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Lubrication Model for the flow of several liquids through an annular space with varying eccentricity

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Abstract. *The cementation process is an important step in the construction of oil and gas wells. It provides zonal isolation and support for the well bore. During the cementation, it is necessary to displace the drilling mud by the cement slurry. To avoid mixing of these liquids, spacer fluids are usually used. Therefore, it is common to have three or more liquids flowing through the eccentric annular space.*
A complete analysis of the flow in the annular space that occurs during cementation is extreme complex, because of the presence of different liquids, that often present non-Newtonian characteristic and the flow is three dimensional and transient. Thus, a complete model has a prohibitive high computational cost. Simplified models are available in the literature and are used by the oil industry in commercial simulation software for cementation. However, the strong simplifying assumptions limit the range of parameters at which these models are accurate.
In this work, a 2D model for this 3D flow was developed using lubrication theory. The developed model considered the variation of the inclination and eccentricity along the well and the non-Newtonian behavior of the flowing liquids. The results show condition at which the displacement process is far from ideal which may lead to poor cementing job.

Keywords: *Cementation Process, Lubrication Model, Annular Space*

1. INTRODUCTION

The process of drilling a new well is dividede in different at some stages. One of these stages is the cementation process. After positioning the steel casing into the wellbore, it is necessary to fill the space that it left between the casing and rock wall. The cement has the following objectives: fill the space between the casing and well, to promote adhesion between the wall (rock) and the casing; provide mechanical support for the coating; and to isolate the formation to prevent fluid loss.

For the success of the cementation operation it is necessary a complete remove the drilling fluid or spacer fluid and not allow mixing between them. Issues or failures that occur during cementation can affect the hydraulic isolation of the well leading to the migration of gases or liquids from rock to annular and can cause severe productivity problems, but also endanger the safety of operation and cause environmental damage.

There are many studies in the literature on cementation. Some factors strongly influence the final outcome of the cementation process, among them there is the rheology of the fluids pumped, the geometry of the wells, the flow rate and the pumped volume of each fluid.

Bittleston [4] developed a model that considers the eccentricity of well and use a cartesian coordinate system to represent the geometry of the annular space. An asymptotic method the 2D problem as a sequence of 1D problems. As discussed in work, the main focus was to resolve the problem with low computational cost without a major concern with the accuracy of the solution.

The work of Pilipenko [3] was focused on viscoplastic fluids (Heschel-Bulkley). The model determines the areas where the stress is smaller than the yield stress and fluid does not flow. The model is tested for problems with steady-state fluid displacements in a reference frame attached to the interface. The eccentricity is taken to be constant and small.

Pina [1] developed a model using the lubrication theory and a cylindrical coordinate system. The effect of curvature of the straight section of the annular was not neglected. Thus, the model produces very accurate results, for any radii ratio R_i/R_0. However, the analysis was restricted to the flow of a Newtonian fluid.

The objective of this work is to develop an asymptotic model based on the theory of lubrication to study the displacement of different liquids through an annular space with variable eccentricity along well. The model, as well as the work discussed above, disregards the effect of curvature in the annular and describe the annular space by a cartesian coordinate system. Thus, the accuracy of the results is greater for radii ratio close to the unity. The model considers that the inclination of the well can variable along the length. Unlike the work the literature, the model is not limited to small eccentricities.

2. MATHEMATICAL FORMULATION

To model the flow in the annular space formed by two cylinders with eccentricity varying along the axial direction, we chose to discard the curvature of the cylinder wall by adopting a cartesian system of coordinates (z, y, x), where z is the coordinate in main flow direction (axial), y in the radial direction and x tangential direction.

The origin of the coordinate system in each straight section of the well is located at the center of the smaller cylinder which has radius R_i. The position of the center of the outer cylinder is defined by the eccentricity $e(z) = (e_1^2 + e_2^2)^{0.5}$, where e_1 and e_2 are respectively, horizontal and vertical (orthogonal functions) eccentricity. The coordinate of the wall of the outer cylinder is defined in terms of the larger cylinder radius R_0 and the eccentricity:

$$R(z,x) = e(z)cos(\frac{x}{R_i} - \gamma) + \sqrt{R_0^2 - e^2(z)sin^2(\frac{x}{R_i} - \gamma)} \tag{1}$$

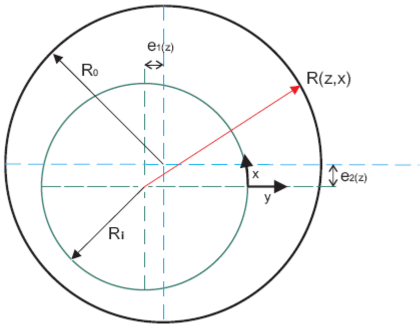

Figure 1. Geometry detail

Using the cartesian system of coordinates, the momentum conservation equations of the flow in the annular space is:

1. z direction:

$$\rho\left\{u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial x}\right\} =$$
$$-\frac{\partial p}{\partial z} + \rho g_z + \left[\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)\right] \tag{2}$$

2. y direction:

$$\rho\left\{u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial x}\right\} =$$
$$-\frac{\partial p}{\partial y} + \rho g_y + \left[\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)\right] \tag{3}$$

3. x direction:

$$\rho\left\{u\frac{\partial w}{\partial z} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial x}\right\} =$$
$$-\frac{\partial p}{\partial x} + \rho g_x + \left[\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right)\right] \tag{4}$$

where u, v and w are the axial, radial and tangential velocity components.

The complete solution of this problem has a prohibitive high computational cost. This system will be simplified in this work using lubrication theory.

A dimensional analysis can be used to eliminate some terms in this equation, this procedure is known as lubrication approximation.

The length scale in the axial direction is much larger than the length on the radial and azimuthal directions. Consequently, from the continuity equation, we can show that the velocity along the axial direction is much larger than the velocity components on the y and z directions. Furthermore, because the small annular space, the variation of the velocity components in the axial and tangential direction are much smaller than in the radial direction thus the second derivates are

$$\frac{\partial^2 u}{\partial y^2} \gg \frac{\partial^2 u}{\partial z^2}, \frac{\partial^2 u}{\partial x^2} \tag{5}$$

$$\frac{\partial^2 w}{\partial y^2} \gg \frac{\partial^2 w}{\partial z^2}, \frac{\partial^2 w}{\partial x^2} \tag{6}$$

Using the simplifications proposed by the lubrication approximation and neglecting the change in hydrostatic pressure in the radial direction, the equations of momentum reduce to:

$$-\frac{\partial p}{\partial z} + \left[\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)\right] - \rho g \sin\alpha = 0 \tag{7}$$

$$-\frac{\partial p}{\partial x} + \left[\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)\right] - \rho g \cos\left(\frac{x}{R_i}\right)\cos\alpha = 0 \tag{8}$$

$$-\frac{\partial p}{\partial y} = 0 \tag{9}$$

Velocity profile in axial direction can be obtained integrating equation (7). It can be done because the pressure gradient its not function of y coordinate.

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial z}y^2 + \frac{\rho g \sin\alpha}{2\mu}y^2 + C_1 y + C_2 \tag{10}$$

Appropriate boundary conditions are zero axial velocity in outer and inner cylinder wall and defining $H(z,x) = R(z,x) - R_i$, we can be obtain the integration constants:

$$C_1 = -\frac{H^3}{12\mu} \tag{11}$$

$$C_2 = -\frac{H^3}{12\mu}\rho g \sin\alpha = \rho g \sin\alpha C_1 \tag{12}$$

For the tangential direction, the same procedure can be done.

$$w = \left[\frac{1}{\mu}\frac{\partial p}{\partial x} + \rho g \cos\alpha \cos\left(\frac{x}{R_i}\right)\right]\frac{y^2}{2} + C_3 y + C_4 \tag{13}$$

$$C_3 = -\frac{H^3}{12\mu} \tag{14}$$

$$C_4 = -\frac{H^3}{12\mu}\rho g \sin\alpha = \rho g \sin\alpha C_3 \tag{15}$$

Observe that the integral of the axial and tangential velocity profiles along the y-direction are given by:

$$\int_0^H u \, dy = C_1\frac{\partial p}{\partial x} + C_2 \tag{16}$$

$$\int_0^H w \, dy = C_3\frac{\partial p}{\partial x} + C_4 \tag{17}$$

A differential equation for the pressure fluid can be obtained by integration the mass conservation equation along the y-direction.

$$\int_0^H \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\right) dy = 0 \tag{18}$$

Integrating each of the terms separately using a Leibnitz's rule and considering the no slip boundary conditions on the walls, we obtain:

$$\frac{\partial}{\partial z}\left[C_1\frac{\partial p}{\partial z}\right] + \frac{\partial}{\partial x}\left[C_3\frac{\partial p}{\partial x}\right] = -\left[\frac{\partial C_2}{\partial z} + \frac{\partial C_4}{\partial x}\right] \tag{19}$$

A solution of equation (19) gives the pressure field $p(z,x)$. With the pressure field, the velocity fields $u(x,y,z)$ and $w(x,y,z)$ can be obtained from equations (15) and (13). Its important to note that the coefficients $C_1(z,x)$, $C_2(z,x)$, $C_3(z,x)$, $C_4(z,x)$ depend on the geometry of annular space and the properties of liquid that occupies the point (z,x). Consequently, these coefficients vary with time as the fluid is replaced by another during the process of displacement.

To solve the equation (19), it is necessary to define the boundary conditions. For the well exit ($z = L$) we set a value of pressure P_s. Along left and right boundaries, $x = 0$ and $x = 2\pi R_i$ a periodic conditions are considered.

Two different conditions of entry ($z = 0$) were considered. The first was to impose a pressure input P_e and the second, more used, was to define a input flow rate.

To define the liquid properties at each point during the displacement process, we use a pure convective transport equation of a color function ϕ:

$$\phi \cdot \nabla\phi = 0 \tag{20}$$

each fluid has a different value of ϕ.

3. SOLUTION METHOD

The differential Eq. (19) was solve by finite difference method. To docretoze the domain a rectangle grid is created with NZ nodes in axial direction and NX in another direction.

$$\frac{\partial}{\partial z}\left[C_1\frac{\partial p}{\partial z}\right] = \frac{2}{\Delta z(i) + \Delta z(i-1)} \left\{ C_1(i+1,j)\frac{P(i+1,j) - P(i,j)}{\Delta z(i)} - C_1(i,j)\frac{P(i,j) - P(i-1,j)}{\Delta z(i-1)} \right\} \tag{21}$$

$$\frac{\partial}{\partial x}\left[C_3\frac{\partial p}{\partial x}\right] = \frac{2}{\Delta x(j) + \Delta x(j-1)} \left\{ C_3(i,j+1)\frac{P(i,j+1) - P(i,j)}{\Delta x(j)} - C_3(i,j)\frac{P(i,j) - P(i,j-1)}{\Delta x(j-1)} \right\} \tag{22}$$

$$\frac{\partial C_2}{\partial z} = \frac{C_2(i+1,j) - C_2(i,j)}{\frac{\Delta z(i) + \Delta z(i-1)}{2}} \tag{23}$$

$$\frac{\partial C_4}{\partial x} = \frac{C_4(i,j+1) - C_4(i,j)}{\frac{\Delta x(j) + \Delta x(j-1)}{2}} \tag{24}$$

Means velocities can be calculated for each face of the model.

$$\overline{U} = \frac{2}{H(i-1,j) + H(i,j)}\left[C_1\frac{P(i,j) - P(i-1,j)}{\Delta z(i-1)} + C_2(i,j)\right] \tag{25}$$

$$\overline{W} = \frac{2}{H(i,j-1) + H(i,j)}\left[C_3\frac{P(i,j) - P(i,j-1)}{\Delta x(j-1)} + C_4(i,j)\right] \tag{26}$$

4. RESULTS

4.1 Analytical Validation

In the first test is use a concentric annular space and pumped one Newtonian fluid. Thus it is possible to compare the results with analytical solutions. The goal of the test was to verify the accuracy of the user of cartesian coordinate system to describe the annular flow.

The results of cases 1, 2 and 3 in table 2 show, as expected, that the model developed for the Newtonian fluid works well when the ratio of radios $\frac{R_i}{R_0}$ approaches unity. The error found is related to the curvature term neglected in the model.

-	Case 1	Case 2	Case 3
L [m]	100.0	100.0	100.0
Nz	21	21	21
Nx	21	21	21
R_0 [m]	0.1524	0.1524	0.1524
R_i [m]	0.1143	0.12192	0.13716
R_i/R_0	0.75	0.8	0.9
$\mu[Pa.s]$	0.02	0.02	0.02
$\rho[kg/m^3]$	1400	1400	1400
$Q[m^3]$	0.002649788	0.002649788	0.002649788

Table 1. Data for Analytical Validation

-	Case 1	Case 2	Case 3
ΔP_{teo}	1601.110	2931.721	20847.794
ΔP_{sim}	1347.706	2589.841	20195.819
Erro [%]	18.8	13.2	3.2

Table 2. Results for Analytical Validation

4.2 Horizontal Well

A more recently developed technique known as directional drilling is aimed at extending horizontally in some directions the well, to improve productivity. Inclined wells may lead to a gravitational stratification of fluids, if the difference in density is high.

To simulate the directional drilling in an extreme case, a example of an horizontal concentric annular with the geometry defined as show in the table 3. Two Newtonian fluids with properties specified in table 4 are pumped. Fluid 2 has a density greater than fluid 1, which originally occupied the annular space.

Nz	201
Nx	21
L [m]	100.0
R_0 [m]	0.1524
R_i [m]	0.1143
e_1 [m]	0.0
e_2 [m]	0.0
$\alpha[deg]$	90.0

Table 3. Geometry Data – Example Horizontal Well

-	$\mu[Pa.s]$	$\rho[kg/m^3]$	$Q[m^3/s]$	$\forall[m^3]$
Fluid 1	0.02	1400.00	0.007949364	0.0
Fluid 2	0.03	3000.00	0.007949364	5.0

Table 4. Fluids Data – Example Horizontal Well

The plots with the evolution of interface (Figure 2) show that as the heavier fluid 2 is injected, it moves to the bottom of the annular (270^0), leaving the lighter fluid 1, on the upper part of the annular (90^0).

This stratification reduces the efficiency of the displacement and can bring serious problems for the process of cementation.

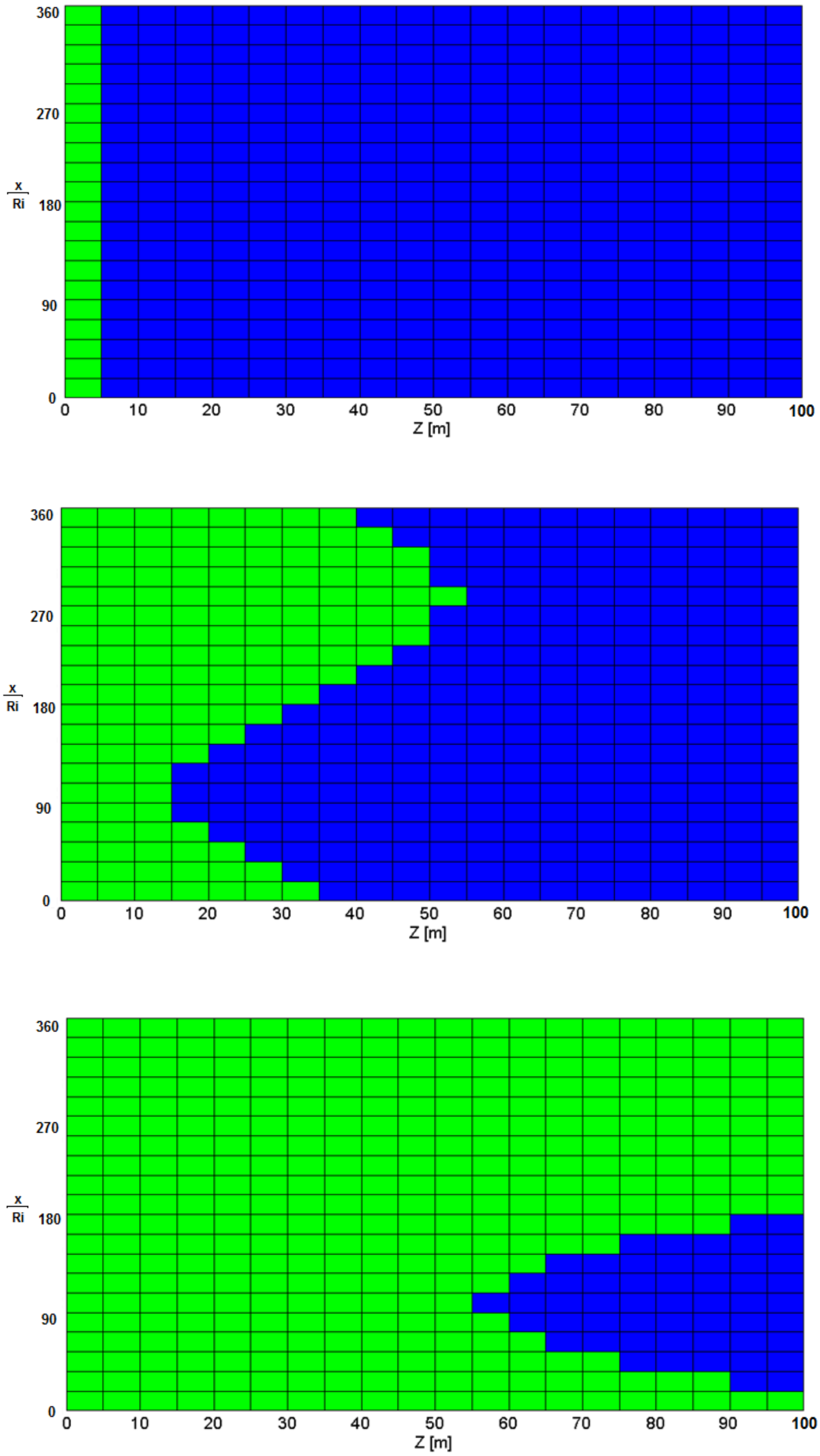

Figure 2. Evolution of the interface on a horizontal well

4.3 Directional Well

The second example is with a more complex geometry, close to a real case. The sketch of the geometry is shown in Fig. 3. The well was divided in seven segments, as shown in table 5, each segment has length, slope and different number of nodes. In each segment we tried to keep the nodes spaced by about 10 meters. The cylinders were concentric with external radius of $R_0 = 0.122238m$ and the internal radius of the cylinder $R_i = 0.10759m$.

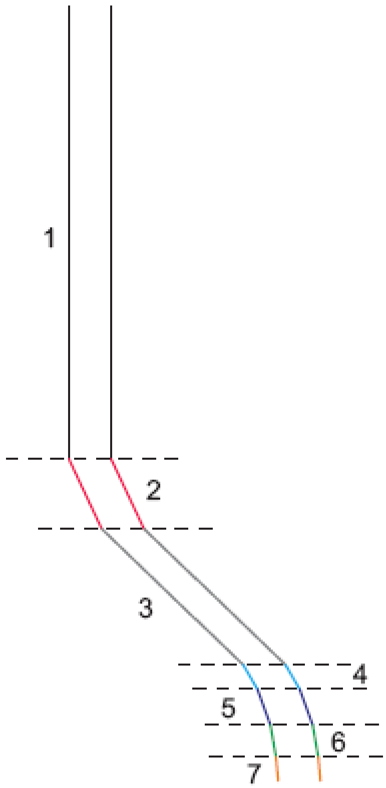

Figure 3. Sketch of the geometry used in example 2

Segment	1	2	3	4	5	6	7
Nz	370	63	111	23	31	27	19
Nx	21	21	21	21	21	21	21
L [m]	3674.0	626.0	1161.0	230.0	309.0	268.0	190.0
α [deg]	0.0	25.0	46.0	30.0	20.0	10.0	5.0

Table 5. Segments Data – Example Real Geometry Well

Seven fluids are pumped with differents properties, flow rates and volumes, as shown in table 6. Each fluid has characteristics close to the fluids used in a real operation and are called as follows: 1 is drilling fluid that is filling the annular initially, fluid 2 is the washer fluid and the fluid 3 is a spacer fluid. Fluid 4 and 5 have property closed to cement and the fluid 6 and 7 are displacement fluids, with the same properties, but pumped with different flow rate.

Fluid	1	2	3	4	5	6	7
$\mu[Pa.s]$	0.045	0.005	0.050	0.030	0.025	0.045	0.045
$\rho[kg/m^3]$	1342.0	899.0	1258.0	1534.0	1893.0	1342.0	1342.0
$Q[m^3/s]$	0.0	0.21198	0.26498	0.26498	0.07949	0.13249	0.26498
$\forall[m^3]$	0.0	7.94	15.90	17.49	27.03	9.54	55.65

Table 6. Fluids Data – Example Real Geometry Well

Figure 4 shows the evolution of the interface throughout the process. The low density and viscosity of fluid 2 shifts to the upper part of the annular (90^0) as the well becomes more oblique. We can observe that the lower part of the annular never came into contact with fluid 2. Next fluid (3) has a higher density and viscosity than fluid 2 and shows a more uniform displacement. The properties the other fluids were properly selected for an almost uniform displacement, with high efficiency.

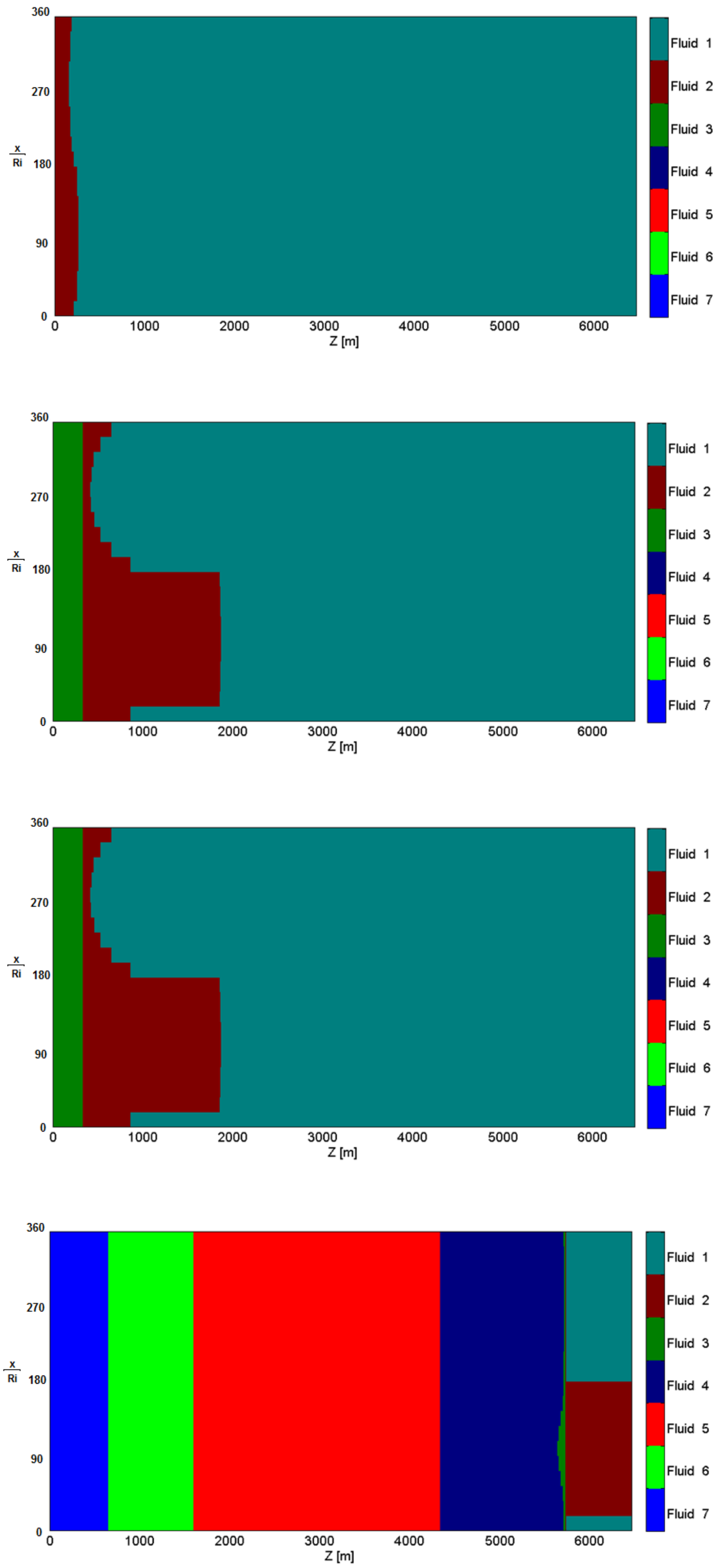

Figure 4. Evolution of interface in example 2

5. FINAL REMARKS

This work show a lubrication model developed to study the displacement process in annular space to study the displacement process in annular space with varying eccenticity and inclination. It can be used to study and design cementation processes. It is able to simulate the process in real geometry of wells used in the oil industry at relatively small computational time.

This model is being extended to include the curvature terms in calculation of the velocity profiles at each cross section, so it can obtain accurate prediction even at low raddi ratios.

6. ACKNOWLEDGEMENTS

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8. Responsibility notice