented for the Flutter speed calculations – Method K and *Structured Singular Values* – presented results coherent with each other. Thus, the state space system modeling by the *Linear Fractional Transformation* to the inclusion of aerodynamic uncertainties in the model aiming the *Structured Singular Value* Method was validated.

The Flutter speed calculations by the Structured Singular Value presented very precise results (almost the same as that obtained from Method K and from the variation of eigenvalues of the state matrix with the speed). The use of this method and modeling assumptions was validated to allow the inclusion of parametric uncertainties in the stiffness coefficients.

Besides that, this stability analysis method can be use to aid the design of control systems and also to analyze the closed loop robust stability. The potential of this method will be better exploited in another work.

The focus of this work consisted in the validation of the modeling by the *Linear Fractional Transformation* and the results obtained by the *Structured Singular Value* method. This method allows the inclusion of simultaneous variation in several parameters of the model in one stability analysis, which can reduce considerably the time of analysis if adequately employed.

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