INFLUENCE OF THE CARDANIC SUSPENSION IN THE DYNAMICS OF A GYROSCOPE

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Abstract. *The dynamics of a non axi-symmetrical body in space was introduced in previously [Dynamics of bodies in space rotating across stability borders, Zambrano. & Weber., 2009] focusing the calculation of the minimal kinetic energy for starting the change of basins of attraction, handling of singular points during the integration of the movement equations for cardanic angles, and investigating stability conditions of the movement for different inertial configurations. The present work is a continuation of the study of the dynamics of a body in space using tools from Nonlinear Dynamics applied to the understanding of the motion of a gyroscope on a cardanic suspension. In the study the influence of the inertia of the gimbals during large excursion motions are considered, crossing the stability borders in the concept of Lyapunov. Non axi-symmetric rotors are considered, and the necessary conditions in angular momentum and kinetic energy for the escape of a basin of attraction are obtained. Due to the influence of the gimbals inertia, a significant variation in the value of minimal energy for this escape is obtained, in spite that qualitative results of the free body are maintained. The problem of singularity which existed in the case of a body in space was eliminated due to the inertia of the gimbals, then, the numerical integration of movement equations using cardanic angles does not present any problems. The model in this paper does not consider energy loss due to rubbing between the components of the gyroscope: the total kinetic energy of the gyroscope is constant, but there exists an exchange of energy between the rotor and the gimbals, whose complexity depends on the initial conditions, as is seen clearly in the results. It is shown how the change of moments of inertia (the gimbals), influences the stability of the movement. The contribution of this work is to evaluate the importance of the configuration of the inertia of the frames which must be done very carefully (it is possible to show significant differences in the dynamics of the body due to influence of the small inertia of the gimbals), and to characterize the gyroscope with cardanic suspension using tools from non-linear dynamics and chaos theory.*

Keywords*: Dynamics, Limits of Stability, Inertia, Orientation in space.*

1. INTRODUCTION

The topic of rotation in space continues receiving attention along the modern development of science: traditional mechanics finds new ways using modern computational and visualization techniques. Even so, to understand basic rotational phenomena of a free body in space it is very helpful to use a gyroscope on cardanic suspension. Numerous important gyroscopic problems that occur in applications can be simulated qualitatively in the experiment.

When the attitude of a body in space is studied, it can be characterized by different differential equations according to the used tools: there are several parameterizations possible: among most commonly used, there are cardanic angles, Euler angles, Euler Rodrigues parameters, Quaternions, etc. The best parameterization depends on the amplitudes of the displacements to be studied. In the solution of differential equations of motion there may exist singularity points like it happens using cardanic angles (Zambrano, H. & Weber, H.I., 2009) demanding special care. On the other hand, the Gyroscope with cardanic suspension and no restriction for the gimbals rotation will reproduce quite well the motion characteristics of a body in space. This is true for small amplitude nutations, but there exist differences due to the presence of the inertia of the gimbals. This inertia may exert considerable influence in the dynamics of the movement modifying the nonlinear structure of the equations but, nevertheless, it presents a more realistic description of the real phenomena.

In this work the dynamics of this Gyroscope is studied using cardanic angles, the nonlinear characterization of the movement is done, basins of attraction are generated and stay defined until certain conditions, given by the Lyapunov definition of the stability of motion; are reached. It will be shown that in this case singularity points do not exist as in the solutions of a previous work (Zambrano, H. & Weber, H., 2009): these singularity points disappear due to the inertia of the gimbals. A random looking behavior of the deterministic systems will receive the name of Chaos (Savi, 2006). Defining slightly different initial conditions will give rise to a seemingly random response and it will be shown that the Gyroscope has chaotic characteristics resulting in unexpected movements.

2. EQUATIONS OF MOTION

The equations were obtained using Lagrange equations. The kinematics is done defining sequential rotations with cardanic angles, each rotation is associated to a component of the gyroscope: angle α is used to define the position of the external gimbal *(Qex)*, angle β defines the motion of the internal gimbal *(Qin)* in the external gimbal and the angle γ

gives the spin of the rotor in the internal gimbal (Fig. 1). Since there are only rotations in the dynamics of the gyroscope with gimbals, three generalized coordinates are enough to study the problem. We choose cardanic angles. To define the sequential rotations it is necessary to work with 4 reference systems (SR), one fixed in space and the others attached to the components of the gyroscope (in movement). A short form for the notations is:

$$\underset{(x,y,z)}{F} \xrightarrow{\alpha_{(x)}} \underset{(x,y',z')}{Q} \xrightarrow{\beta_{(y')}} \underset{(x'',y',z'')}{R} \xrightarrow{\gamma_{(z'')}} \underset{(x''',y''',z'')}{S} \tag{1}$$

The gyroscope is composed by three bodies; the total kinetic energy is the sum of the energies of each body. This kinetic energy is independent of the coordinate system where it is calculated, allowing calculating the energy of each component in its respective SR which makes the calculation easier, since the inertia matrix is constant.

$$E_C = E_{C\,Qex} + E_{C\,Qin} + E_{C\,Rot} \tag{2}$$

The total kinetic energy of the gyroscope is then:

$$\begin{aligned} E_C = & \frac{1}{2}\dot{\alpha}^2 I_x + \frac{1}{2}\dot{\alpha}^2 \cos(\beta)^2 I_{X\_R} + \frac{1}{2}\dot{\beta}^2 I_{Y\_R} + \frac{1}{2}\dot{\alpha}^2 \sin(\beta)^2 I_{Z\_R} + \\ & + \frac{1}{2}\left(\cos(\gamma)\,\dot{\alpha}\,\cos(\beta) + \sin(\gamma)\,\dot{\beta}\right)^2 I_1 + \frac{1}{2}\left(-\sin(\lambda)\,\dot{\alpha}\,\cos(\beta) + \cos(\gamma)\,\dot{\beta}\right)^2 I_2 + \frac{1}{2}\left(\dot{\alpha}\,\sin(\beta) + \dot{\gamma}\right)^2 I_3 \end{aligned} \tag{3}$$

Where: I_x is the moment of inertia of the external gimbal corresponding to axle x of SR(Q), moments I_{X_R}, I_{Y_R} and I_{Z_R} are the principal moments of inertia of the internal gimbal in its respective axles of SR(R), the moments, I_1, I_2, I_3 are the principal moments of inertia of the rotor in axles x, y, z of SR(S) respectively.

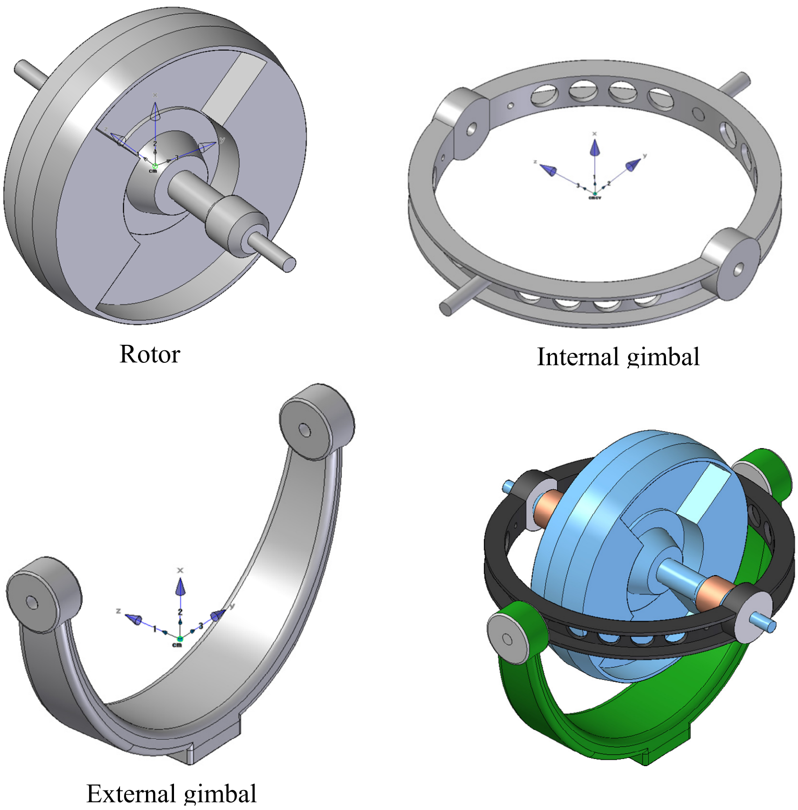

Figure 1. Diagram of components of the gyroscope

The studied problem consists in: a rotor which initially is turning with constant angular momentum, an external impact force on one of the gimbals, i.e. one of the axles, x or y, of SR(R), this impact generates an instantaneous change in the angular momentum, starting a nutation motion.

NORMALIZATION OF THE EQUATIONS

To better analyze the results, and also to diminish one parameter in the motion equations, thus diminishing the complexity of the mathematical calculation, it is advisable to count on non-dimensional motion equations: therefore the initial value (after the impact) of the angular momentum $h_G = I_3\, v$ will be considered for the normalization. The time will be changed by the non-dimensional value $\tau = v\, t$, where v represents an angular speed (instantaneously after the impact); the original conception of this normalization is presented by (Weber, H.I., 2005). The temporal derivatives will be changed to:

$$v = \frac{h_G}{I_3} \qquad \dot{\alpha} = v\alpha' \quad \dot{\beta} = v\beta' \quad \dot{\gamma} = v\gamma' \tag{4}$$

Another step of the normalization process is directed to the moments of inertia: since the original rotation axis is (z) Fig. (1) moment of inertia I_3 was used to define the parameter τ, and all other moments of inertia will be normalized with respect to I_3. The non-dimensional moments of inertia of the rotor and of the external gimbal are:

$$\mu_1 = I_1/I_3 \qquad \mu_2 = I_2/I_3 \qquad \mu_x = I_x/I_3$$

The internal gimbal is considered as a ring, to diminish the number of parameters; therefore we only have the polar moment I_{X_R} and two diagonal moments $I_{Y\_R} = I_{Z\_R}$ which are equal. In the normalized form we get:

$$\mu_p = I_{X\_R}/I_3 \qquad \mu_d = I_{Y\_R}/I_3 = I_{Z\_R}/I_3$$

After the impact, the of the gyroscope remains constant; in its normalized form it is represented as:

$$EE = 2E_C/h_G v \tag{5}$$

LAGRANGE EQUATIONS

The movement is conservative; therefore it is easy to apply Lagrange equations. Since there is only kinetic energy variation, Lagrange equations taking Eq. (3), are used with cardanic angles as generalized coordinates.

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{\alpha}}\right) - \frac{\partial E_C}{\partial \alpha} = 0 \qquad \frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{\beta}}\right) - \frac{\partial E_C}{\partial \beta} = 0 \qquad \frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{\gamma}}\right) - \frac{\partial E_C}{\partial \gamma} = 0 \tag{6}$$

Equation (6) will result in differential equations of second order; the earlier described normalization process is applied in these second order equations to obtain non-dimensional equations of movement. The kinetic energy of every body can also be normalized following the same normalization process.

2.1. Analysis of initial conditions

In an initial instant before the impact, the rotor has a constant angular momentum $\mathbf{H}_G$ in direction of the axle z_0 of SR(x_0, y_0, z_0). With the impact it changes in module and direction to a new angular momentum $\mathbf{h}_{G_0}$: it now has the direction of axle z of SR(x, y, z) formerly defined as (F).

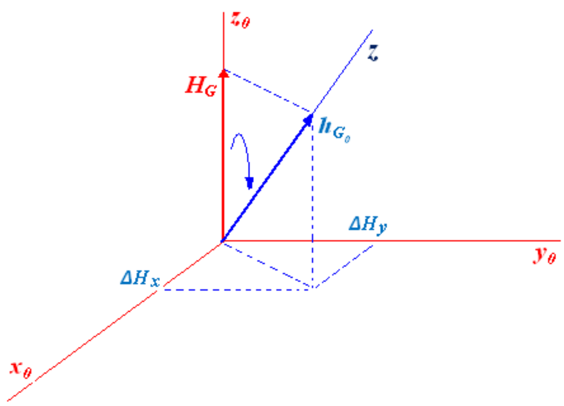

Figure 2. Angular momentum change due to impact

Two coordinate systems are considered to understand the impact process:

First (R) with axles (x_0, y_0, z_0) defined in Eq. (1) in its position at a time $\tau = 0$ and second F(x, y ,z) where the angular momentum ($\mathbf{h}_{G0}$) lines up with the axle z. Then, $\mathbf{h}_{G0}$ in (R) can be written as:

$$ {}^{R}H_0 = \begin{bmatrix} -\sin(\beta)\cos(\alpha)\, h_0 \\ \sin(\alpha)\, h_0 \\ \cos(\beta)\cos(\alpha)\, h_0 \end{bmatrix} $$

On the other hand, the total angular momentum of the gyroscope can be calculated summing up the angular momentum of each component:

$$ \mathbf{H}_0 = \mathbf{H}_{0\_Qext} + \mathbf{H}_{0\_Qin} + \mathbf{H}_{0\_Rotor} $$

In this way we arrive to another mathematical expression for $\mathbf{h}_{G0}$, giving:

$$ \begin{bmatrix} -\sin(\beta)\cos(\alpha)\, h_0 \\ \sin(\alpha)\, h_0 \\ \cos(\beta)\cos(\alpha)\, h_0 \end{bmatrix} = I_3 \begin{bmatrix} \cos(\beta)\,\dot{\alpha}\,(\xi_1 + A) + B\,\dot{\beta} \\ (\xi_2 + C)\,\dot{\beta} + B\,\dot{\alpha}\cos(\beta) \\ (\xi_3 + 1)\,\dot{\alpha}\sin(\beta) + \dot{\gamma} \end{bmatrix} \qquad \begin{matrix} A = \cos(\gamma)^2\,\mu_1 + \sin(\gamma)^2\,\mu_2 \\ B = (\mu_1 - \mu_2)\cos(\gamma)\sin(\gamma)\,, \\ C = \sin(\gamma)^2\,\mu_1 + \cos(\gamma)^2\,\mu_2 \end{matrix} \tag{7} $$

where:

$$ \xi_1 = \frac{I_x + I_{X\_R}}{I_3} = \mu_x + \mu_p \qquad \xi_2 = \frac{I_{Y\_R}}{I_3} = \mu_d \qquad \xi_3 = \frac{I_x + I_{Z\_R}}{I_3} = \mu_x + \mu_d $$

The expressions for the initial angular speeds (after the impact) in function of cardanic angles are obtained by the solution of Eq. (7). Also here the normalization is applied to obtain non dimensional initial angular speeds. These procedure shows that it is possible to define the whole problem starting with initial angles, in spite the fact that the initial condition was generated by an impact perpendicular to the axle of the original angular momentum of the rotor.

In this work only two general initial conditions are studied: when the impact occurs in y direction of SR(R) then { $\alpha \neq 0, \beta = 0, \gamma = 0$ } is taken, and when the impact occurs in x direction of SR(R) then { $\alpha = 0, \beta \neq 0, \gamma = 0$ }. Knowing the initial angles it is possible to calculate initial angular speeds, and, in this way, all initial conditions for the differential equations of second order are defined.

3. INFLUENCE OF THE INERTIA OF THE CARDANIC SUSPENSION

It was mentioned before that this work gives sequence to a recent work (Zambrano, H. & Weber, H.I., 2009), where only the dynamics of the rotor in space was analyzed (without a cardanic suspension). Basins of attraction were defined using angles α and β, and the change of basin during large amplitude motions may happen in the direction of α, but the change of basin in direction of β is only possible at two points. This behavior can be confirmed by Fig. (3).

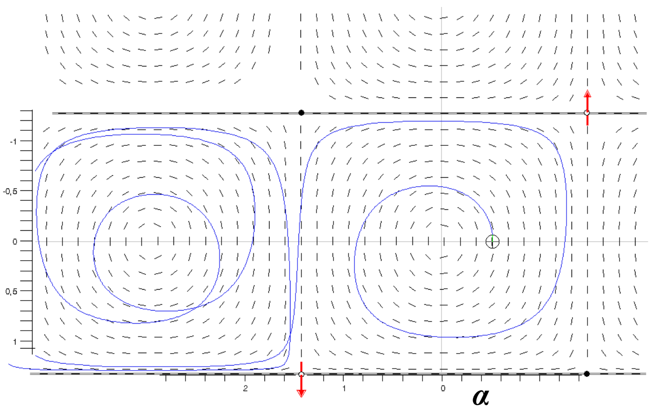

Figure 3. Diagram showing motion across stability borders (rotor in space)

Figure (3) presents the behavior of a non axi-symmetric rotor (rotor in space): it is an easy task to calculate the necessary minimum energy for a change in the basin of attraction to happen. When considering the influence of gimbals with inertia, it is only possible to calculate of the minimum energy using a numerical method.

The rotor presented in Fig. (1) is part of the Magnus gyroscope (Magnus, 1965); some physical characteristics of the components of the gyroscope obtained from a CAD fitting to the object are shown in the following table.

Table 1. Moments of inertia of the Magnus gyroscope.

Axle	Bodies	Density (kg/m^3)	Moment of inertia (kg mm^2)		μ (Dimensionless)	mass (kg)
z	Rotor (aço inox. AISI 316L)	8000	4686,234	μ_3	1	2,340
x	Rotor (aço inox. AISI 316L)	8000	3632,807	μ_1	0,775208195	2,340
y	Rotor (aço inox. AISI 316L)	8000	2291,235	μ_2	0,48892885	2,340
x' = x (p)	Internal gimbal (ALUMOLD)	2700	544,265	μ_p	0,116141234	0,104
z' = z (d)	Internal gimbal (ALUMOLD)	2700	218,190	μ_{d1}	0,046559775	0,104
y' = y (d)	Internal gimbal (ALUMOLD)	2700	334,908	μ_{d2}	0,071466342	0,104
x'' = x	External gimbal (ALUMOLD)	2700	1209,290	μ_x	0,258051561	0,311

The influence of the inertia of the gimbals changes the geometry of the basins of attraction shown in Fig. (3); the singularity point that existed in the study of the rotor in space will not exist any more when the inertia of the gimbals are considered, even for very small values of this inertia, the effect in the dynamics of the system is considerable.

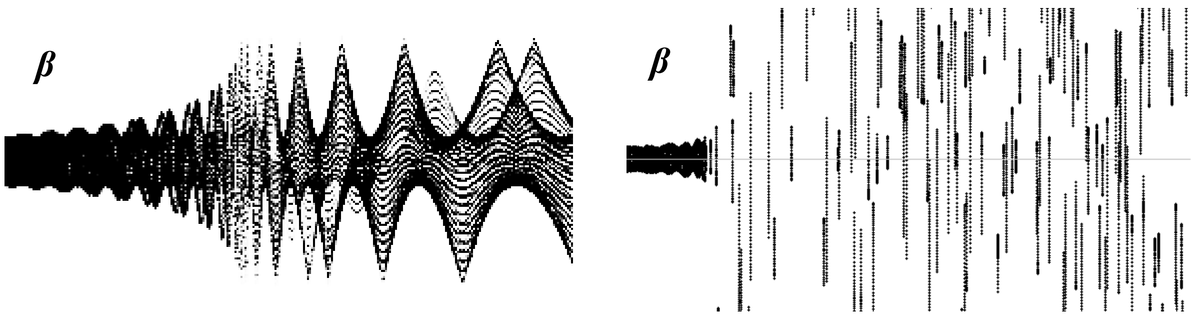

Figure 4. Bifurcation Diagram for μ_1, without gimbals on left, with gimbals on right.

The first important difference when there are gimbals is the facility of changing the basin of attraction in β direction, therefore in the motion equations the denominator will never be zero. In Fig. (4) there are presented two cases where the control variable is μ_1 , in the figure on the left for a single rotor it does not exist change of basin of attraction in β direction, therefore the angle β remains inside $< -\pi/2 , \pi/2 >$. Otherwise when the inertia of the gimbals are considered the angle β can reach any value without problem (Fig. (4) on the right).

3.1. Sensitivity of the dynamics to the inertia of the gimbals.

The sensitivity to the inertia of the gimbals in dynamics is very high. Even for values of μ_x smaller than 10^{-5}, Fig.5 and 6, there are regions with very irregular characteristics. The figures consider the same initial condition that will lead to instability in the linear region and solution points between $40 < \tau < 42$ are represented.

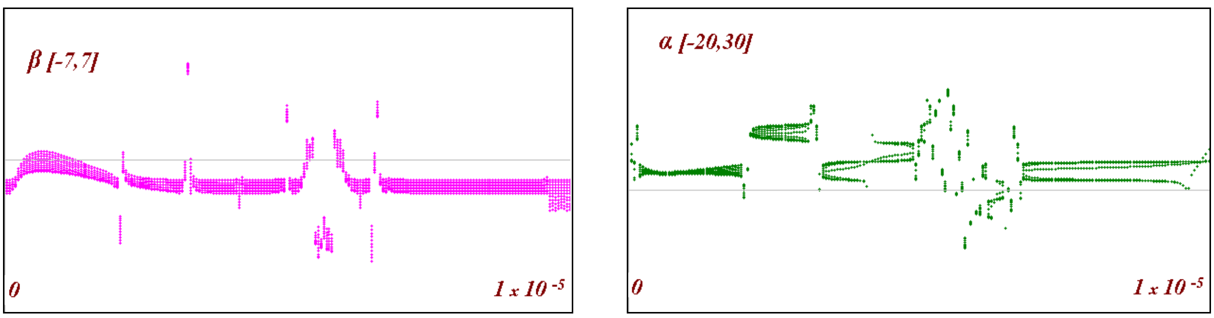

Figure 5. Bifurcation Diagram for μ_x very small.

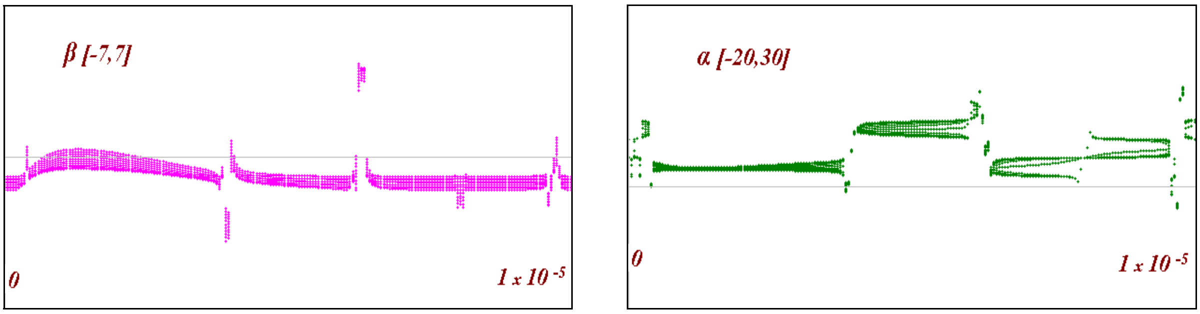

Figure 6. Bifurcation Diagram for μ_p and μ_d very small.

In conclusion of the previous results, the dynamics a body in space is affected very much by the inertia of the external gimbal as also by the inertia of the internal gimbal, considered individually and also simultaneously.

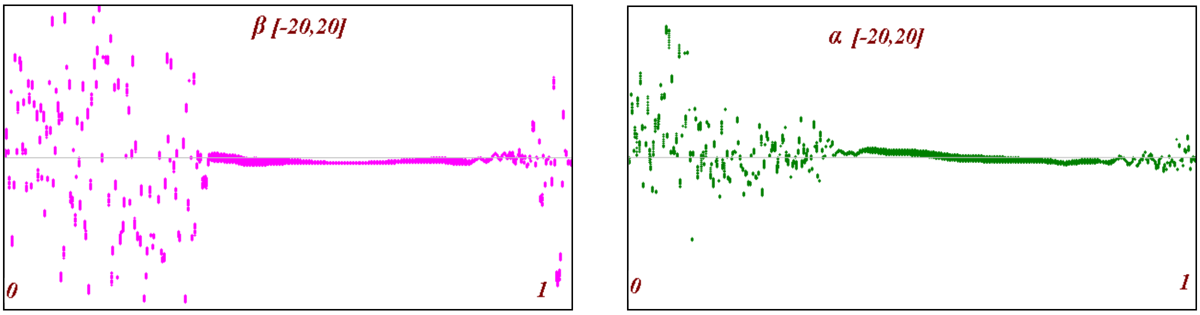

Figure 7. Bifurcation Diagram for μ_x .

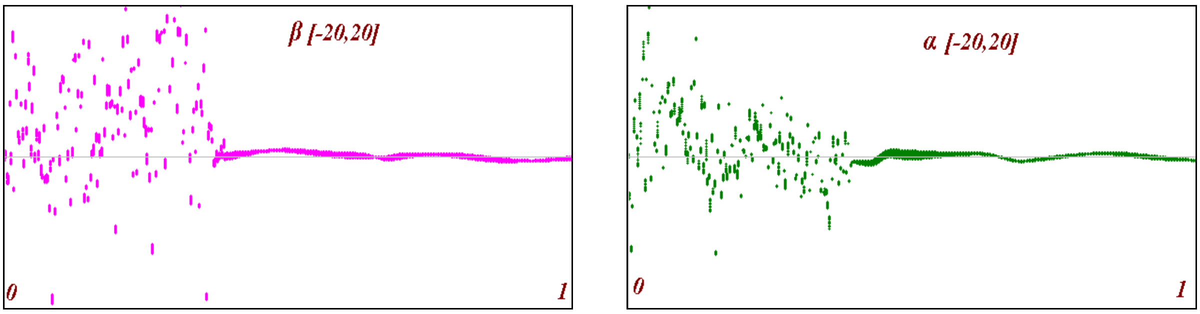

Figure 8. Bifurcation Diagram for μ_p and μ_d .

Figure (7) shows that there exists an intermediate region when varying the inertia value of the external gimbal: in this region the movement is stable for the initial condition (α_0=0.555), but the position and the length of this stable region in the figure depends on the initial condition. In Fig. (8) two regions appear, one unstable and the other stable. The increase of the inertia leads to an increase of stability since the gimbal with bigger inertia behaves in a more stable way for the same disturbance. Considering both gimbals simultaneously we obtain a greater stable region but also that for greater inertia again a region of instability appears, meaning that there is an optimal intermediate region for the moments of inertia.

Finally, considering gimbals with inertia, makes the dynamics of the free rotor to cover a greater range of amplitudes, it is possible to move in direction of the angle α and also in the angle β crossing basins of attraction. Points of singularity do not exist; when the inertias grow and the body turns heavy, more energy is needed from the impact to cross borders of the basin. In this aspect there is a big difference to the movement of a pure rotor in space.

The analysis done above studies the impact in direction of axle y of SR(R) in its initial position. The case of the impact in direction of axle x of SR(R) is similar, the movement of body in space is more stable when the impact is in this direction, and the movement does not change basin, but due to the inertias of the gimbals, the gyroscope changes basin of attraction, and the dynamics of the movement is similar to the case studied here.

4. GYROSCOPE DYNAMICS

In this part of work, the behavior of the Magnus gyroscope is analyzed, Table (1). According to the work of Zambrano, H. & Weber, H.I., 2009, if the gimbals are not considered, the minimum energy for the change of basin of attraction and also the maximum energy that can be applied by the impact (in axle y of initial SR(R)) are:

$EE_{\min\,(\text{cambio vasija})} = 1{,}289$ (α_0=0,5547) $EE_{\max} = 2{,}045$ (α_0=π/2)

Now, considering the influence of the gimbals, the new values of the energies are:

$EE_{\min\,(\text{cambio vasija})} = 1{,}0023$ (α_0=0,05238) $EE_{\max} = 1{,}8282$ (α_0=π/2)

An interesting fact when considering the inertias of the gimbals is that the new basin of attraction is not any more a square with π/2 of side, as in the case of the body in space, but an ellipse with approximately π/8 of major axis, as is shown in the following figure:

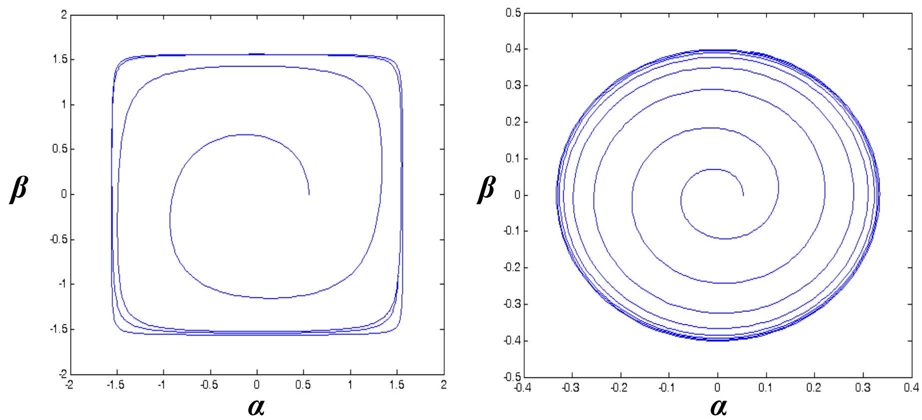

Figure 9. Form of limits of the basins of attraction, without gimbals on left, with gimbals on right.

Other interesting fact in respect to the initial energy is that in case of body in space (without gimbals) the body does not change basin of attraction when the impact occurs in direction of axle x, but due to the inertias of the gimbals the limit of the basin of attraction is equal to the outer ellipse of Fig. (9) right, which was drawn for an impact in y.

The value of the initial energies for an impact in direction x of the initial SR(R) is:

$EE_{min} = 1{,}072$ (β_0=0,39949) EE_{max} = infinite (β_0=π/2)

The condition of infinite energy means that a very strong impact is necessary to obtain a big initial angle of β_0. One better presentation of the initial energies appears in the following figure:

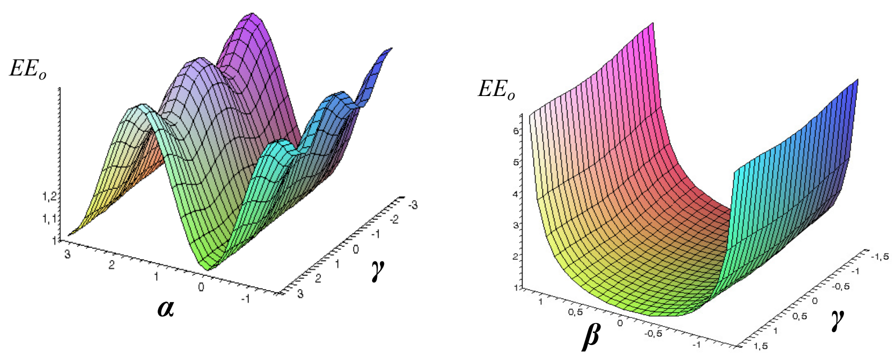

Figure 10. Initial kinetic energy: left, impact in y; right, impact in x.

Beyond changing the form of the limits of the basin of attraction, which is now elliptical, the centers also move. This meaning that, when the body in space is studied, the centers of the basins of attraction are fixed equal to a multiple of π, but due to the inertias of the gimbals the centers do not remain fixed during the movement; for each initial condition different centers of basins of attraction are obtained. Considering all aspects, it is difficult to identify all points that belong to a specific basin. Using a numerical method it is possible to calculate the limits of the basins of attraction in elliptic form, the maximum and minimum axes of the ellipse can be defined during the first change of basin of attraction (for the inertial configuration of the gyroscope of Magnus, major axis $\approx \pi/8$, and minor axis $\approx \pi/10$).

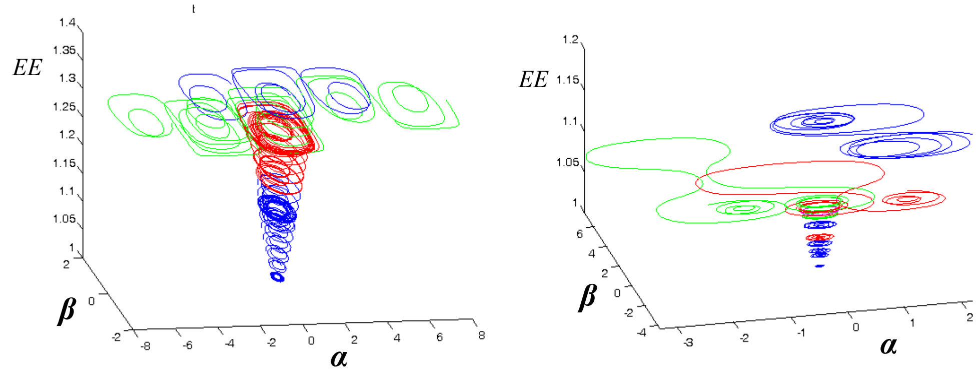

Figure 11. Basins of attraction, without gimbals on left, with gimbals on right.

Figure (12) presents the kinetic energies of each gimbal and of the rotor, which interact conserving a constant value in the sum. The smallest kinetic energy belongs to the internal gimbal motion, due to the fact that it has a smaller inertia. In Fig. (13) some basins of attraction can be seen, in the direction of the angle α and β, the behavior of the angles are also observed in Fig. (12), obtaining an almost linear behavior of γ in comparison to other angles.

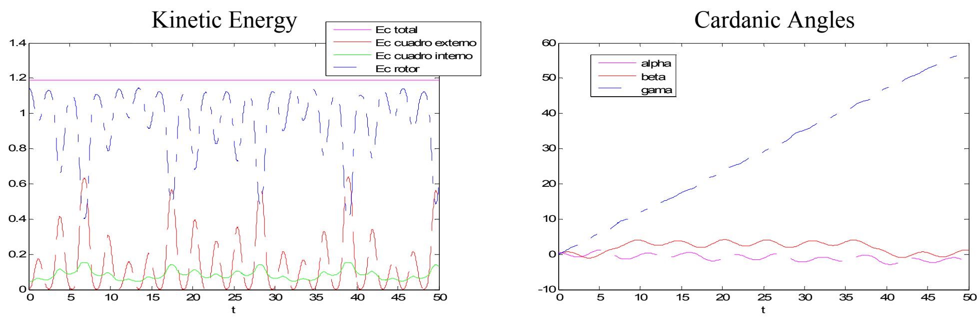

Figure 12. Energy and angles in function of τ , with α_0=*0.5*

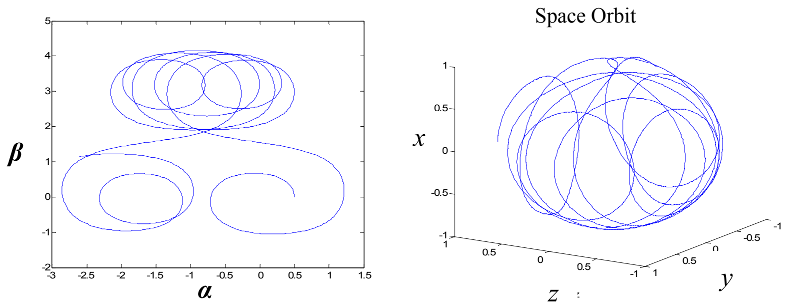

Figure 13. Orbits of the movement, with α_0=0.5.

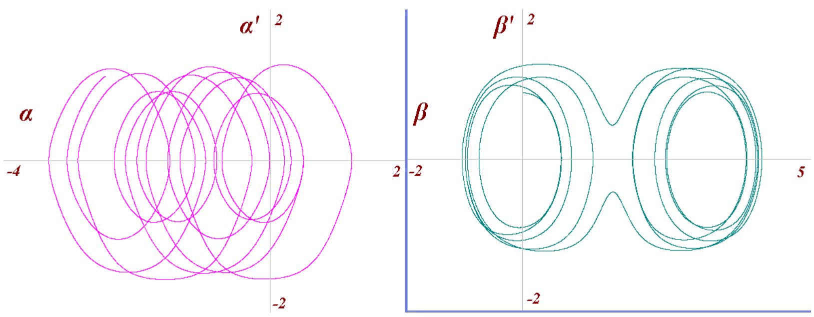

Figure 14. Phase-space representation, with α_0=0.5.

In the case of body in space, in the spatial orbit two hemispheres appear: it is true that several basins of attraction existed and appeared in the figures α vs β, but when the motion is represented in space (on a sphere of unitary radius) they remained superposed in only two centers or two hemispheres. When the inertia of the gimbals are considered, in the space orbit more centers appear, Fig. (13), and it is not possible to work with hemispheres to understand the movement, this behavior can much easily be observed experimentally.

The Phase-Spaces in Fig. (14), the angular speeds stay in a small region when compared to the speeds in the case of the body in space, since in this case when some change of basin happens the speeds are very high (greater than 60), this fact is tranquilizing since very great speeds are not feasible.

5. STUDY OF CHAOS

The investigation of chaos in this topic was initiated when studying the body in space; initially we believed that the jumps of basins of attraction had a chaotic behavior, which was easy to see in numerical results. After calculating the Lyapunov exponent of this movement, the chaotic tendency was verified. The influence of the inertias of the gimbals also are important in this analysis, the motion equations of the gyroscope are more complex than those of the body in space, therefore one expects also to get chaos. As the behavior of γ is almost linear, it is better to take periodic values of α, β angles for a specific condition and draw Poincare maps. In the following figure angles are represented for a specific speed condition.

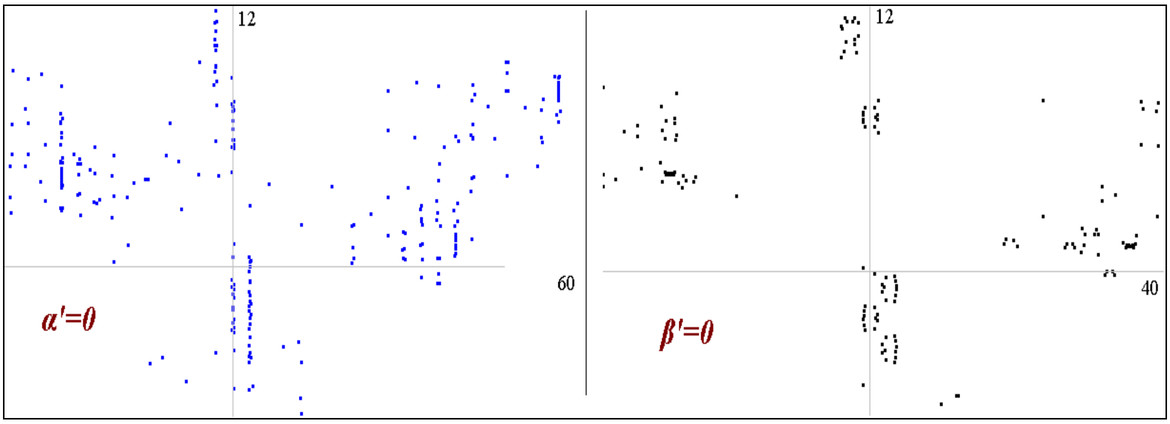

Figure 15. Poincare maps, β vs α for zero speed values.

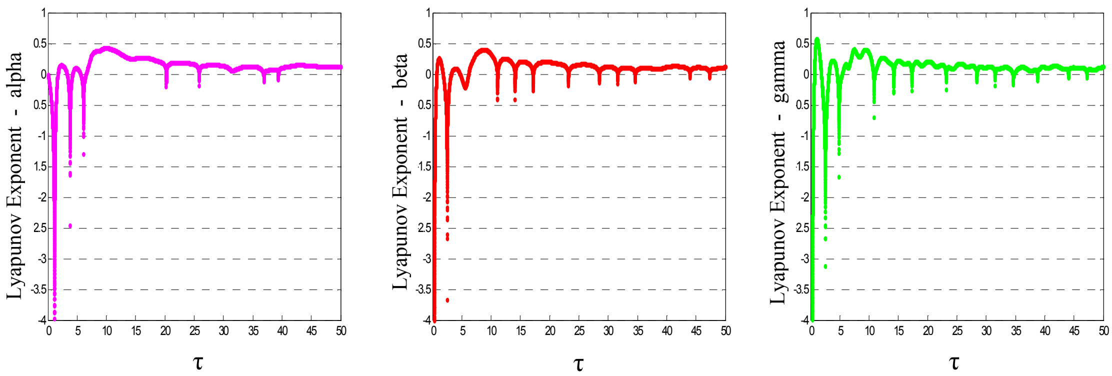

Figure 16. Lyapunov Exponent, considering the variation in initial condition $\Delta\alpha_0=10^{-6}$.

Figure (15) characterizes a chaotic behavior of the gyroscope, reaffirming the conclusion of Fig. (14) which also induces the believe of chaotic motion, since there is an infinite number of superimposed unstable orbits.

Figure (16) is used for a quantitative evaluation of the chaotic behavior, considering an initial condition of $\alpha_0=0,5$ and changing in a small amount $\Delta\alpha_0=10^{-6}$ to a nearby orbit. These figures depend on the initial condition, but the conclusion on the chaos will be the same, the positive value of the exponent will be always reached.

6. CONCLUSION

In the book of (Magnus, 1965) several experiments are proposed to evaluate the motion of a non axi-symmetric rotating body in space using the phenomena present in a gyroscope on a cardanic suspension. In fact, the inclusion of the inertia of the gimbals will give a much richer problem and, even very small inertias for the gimbal may alter the behavior considerably. When the system is subjected to impacts that inject sufficient energy for a change in the basin of attraction, the rotor without gimbals may only do it in α direction; in the β direction there is a singularity in the equations obliging us to work with an alternative parameterization, like quaternions. That is not the case for the gyroscope with gimbals.

The minimum kinetic energy to get a crossing over the borders defined by the limits of the Lyapunov stability and reach another basin of attraction, is lesser when including the gimbals, and the maximum energy transferred to the rotating body by the impact diminishes also a little. There are some differences according to which axis the impact is considered and the above mentioned is valid for the axis of minimum inertia. When impacting along the axis of maximum inertia (x) the minimum kinetic energy is also smaller but the maximum transferred energy by the impact tends to infinity of beta becomes greater as $\beta_0=1,25$.

The basin of attraction considered in α, β plane changes its geometrical appearance, from something like a square (without gimbals) to an elliptic form (considering gimbals). In this case there are neutral regions between distinct basins, where the motion crosses by when moving from one to the other. Sometimes the basins are superposed, inducing us to consider moving centers along the time. When representing the rotational orbit over a sphere with unitary radius we get the impression of having several basins over it and that the centers where during some time the motion keeps oscillating around also move with the time.

Looking at the phase planes and at the Poincaré maps, one may conclude on the presence of chaos in the motion. One may certify this conclusion drawing the diagram to obtain the Lyapunov exponent.

The investigated case considers a non axi-symmetric rotor where the biggest inertia is the polar inertia of the rotor ($1 \geq \mu_1$, μ_2) and where the initial impact that disturbs the motion occurs along the direction of y axis (μ_2) and along x axis (μ_1). For the axi-symmetric case ($\mu_1 = \mu_2$) the dynamic behavior including or not the gimbals is the same and the stability is kept changing the strength of the impact. In the non axi-symmetric case if the rotor is by itself unstable for a certain impact, it also will be with the gimbals but, there can be a case when the rotor is stable without considering the gimbals and turns out unstable due to its effect.

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