ADAPTIVE FUZZY SLIDING MODE CONTROL OF A CHAOTIC PENDULUM CONSIDERING NOISY INPUT SIGNALS

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Abstract. *Chaos control may be understood as the use of tiny perturbations for the stabilization of unstable periodic orbits embedded in a chaotic attractor. The idea that chaotic behavior may be controlled by small perturbations of physical parameters allows this kind of behavior to be desirable in different applications. In this work, a variable structure controller is proposed for n-th order chaotic systems. The approach is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm to cope with modeling inaccuracies and external disturbances. The convergence properties of the closed-loop signals are analytically proven using Lyapunov's direct method and Barbalat's lemma. The general procedure is applied to a nonlinear pendulum and numerical results are presented in order to demonstrate the control system performance. Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. This work also investigates the effect of noise on the proposed control scheme, verifying the influence on the system stabilization and on the required control action.*

Keywords: *Adaptive algorithms, Chaos control, Fuzzy logic, Nonlinear pendulum, Noisy signal, Sliding modes*

1. INTRODUCTION

Chaotic response is related to a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaos has sensitive dependence to initial conditions, which implies that the system evolution may be altered by small perturbations. Chaos control is based on the richness of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of an UPO embedded in a chaotic attractor. It makes this kind of behavior to be desirable in a variety of applications, since one of these UPO can provide better performance than others in a particular situation.

The first chaos control method has been proposed by Ott et al. (1990), nowadays known as the OGY (Ott-Grebogi-Yorke) method. This is a discrete technique that considers small perturbations applied in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section. The delayed feedback control (Pyragas, 1992), on the other hand, was the first continuous method proposed for controlling chaos, which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and the delayed state of the system.

Since the beginning of chaos control studies in the 1990's, many alternative methods were proposed in order to overcome some limitations of the original techniques. Pyragas (2006) presents a review about improvements and applications of time-delayed feedback control. Based on OGY method, Dressler and Nitsche (1992), Hübinger et al. (1994), de Korte et al. (1995), Otani and Jones (1997), So and Ott (1995) and De Paula and Savi (2009b) suggest some improvements. Savi et al. (2006) discusses some of these alternatives.

Literature presents some contributions related to the analysis of chaos control in mechanical systems. Andrievskii and Fradkov (2004) present an overview of applications of chaos control in various scientific fields. Mechanical systems are included in this discussion presenting control of pendulums, beams, plates, friction, vibroformers, microcantilevers, cranes, and vessels. Savi et al. (2006) also present an overview of some mechanical system chaos control that includes system with dry friction (Moon et al., 2003), impact (Begley and Virgin, 2001) and system with non-smooth restoring forces (Hu, 1995). Spano et al. (1990) explores the idea of chaos control applied to intelligent systems while Macau (2003) shows that chaos control techniques can be used in spacecraft orbits. Pendulum systems are analyzed in (Pereira-Pinto et al., 2004, 2005; Wang and Jing, 2004; Yagasaki and Yamashita, 1999) using different approaches. De Paula and Savi (2009b) propose a multiparameter semi-continuous method based on OGY approach to perform the chaos control of a nonlinear pendulum. Afterwards, De Paula and Savi (2009a) use a continuous delayed-feedback scheme and Bessa et al. (2009a) propose an adaptive fuzzy sliding mode based approach to control chaos in the same nonlinear pendulum.

In this work, a generalization of the control scheme proposed in (Bessa et al., 2009a) is presented to chaos control. Bessa et al. (2009a) used an adaptive fuzzy inference system to approximate the unknown system dynamics within boundary layer of smooth sliding mode controllers. A drawback of this approach is the adoption of the state variables in the premise of the fuzzy rules. For higher-order systems the number of fuzzy sets and fuzzy rules becomes incredibly large, which compromises the applicability of this technique. In this paper, in order to reduce the number of fuzzy sets and rules and consequently simplify the design process, the switching variable s, instead of the state variables, is considered in the premise of the fuzzy rules. Using Lyapunov's second method and Barbalat's lemma, the boundedness of all closed-loop signals and some convergence properties of the tracking error are analytically proven. As an application of the general procedure, the chaos control of a nonlinear pendulum that has a rich response, presenting chaos and transient chaos (De Paula et al., 2006), is treated. Numerical simulations are carried out illustrating the stabilization of some UPOs of the chaotic attractor showing an effective response. Unstructured uncertainties related to unmodeled dynamics and structured uncertainties associated with parametric variations are both considered in the robustness analysis. Moreover, the analysis from noisy time series is conducted showing the effectiveness of the controller to stabilize unstable orbits. A comparison between the stabilization of general orbits and unstable periodic orbits embedded in chaotic attractor is performed showing the less energy consumption related to UPOs.

2. ADAPTIVE FUZZY SLIDING MODE CONTROL

As demonstrated by Bessa and Barrêto (2009), adaptive fuzzy algorithms can be properly embedded in smooth sliding mode controllers to compensate for modeling inaccuracies, in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that adaptive fuzzy sliding mode controllers are suitable for a variety of applications ranging from remotely operated underwater vehicles (Bessa et al., 2008) to electro-hydraulic servo-systems (Bessa et al., 2009b).

On this basis, let us consider a second order dynamical system represented by the following equation of motion:

Consider a class of n^{th}-order nonlinear systems:

$$x^{(n)} = f(\mathbf{x},t) + h(\mathbf{x},t)u + p \tag{1}$$

where u is the control input, the scalar variable x is the output of interest, $x^{(n)}$ is the n-th time derivative of x, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, p represents external disturbances and unmodeled dynamics, and $f, h : \mathbb{R}^{n+1} \to \mathbb{R}$ are both nonlinear functions.

In respect of the dynamic system presented in equation (1), the following assumptions will be made:

Assumption 1 *The function f is unknown but bounded, i.e. $|\hat{f}(\mathbf{x},t) - f(\mathbf{x},t)| \le \mathcal{F}$, where $\hat{f}$ is an estimate of f.*

Assumption 2 *The input gain h is unknown but positive and bounded, i.e. $0 < h_{\min} \le h(\mathbf{x},t) \le h_{\max}$.*

Assumption 3 *The disturbance p is unknown but bounded, i.e. $|p| \le \mathcal{P}$.*

The proposed control problem is to ensure that, even in the presence of external disturbances and modeling imprecisions, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the state space.

Regarding the development of the control law the following assumptions should also be made:

Assumption 4 *The state vector $\mathbf{x}$ is available.*

Assumption 5 *The desired trajectory $\mathbf{x}_d$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_d$, as well as $x_d^{(n)}$, is available and with known bounds.*

Now, let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable x, and

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$$

as the tracking error vector.

Consider a sliding surface S defined in the state space by the equation $s(\tilde{\mathbf{x}}) = 0$, with the function $s : \mathbb{R}^n \to \mathbb{R}$ satisfying

$$s(\tilde{\mathbf{x}}) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} \tag{2}$$

or conveniently rewritten as

$$s(\tilde{\mathbf{x}}) = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{3}$$

where $\mathbf{c} = [c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda, c_0]$ and c_i states for binomial coefficients, i.e.

$$c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)!\, i!}\,, \qquad i = 0, 1, \ldots, n-1 \tag{4}$$

which makes $c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$ a Hurwitz polynomial.

From equation 4, it can be easily verified that $c_0 = 1$, for $\forall n \geq 1$. Thus, for notational convenience, the time derivative of s will be written in the following form:

$$\dot{s} = \mathbf{c}^{\mathrm{T}}\dot{\tilde{\mathbf{x}}} = \tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{5}$$

where $\bar{\mathbf{c}} = [0, c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda]$.

Now, let the problem of controlling the uncertain nonlinear system (1) be treated in a Filippov's way (Filippov, 1988), defining a control law composed by an equivalent control $\hat{u} = \hat{h}^{-1}(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \hat{h}^{-1}\left(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}\right) - K\,\mathrm{sgn}(s) \tag{6}$$

where $\hat{p}$ is an estimate of p, $\hat{h} = \sqrt{h_{\max}h_{\min}}$ is an estimate of h, K is a positive gain and sgn$(\cdot)$ is defined as

$$\mathrm{sgn}(s) = \begin{cases} -1 & \text{if} \quad s < 0 \\ 0 & \text{if} \quad s = 0 \\ 1 & \text{if} \quad s > 0 \end{cases} \tag{7}$$

Based on Assumptions 1–3 and considering that $\mathcal{H}^{-1} \leq \hat{h}/h \leq \mathcal{H}$, where $\mathcal{H} = \sqrt{h_{\max}/h_{\min}}$, the gain K should be chosen according to

$$K \geq \mathcal{H}\hat{h}^{-1}(\eta + \mathcal{P} + |\hat{p}| + \mathcal{F}) + (\mathcal{H} - 1)|\hat{u}| \tag{8}$$

where η is a strictly positive constant related to the reaching time.

Based on the sliding mode methodology (Slotine and Li, 1991), it can be easily verified that (6) is sufficient to impose the sliding condition:

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}s^2 &= s\dot{s} = (\tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s = (x^{(n)} - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s = (f + hu + p - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s \\ &= \left[f + h\hat{h}^{-1}(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}) - hK\,\mathrm{sgn}(s) + p - (x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})\right]s\end{aligned}$$

Recalling that $\hat{u} = \hat{h}^{-1}(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$, and noting that $f = \hat{f} - (\hat{f} - f)$ and $p = \hat{p} - (\hat{p} - p)$, one has

$$\frac{1}{2}\frac{d}{dt}s^2 = -\left[(\hat{f} - f) + (\hat{p} - p) + \hat{h}\hat{u} - h\hat{u} + hK\,\mathrm{sgn}(s)\right]s$$

Thus, considering assumptions 1–3 and defining K according to (8), it follows that

$$\frac{1}{2}\frac{d}{dt}s^2 = s\dot{s} \leq -\eta|s|$$

Then, dividing by $|s|$ and integrating both sides over the interval $0 \leq t \leq t_s$, where t_s is the time required to hit S, gives

$$\int_0^{t_s} \frac{s}{|s|}\dot{s}\,dt \leq -\int_0^{t_s} \eta\,dt$$

$$|s(t = t_s)| - |s(t = 0)| \leq -\eta\, t_s$$

In this way, noting that $|s(t = t_s)| = 0$, one has

$$t_s \leq \frac{|s(t = 0)|}{\eta}$$

and, consequently, the finite time convergence to the sliding surface S.

In order to obtain a good approximation to the disturbance p, the estimate $\hat{p}$ will be computed directly by an adaptive fuzzy algorithm.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang) (Jang et al., 1997), whose rules can be stated in a linguistic manner as follows:

$$\textit{If } s \textit{ is } S_r \textit{ then } \hat{p}_r = \hat{P}_r \quad ; \quad r = 1, 2, \ldots, N$$

where S_r are fuzzy sets, whose membership functions could be properly chosen, and $\hat{P}_r$ is the output value of each one of the N fuzzy rules.

At this point, it should be highlighted that the adoption of the switching variable s in the premise of the rules, instead of the state variables as in (Bessa et al., 2009a), leads to a smaller number of fuzzy sets and rules, which simplifies the design process. Considering that external disturbances are independent of the state variables, the choice of a combined tracking error measure s also seems to be more appropriate in this case.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{p}(s) = \hat{\mathbf{P}}^{\mathrm{T}}\mathbf{\Psi}(s) \tag{9}$$

where, $\hat{\mathbf{P}} = [\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_N]$ is the vector containing the attributed values $\hat{P}_r$ to each rule r, $\mathbf{\Psi}(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]$ is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

To ensure the best possible estimate $\hat{p}(s)$ to the disturbance p, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{P}}} = \vartheta s \mathbf{\Psi}(s) \tag{10}$$

where ϑ is a strictly positive constant related to the adaptation rate.

Equation (10) also shows that there is no adaptation when states are on the sliding surface, $\dot{\hat{\mathbf{D}}} = 0$ for $s = 0$.

It is important to emphasize that the chosen adaptation law, Eq. (10), must not only provide a good approximation to p but also assure the convergence of the tracking error to the sliding surface $S(t)$, for the purpose of trajectory tracking. In this way, in order to evaluate the stability of the closed-loop system, let a positive-definite function V be defined as

$$V(t) = \frac{1}{2}s^2 + \frac{1}{2\vartheta}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$$

where $\delta = \hat{\mathbf{P}} - \hat{\mathbf{P}}^*$ and $\hat{\mathbf{P}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{p}^*(s)$. Thus, the time derivative of V is

$$\begin{aligned}
\dot{V}(t) &= s\dot{s} + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}} \\
&= (\tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}} \\
&= (x^{(n)} - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}} \\
&= \left(f + hu + p - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}\right) s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}} \\
&= \big[f + h\hat{h}^{-1}(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}) - hK\,\mathrm{sgn}(s) + p - (x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})\big]s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\delta}}
\end{aligned}$$

Defining the minimum approximation error as $\varepsilon = \hat{p}^*(s) - p$, recalling that $\hat{u} = \hat{h}^{-1}(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$, and noting that $\dot{\boldsymbol{\delta}} = \dot{\hat{\mathbf{P}}}$, $f = \hat{f} - (\hat{f} - f)$ and $p = \hat{p} - (\hat{p} - p)$, $\dot{V}$ becomes:

$$\begin{aligned}\dot{V}(t) &= -\big[(\hat{f} - f) + \varepsilon + (\hat{p} - \hat{p}^*) + \hat{h}\hat{u} - h\hat{u} + hK\,\mathrm{sgn}(s)\big]s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\hat{\mathbf{P}}} \\ &= -\big[(\hat{f} - f) + \varepsilon + (\hat{\mathbf{P}} - \hat{\mathbf{P}}^*)^{\mathrm{T}}\boldsymbol{\Psi}(s) + \hat{h}\hat{u} - h\hat{u} + hK\,\mathrm{sgn}(s)\big]s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\dot{\hat{\mathbf{P}}} \\ &= -\big[(\hat{f} - f) + \varepsilon + \hat{h}\hat{u} - h\hat{u} + hK\,\mathrm{sgn}(s)\big]s + \vartheta^{-1}\boldsymbol{\delta}^{\mathrm{T}}\big[\dot{\hat{\mathbf{P}}} - \vartheta s\boldsymbol{\Psi}(s)\big]\end{aligned}$$

Thus, by applying the adaptation law (10) to $\dot{\hat{\mathbf{D}}}$:

$$\dot{V}(t) = -\big[(\hat{f} - f) + \varepsilon + \hat{h}\hat{u} - h\hat{u} + hK\,\mathrm{sgn}(s)\big]s$$

Furthermore, considering assumptions 1–3, defining K according to (8) and verifying that $|\varepsilon| = |\hat{p}^* - p| \leq |\hat{p} - p| \leq |\hat{p}| + \mathcal{P}$, it follows

$$\dot{V}(t) \leq -\eta|s| \tag{11}$$

which implies $V(t) \leq V(0)$ and that s and $\boldsymbol{\delta}$ are bounded. Considering that $s(\tilde{\mathbf{x}}) = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}}$, it can be verified that $\tilde{\mathbf{x}}$ is also bounded. Hence, equation (5) and Assumption 5 implies that $\dot{s}$ is also bounded.

Integrating both sides of (11) shows that

$$\lim_{t\to\infty}\int_0^t \eta|s|\,d\tau \leq \lim_{t\to\infty}\,[V(0) - V(t)] \leq V(0) < \infty$$

Since the absolute value function is uniformly continuous, it follows from Barbalat's lemma (Khalil, 2001) that $s \to 0$ as $t \to \infty$, which ensures the convergence of the tracking error vector to the sliding surface S.

However, the presence of a discontinuous term in the control law leads to the well known chattering phenomenon. To overcome the undesirable chattering effects, Slotine (1984) proposed the adoption of a thin boundary layer, S_ϕ, in the neighborhood of the switching surface:

$$S_\phi = \big\{\tilde{\mathbf{x}} \in \mathbb{R}^n \;\big|\; |s(\tilde{\mathbf{x}})| \leq \phi\big\}$$

where ϕ is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϕ. It should be noted that this smooth approximation, which will be called here $\varphi(s, \phi)$, must behave exactly like the sign function outside the boundary layer. There are several options to smooth out the ideal relay but the most common choices are the saturation function:

$$\mathrm{sat}(s/\phi) = \begin{cases} \mathrm{sgn}(s) & \text{if} \quad |s/\phi| \geq 1 \\ s/\phi & \text{if} \quad |s/\phi| < 1 \end{cases}$$

and the hyperbolic tangent function $\tanh(s/\phi)$.

In this way, to avoid chattering, a smooth version of Eq. (6) can be adopted:

$$u = \hat{h}^{-1}\left(-\hat{f} - \hat{p} + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}\right) - K\varphi(s, \phi) \tag{12}$$

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation inside the boundary layer turns the perfect tracking into a tracking with guaranteed precision problem, which actually means that a steady-state error will always remain.

Remark 1 *It has been demonstrated by Bessa (2009) that by adopting a smooth sliding mode controller, the tracking error vector will exponentially converge to a closed region* $\Phi = \{\tilde{\mathbf{x}} \in \mathbb{R}^n \mid |s(\tilde{\mathbf{x}})| \leq \phi$ *and* $|\tilde{x}^{(i)}| \leq \xi_i\lambda^{i-n+1}\phi, i = 0, 1, \ldots, n-1\}$*, with* ξ_i *defined as*

$$\xi_i = \begin{cases} 1 & \textit{for} \quad i = 0 \\ 1 + \sum_{j=0}^{i-1}\binom{i}{j}\xi_j & \textit{for} \quad i = 1, 2, \ldots, n-1. \end{cases}$$

3. Controlling a nonlinear pendulum

As an application of the control procedure, a nonlinear pendulum is investigated. This pendulum is based on an experimental set up, previously analyzed by Franca and Savi (2001) and Pereira-Pinto et al. (2004). De Paula et al. (2006) presented a mathematical model to describe the dynamical behavior of the pendulum and the corresponding experimentally obtained parameters.

The schematic picture of the considered nonlinear pendulum is shown in Fig. 1. Basically, the pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). This assembly is driven by a string-spring device (6) that is attached to an electric motor (7) and also provides torsional stiffness to the system. A magnetic device (3) provides an adjustable dissipation of energy. An actuator (5) provides the necessary perturbations to stabilize this system by properly changing the string length.

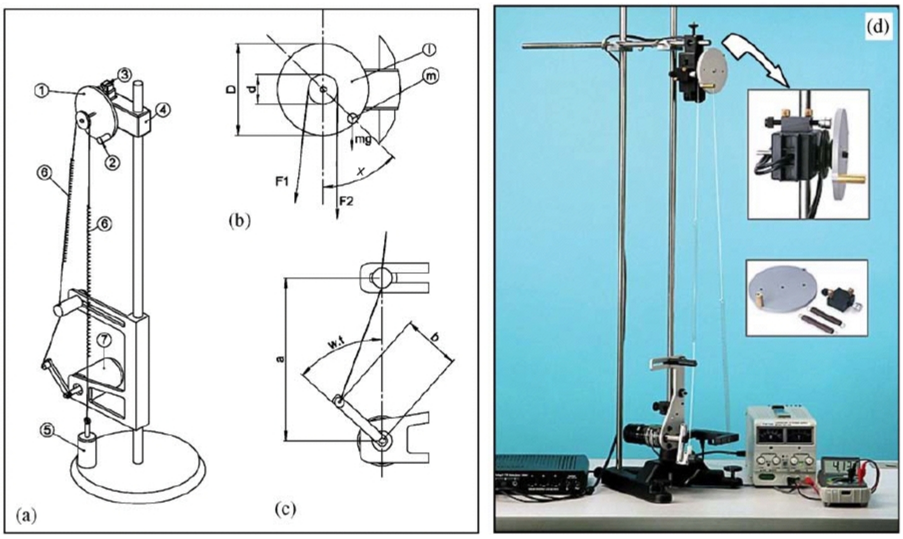


Figure 1. (a) Nonlinear pendulum – (1) metallic disc; (2) lumped mass; (3) magnetic damping device; (4) rotary motion sensor (PASCO CI-6538); (5) anchor mass; (6) string-spring device; (7) electric motor (PASCO ME-8750). (b) Parameters and forces on metallic disc. (c) Parameters from driving device. (d) Experimental apparatus.

In order to obtain the equations of motion of the experimental nonlinear pendulum it is assumed that system dissipation may be expressed by a combination of a linear viscous dissipation together with dry friction. Therefore, denoting the angular position as ϕ, the following equation is obtained De Paula et al. (2006):

$$\ddot{x} + \frac{\zeta}{I}\dot{x} + \frac{\mu\,\mathrm{sgn}(\dot{x})}{I} + \frac{kd^2}{2I}x + \frac{mgD\sin(x)}{2I} = \frac{kd}{2I}\left(\sqrt{a^2+b^2-2ab\cos(\omega t)} - (a-b) - \Delta l\right) \quad (13)$$

where ω is the forcing frequency related to the motor rotation, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation crank of the motor, D is the diameter of the metallic disc and d is the diameter of the driving pulley, m is the lumped mass, ζ represents the linear viscous damping coefficient, while μ is the dry friction coefficient; g is the gravity acceleration, I is the inertia of the disk-lumped mass, k is the string stiffness and Δl is the length variation in the spring provided by the linear actuator (5).

De Paula et al. (2006) show that this mathematical model presents results that are in close agreement with experimental data. The pendulum equation can be expressed in terms of Eq. (1) by assuming that $\mathbf{x} = [x, \dot{x}]$, $h = kd/2I$, $u = -\Delta l$, f can be obtained from Eq. (1) and Eq. (13), and the term p represents modeling inaccuracies and external disturbances.

In this way, according to the previously described scheme and considering $s = \dot{\tilde{x}} + \lambda\tilde{x}$, a smooth control law can be chosen as follows

$$u = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{x}_d - \lambda\dot{\tilde{x}}) - K\,\mathrm{sat}(s/\phi) \quad (14)$$

The controller capability is now investigated by considering numerical simulations. The fourth order Runge-Kutta method is employed and sampling rates of 107 Hz for control system and 214 Hz for dynamical model are assumed. The model parameters are chosen according to De Paula et al. (2006): $I = 1.738 \times 10^{-4}$ kg m^2; $m = 1.47 \times 10^{-2}$ kg; $k = 2.47$ N/m; $\zeta = 2.368 \times 10^{-5}$ kg m^2/s; $\mu = 1.272 \times 10^{-4}$ N m; $a = 1.6e \times 10^{-1}$ m; $b = 6.0 \times 10^{-2}$ m; $d = 4.8 \times 10^{-2}$ m; $D = 9.5 \times 10^{-2}$ m and $\omega = 5.61$ rad/s.

For tracking purposes, different UPOs are identified using the close return method (Pereira-Pinto et al., 2004) and two of these are chosen as desired trajectories in the numerical studies that follows.

In order to demonstrate that the adopted control scheme can deal with both structured (parametric) and unstructured uncertainties (unmodeled dynamics), an uncertainty of $\pm 20\%$ over the value of the viscous damping coefficient, ζ, is considered and the dry friction is treated as unmodeled dynamics and not taken into account within the design of the control law. On this basis, the estimates $\hat{\zeta} = 1.9 \times 10^{-5}$ kg m^2/s and $\hat{\mu} = 0$ are assumed. The other estimates in both $\hat{f}$ and $\hat{h}$ are chosen based on the assumption that model coefficients are perfectly known. The other used parameters are $\mathcal{F} = 1.2$; $\mathcal{P} = 1.1$; $\mathcal{H} = 1.0$; $\phi = 1.0$; $\lambda = 0.8$; $\eta = 0.05$ and $\vartheta = 3.0$.

Concerning the fuzzy system, triangular and trapezoidal membership functions are adopted for S_r, with the central values defined respectively as $C = \{-5.0\ ;\ -1.0\ ;\ -0.5\ ;\ 0.0\ ;\ 0.5\ ;\ 1.0\ ;\ 5.0\} \times 10^{-5}$ (see Fig. 2). It is also important to emphasize that the vector of adjustable parameters is initialized with zero values, $\hat{\mathbf{P}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law, Eq. (10).

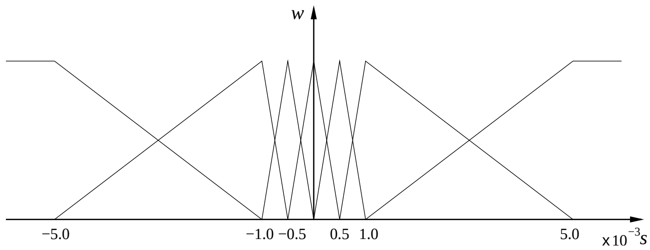


Figure 2. Adopted fuzzy membership functions.

The idea of the UPO control is interesting since these orbits are embedded in the chaotic attractor and, therefore are natural orbits related to the system dynamics. Hence, it is an important task to evaluate a comparison of the control action required to stabilize some UPOs and a general orbit (artificial or non-natural). Basically, three different situations are treated. In the first case, Fig. 3(a) and Fig. 3(d), a general artificial orbit $[\phi_d, \dot{\phi}_d] = [1.0 + 2.35\sin(2\pi t), 4.70\pi\cos(2\pi t)]$ is considered. A second case, on the other hand, stabilizes a period-1 UPO, Fig. 3(b) and Fig. 3(e). Although both orbits are similar, it should be highlighted that the controller requires less effort to stabilize the UPO. Even with more complicated orbits, as is the case of the period-4 UPO shown in Fig. 3(c), the amplitude of the control action, Fig. 3(f), is significantly smaller when compared with the control effort required to stabilize the general orbit. The control of unstable periodic orbits is the essential aspect to be explored in chaos control that can confer flexibility to the system with low energy consumption.

Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. Noise reduction schemes for chaotic noisy time series (Davies, 1994; Enge et al., 1993; Kostelich and Schreiber, 1993; Sauer, 1992; Schreiber and Grassberger, 1991; Schreiber and Richter, 1999; Shin et al., 1999) or Kalman filtering (Lefebvre et al., 2001; H et al., 2000) are alternatives to deal with this kind of situation, however, these subjects are not employed here. Ott et al. (1990) say that the efficiency of the OGY method is close related to the noise level. Spano et al. (1990) also study the effect of noise in OGY method, confirming the previous conclusion. Pereira-Pinto et al. (2004) presents an analysis of noisy signals of the nonlinear pendulum using a semi-continuous method concluding that the increase of the number of control station can compensate the noise effect. This section investigates the effect of noise on the AFSMC method applied to a nonlinear pendulum verifying the influence on the system stabilization and on the required control action.

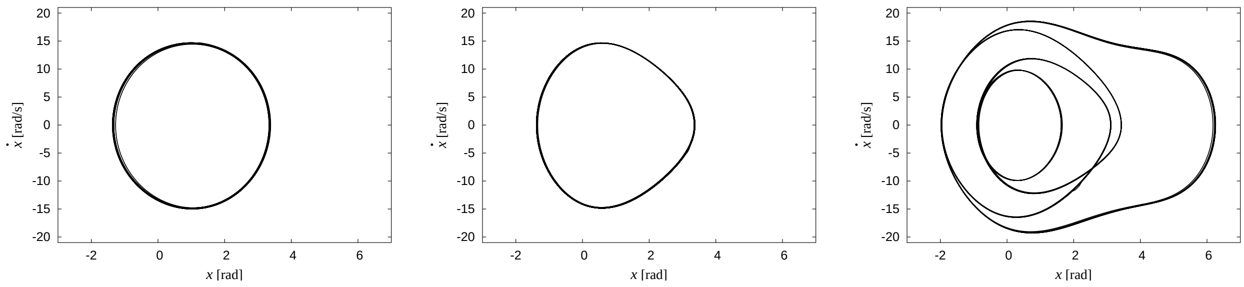
In order to simulate experimental noisy data sets, a white Gaussian noise is introduced in the signal:

$$\bar{x}(t) = x(t) + \epsilon \tag{15}$$

where $\bar{x}$ represents the measured state variable, x the clean signal and ϵ the white Gaussian noise. White Gaussian noise is generated using the polar form of Box-Muller transformation (Box and Muller, 1958). The noise level is parameterized by the standard deviation of the clean signal (S_{signal}). Therefore, the standard deviation of the noise, S_{noise}, is a fraction γ of S_{signal}:

$$\gamma = \frac{S_{\text{noise}}}{S_{\text{signal}}} \times 100 \quad (\%) \tag{16}$$

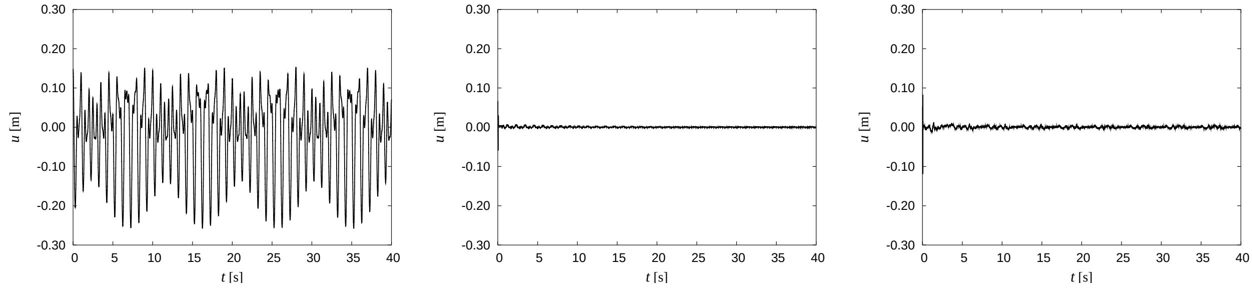
Figures 4–6 shows the stabilization of a period-1 UPO with three different values of γ: 1%, 3% and 5%. The phase space, the control action and the Poincaré section embedded in the related noisy strange attractor are presented.



(a) General orbit.

(b) Period-1 UPO.

(c) Period-4 UPO.



(d) u for general orbit.

(e) u for period-1 UPO.

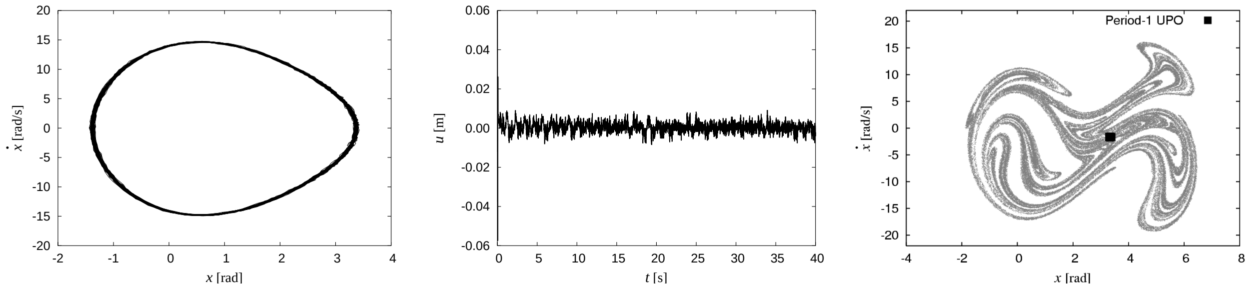
(f) u for period-4 UPO.

Figure 3. Control action required to stabilize a general orbit and 2 different UPOs.

As observed in Figs. 4–6, the proposed control scheme allows the UPOs stabilization even when noisy signals are considered. Nevertheless, it can be verified that the increase of the noise amplitude causes a proportional increase of the control effort and a decrease in the tracking performance.

4. Conclusions

The present contribution presents an adaptive fuzzy sliding mode controller for chaos control. The convergence properties of the tracking error are analytically proven using Lyapunov stability theory and Barbalat's lemma. As an application of the control formulation, numerical simulations of a nonlinear pendulum with chaotic response is of concern. The control system performance is investigated showing the tracking of a generic orbit as well as for UPO stabilization. It is shown that the controller needs less effort to stabilize an UPO when compared with a general non-natural orbit. This is an essential point related to chaos control that can confer flexibility to the system dynamics changing response with low power consumption. The robustness of the proposed control scheme against modeling inaccuracies are investigated evaluating both unstructured and parametric uncertainties. Noisy signals are also investigated showing the controller capability to deal with this kind of uncertainty. In general, the proposed procedure is able to perform chaos control even

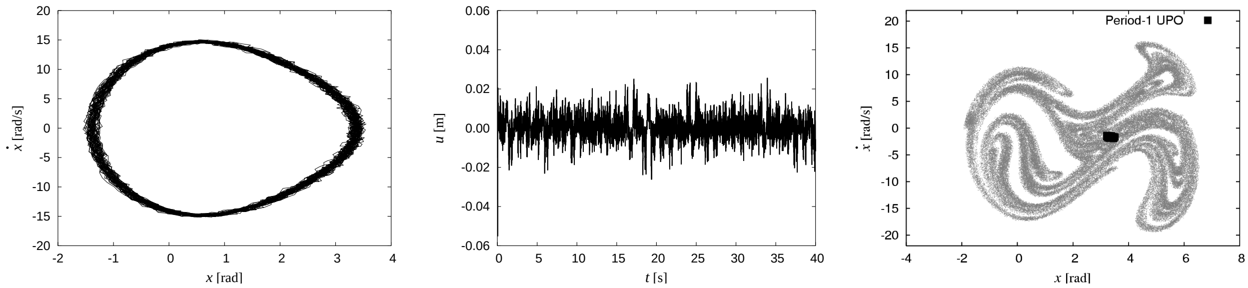


(a) Phase space.

(b) Control action.

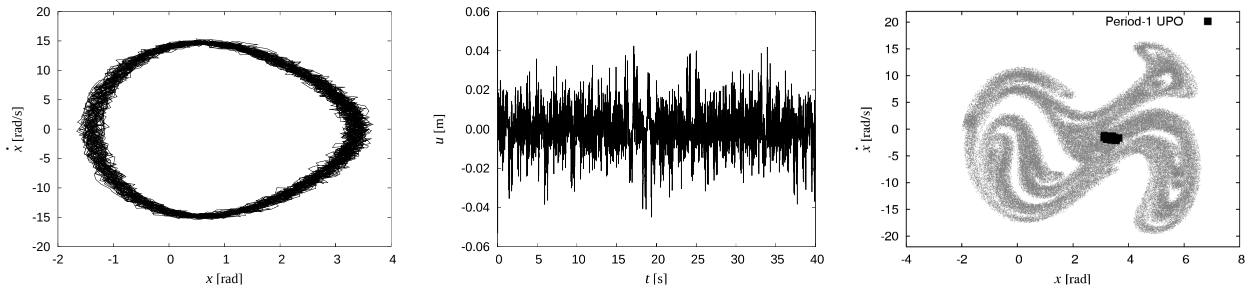
(c) Poincaré section.

Figure 4. Tracking of a period-1 UPO with $\gamma = 1\%$.



(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 5. Tracking of a period-1 UPO with $\gamma = 3\%$.



(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 6. Tracking of a period-1 UPO with $\gamma = 5\%$.

in situations where high uncertainties are involved.

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