OPTIMIZATION OF LAMINATED COMPOSITE PLATES AND SHELLS USING GENETIC ALGORITHMS, NEURAL NETWORKS AND FINITE ELEMENTS

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Abstract. *Structural optimization using computational tools has become a major research field in recent years. Methods commonly used in structural analysis and optimization may demand considerable computational cost, depending on the problem complexity. Therefore, many techniques have been evaluated in order to diminish such impact. Among these various techniques, artificial neural networks may be considered as one of the main alternatives, when combined with classic analysis and optimization methods, to reduce the computational effort without affecting the final solution quality. Use of laminated composite structures has been continuously growing in the last decades due to the excellent mechanical properties and low weight characterizing these materials. Taken into account the increasing scientific effort in the different topics of this area, the aim of the present work is the formulation and implementation of a computational code to optimize manufactured complex laminated structures with a relatively low computational cost by combining the Finite Element Method (FEM) for structural analysis, Genetic Algorithms (GA) for structural optimization and Artificial Neural Networks (ANN) to approximate the finite element solutions. The modules for linear and geometrically non-linear static finite element analysis and for optimize laminated composite plates and shells, using GA, were previously implemented. Here, the finite element module is extended to analyze dynamic responses to optimize problems based in frequencies and modal criteria, and a module with perceptron ANN is added to approximate finite element analyses. Several examples are presented to show the effectiveness of ANN to approximate solutions obtained using the FEM and to reduce significatively the computational cost.*

Keywords: *Laminated composite plates and shells, Artificial Neural Networks, Optimization, Genetic Algorithms, Finite Element*

1. INTRODUCTION

In composite materials structures, some experiences has shown that the GA performs better than traditional gradient based techniques due to the discrete approach of the design variables.

A problem that arises when GA are used is the high computational cost demanded by this method. For this reason some techniques to reduce this cost have been tested. One of them, consist to replace the complete FEM analyses by some approximation technique. The ANN have been shown to be a good alternative to avoid the large number of FEM analyses involved in the GA approach.

In this work these two techniques are combined to make the process faster and cheaper in terms of computational cost.

This work is based on Almeida and Awruch (2009) from which some GA parameters, objective functions and results are taken in order to compare the effectiveness of ANN substituting a complete FE analyse.

2. STRUCTURAL OPTIMIZATION

The structural optimization can be understand as a process to search the configuration which gives the best performance, within some criteria and subjected to certain design constraints.

To model the structural optimization as a mathematical optimization problem, the following concepts are used:

Design variables: they are the characteristics that can be modified by the mathematical optimization algorithm to obtain the best structural performance.

Design constrains: are the restrictions applied to the structure, such as a limit to avoid material failure, maximum or minimum value of design variables and others that depend on the problem being analyzed.

Objective function: it is a mathematical expression in which the design variables and constrains are involved. This function represents a number which has to be maximized or minimized during the optimization process.

2.1 Structural Analysis

The analysis of the composite structures is carried out using the FEM. The element used is a triangular flat plate and shell element with 18 degree of freedom called DKT (*Discrete Kirchhoff Triangle*). This element was developed by Bathe and Ho (1981) for isotropic materials and it was extended by Almeida (2006) for laminated composite materials.

For the modal analysis the mass matrix is obtained using the formulation given by Luo and Hutton (2002).

To solve geometrically non linear problems, the generalized displacement control method (GDCM) (Yang and Shieh, 1990) is used.

The Tsai-Wu failure criterion is employed for failure prediction in a ply (Daniel and Ishai, 1994).

2.2 Genetic Algorithms for composite materials

The genetic algorithm approach adopted here was proposed by Soremekun (1997) and extended by Almeida (2006). The classical genetic algorithm is modified to manage composite materials structures. Two genes are used, one for materials and another for the orientation of reinforcement fibers on each ply.

The genetic operations used here are crossover, mutation and gene swap. For the duble break point crossover the chromosome is divided in 3 parts with almost the same size, the first and the last part of the chromosome for the son is taken from the dad 1 and the middle part is taken from the dad 2.

The selection scheme adopted is the Multiple Elitist 1 proposed by Soremekun (1997). In that scheme some of the best individuals from the parents and sons generations are used in the son generation to maintain and improve the evolution this is defined by the N_e number.

The criterion to stop the GA is defined by two parameters. The first one is the number that limits the generations (N_{LG}), which is used to limit the total number of generations in the GA. The second one, is the number of generation with the same optimum design (N_{SD}), which indicates that the convergence rate has been zero for a defined number of generations.

3. ARTIFICIAL NEURAL NETWORKS

Artificial neural networks are mathematical models imitating the human brain. Based on the experience, the ANN can learn complex relationships between inputs and outputs data. The use of ANN to predict structural responses has been growing in the recent years. To continue this research line here, neural networks will be applied to learn the structural response of laminated composite structures (Kovacs, 1996). The neural network model used here is described below.

3.1 Multilayer Perceptrons

The neural network model adopted here is the multilayer perceptron. The architecture is formed by one input layer with the number of nodes defined by the number of genes, two hidden layers with hyperbolic tangent as the activation function and finally one output neuron performing a linear combination of the hidden layers output. So, the architecture, described as 12-18-18-1, indicates that there are 12 input nodes, two hidden layers with 18 nodes each one and 1 output.

The training is made using backpropagation algorithm. To update the weights the generalized delta rule is employed (Haykin, 1998).

The stop criterion is activated when the mean square error of the training set is less than a fixed value. For the training the on-line update method (sample by sample) is used.

3.2 Artificial Neural Networks and Genetic Algorithms

To combine the ANN and GA, the scheme shown in Fig. 1 is adopted. To generate the training set for the ANN in each case, three GA are executed with two generations each one, having large populations. This approach is used to make a randomly distributed generation in the first generation and some elitist generation in the second one. Three executions make the generations disperse and enough to guarantee that almost every zone of the design space is covered.

So, when the ANNs are trained with the training set generated by the previously described process, every structural response is taken from the ANN. In the case of parameters that can be directly calculated from the genetic codification such as cost, and weight, for example, they are calculated directly.

4. NUMERICAL EXAMPLES AND DISCUSSION

To evaluate the quality of the optimization scheme and the time that can be saved, three examples are shown. In each one the GA and the training time as well as the error of the ANN (with respect to the FEM solution) are described, a comparison with GA-FEM with respect to processing time and quality of the results are also presented. All examples has been ran in a 2.4 Ghz Core 2 Quad computer with 4 Gb of RAM.

4.1 Cost and weight minimization of an in-plane loaded composite laminate plate

In this problem the number of plies, the material of each ply and the orientation of reinforcement fibers are the design variables. The right combination of these variables can determine the cheapest and lightest structure. The constraints of

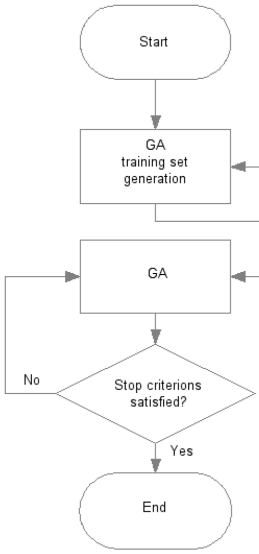

Figure 1. Flowchart of GA-ANN

the problem are the material failure (λ_f) derived from the Tsai-Wu criterion (Daniel and Ishai, 1994), and the structural elastic stability (λ_b), both must be greater or equal to 1.0. The cost is proportional to the material consumption in the laminated, and each material has its own cost per unit weight denoted by C. The plate model with boundary and load conditions is shown in Fig. 2. The finite element mesh has 3000 elements. The objective function is defined in Eq. 1 , where Wt^* and C^* are the normalized weight and cost respectively; this normalization is shown in Eq. 2 and ϕ is the weighting factor of both objectives; in this example the weighting factor is 0.5. W_{min} and W_{max}, are the minimum and maximum weight that the structure can have. C_{min} and C_{max} are the minimum and maximum cost that the structure can have. These parameters are obtained using extreme values of codification.

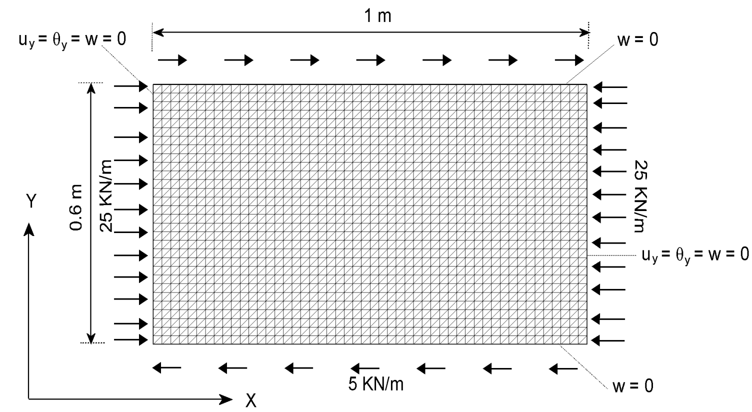

Figure 2. Composite laminated plate with its boundary and load conditions

$$\begin{cases} OBJ = 10 - \sqrt{\left[\phi\left(Wt^*\right)^2\right]^2 + \left[\left(1-\phi\right)\left(C^*\right)^2\right]^2} + 10^{-6}\lambda^* & , if\ \lambda^* \geq 1 \\ OBJ = \left(\lambda^*\right)^2 \left\{10 - \sqrt{\left[\phi\left(Wt^*\right)^2\right]^2 + \left[\left(1-\phi\right)\left(C^*\right)^2\right]^2}\right\} & , if\ \lambda^* < 1 \end{cases} \tag{1}$$

$$C^* = \frac{C - C_{min}}{C_{max} - C_{min}} + 1 \qquad Wt^* = \frac{W - W_{min}}{W_{max} - W_{min}} + 1 \qquad \lambda^* = minimum(\lambda_f, \lambda_b) \tag{2}$$

The elastic constants, strength parameters, specific weight, ply thickness and the parameter of cost per unit weight for the Kevlar-epoxy and Graphite-epoxy are shown in Tab. 1. The elastic constants are the Young's modulus in the fiber direction (E_1) and transverse to the fiber direction (E_2), the shear modulus (G_{12}) and the Poisson's ratio (ν_{12}), respectively. Strength parameters for traction and compression for longitudinal and transversal directions are given by

Table 1. Materials properties - Example 1

Properties	Kevlar-epoxy	Graphite-epoxy
E_1	87.0 GPa	181.0 GPa
E_2	5.5 GPa	10.3 GPa
G_{12}	2.2 GPa	7.17 GPa
ν_{12}	0.34	0.28
t	0.18 mm	0.13 mm
ρ	13.5 KN/m^3	15.7 KN/m^3
C	1.0 uc/N	3.0 uc/N
F_{1t}	1280.0 MPa	1500.0 MPa
F_{1c}	335.0 MPa	1500.0 MPa
F_{2t}	30.0 MPa	40.0 MPa
F_{2c}	158.0 MPa	246.0 MPa
F_6	49.0 MPa	68.0 MPa

F_{1t}, F_{1c}, F_{2t}, and F_{2c}, respectively. The remainder parameters are the shear strength (F_6), the specific weight (ρ) and the thickness (t).

The alphabet used in the GA is shown in Tab. 2; the laminated plate can adopt different number of plies varying from 12 to 24 and the symmetry condition is applied. The size of the design space (SDS) is 55944.

Table 2. Genetic codification alphabet and possible values - Example 1

Angle genes		Material Genes	
code	angle	code	material
1	2 plies at 0^o	1	Kevlar-epoxy
2	2 plies at $\pm 45^o$	2	Graphite-epoxy
3	2 plies at 90^o		

The parameters of the GA are shown in Tab. 3 where P is the size of population, N_e is the elitist scheme parameter, p_{ma} and p_{mm} are the probabilities of angle and material mutation respectively, p_{gs} is the probability of gene swap, p_{pa} and p_{pd} are the probabilities of ply addition and ply deletion, respectively, N_{LG} and N_{SD} are stop criterion parameters.

Table 3. GA parameters - Example 1

P	30	N_{SD}	100	p_{pa}	4%
N_e	4	p_{ma}	4%	p_{pd}	8%
N_{LG}	300	p_{mm}	2%	p_{gs}	80%

The training set consists in 2347 designs and the time used to generate this training set was 1127.74 hours. The neural network architecture used here is 12-18-18-1. Two neural networks are used to approximate (λ_f) and (λ_b) respectively. The cost and the weight are calculated directly from the genetic code of each design. The training times and the error (with respect to the FEM solution) for the neural networks are shown in Tab. 4.

Table 4. Neural networks training time and errors for the Example 1

NN to approximate λ_f		NN to approximate λ_b	
Time	1.81 min.	Time	1.15 min.
Error	0.05	Error	0.03

The optimal design found using GA-ANN and values of the parameters are shown in Tab. 5.

Performing a FE analysis using the design obtained from the GA-ANN optimization, the real differences of the parameters are shown in Tab. 6.

To find the quality of the optimization, the design obtained with GA-ANN is compared to the design found using GA-FEM. This last design and parameters values are shown in Tab. 7.

The real differences between the designs obtained with both methods are shown in Tab.8, where the real difference applying FEM to the optimum design obtained by GA-ANN. This result shows that the design obtained with the GA-ANN is a near optimum design.

Table 5. Results using GA-ANN - Example 1

Results	
Laminate	$\left[\pm 45_4^{ge}, \pm 45^{ke}, 90^{ke}, 0^{ke}\right]_S$
λ_f	16.79
λ_b	1.54
Wt	27.34 N
C	46.93 uc/N
Fitness	9.789
GA Generations	171
Time	0.08 min.

Table 6. Differences between ANN and FEM - Example 1

	λ_f	λ_b	**Fitness**
ANN	16.76	1.39	9.790
MEF	14.63	1.46	9.789
Error	-14.56%	5.07%	-0.01%

Table 7. Results using GA-FEM - Example 1

Results	
Laminate	$\left[\pm 45_4^{ge}, \pm 45^{ke}, 90_4^{ke}\right]_S$
λ_f	14.07
λ_b	1.54
Wt	27.34N
C	46.93N
Fitness	9.789
GA Generations	146
Time	264.73 hours.

Table 8. Real differences between optimum designs in Example 1

	Laminate	λ_f	λ_b	Fitness
GA-ANN	$\left[\pm 45_4^{ge}, \pm 45^{ke}, 90^{ke}, 0^{ke}\right]_S$	16.76	1.39	9.789
GA-FEM	$\left[\pm 45_4^{ge}, \pm 45^{ke}, 90_4^{ke}\right]_S$	14.07	1.54	9.789
Differences		3.80%	-5.76%	0.00%

To evaluate the time saved using ANN, a time comparison is shown in Tab. 9.

Table 9. Time comparison (hours) Example 1

	GA-ANN	GA-FEM
Training set generation	112.74	-
Neural networks training	0.11	-
GA execution	0.001	264.73
Total time	112.851	264.73

The present problem can be solved by a mesh having much less elements. But the aim of this example is to show that notorious gains in time occurs when the FE analysis is time consuming. In this case the difference in processing time is 57.37%.

4.2 Stiffness maximization of a composite laminated shell with geometrically nonlinear behavior

This optimization aim to maximize the stiffness of a composite shallow shell under pressure load. Figure 3 shown the shell with its boundary and load conditions. The mesh has 800 elements. The nonlinear analysis is made using the GDCM method (Yang and Shieh, 1990), with a load increment parameter $\lambda_i = 0.05$. The fitness function is defined in Eq. 3, where NC_{crit} is the critical load level(when curve load level - displacement reaches its first peak), U_{max} which is the maximum displacement which is taken at the end of load or when material failure is observed, NC_{max} is the maximum load level without material failure and V_{nlc} is a penalization for designs that have more than 4 plies with same fiber orientation which is directly obtained from the genetic code without the use of an ANN.

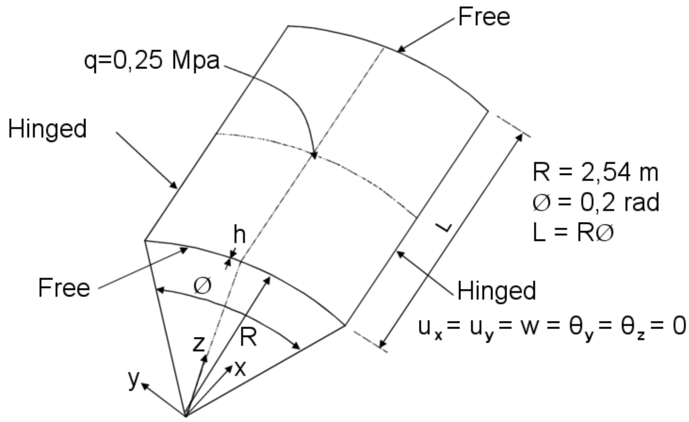

Figure 3. Composite laminated shell whit boundary and load conditions

$$FITNESS = \left(\frac{(NC_{crit}) \cdot (NC_{max}^2)}{(U_{max}) \cdot (V_{nlc} + 1)} \right) \tag{3}$$

It is used Glass-epoxy, which properties are described in Tab. 10.

Table 10. Properties of Glass-epoxy

Properties	Values	Properties	Values
E_1	39.0 GPa	F_{1t}	1080.0 MPa
E_2	8.6 GPa	F_{1c}	620.0 MPa
G_{12}	3.8 GPa	F_{2t}	39.0 MPa
ν_{12}	0.28	F_{2c}	128.0 MPa
ρ	20.6 KN/m^3	F_6	89.0 MPa

The genetic alphabet used is shown in Tab. 11. In this example only the orientation of reinforcement fibers are the design variables, while the material, thickness and number of layers are fixed. The laminated has 28 plies (with $t = 0.45mm$ for each ply), the symmetric condition is imposed, each gene control 2 plies and the chromosome has 7 genes to control the laminate. Due to the long codification, for the crossover is used a double break point. The size of the design space (SDS) is 2187.

Table 11. Genetic codification alphabet - Example 2

Genes of the angles	
code	angle
1	2 plies at 0^o
2	2 plies at $\pm 45^o$
3	2 plies at 90^o

The GA parameters used here are $P = 18$, $N_e = 3$, $p_{ma} = 5\%$, $p_{mm}0\%$, $p_{gs} = 80\%$, $p_{pa} = 0\%$, $p_{pd} = 0\%$, $N_{LG} = 108$, $N_{SD} = 36$ with the same meaning as in the previous example.

In this example, three neural networks are used to approximate the parameters involved in the objective function; each ANN the trained parameters, errors and time are show in Tab. 12. The architecture adopted is 7-15-15-1. The training set has 226 designs and the time to generate this training set was 117.31 minutes.

Table 12. Neural Networks training time and errors for Example 2

NN to approximate (NC_{max})		NN to approximate (NC_{crit})		NN to approximate (U_{max})	
Time	5 seg.	Time	11 seg.	Time	7 seg.
Error	0.001	Error	0.001	Error	0.001

The design obtained using GA-ANN and its parameters values are shown in Tab. 13.

Table 13. Results using GA-ANN - Example 2

Results	
Laminate	$[90_4, \pm 45, (90_2, 0_2)_2]_S$
(NC_{max})	0.989
(NC_{crit})	0.637
(U_{max})	$26.6 \times 10^{-3} m$
Fitness	23.42
GA Generations	47
Time	0.01 min.

To evaluate the quality of the approximation made by the neural networks, the parameters values are compared to the FEM obtained parameter of the same design, these differences are shown in Tab. 14.

Table 14. Differences between ANN and FEM - Example 2

	(NC_{max})	(NC_{crit})	(U_{max})	**Fitness**
ANN	0.989	0.637	$26.6 \times 10^{-3} m$	23.42
FEM	1.000	0.495	$27.4 \times 10^{-3} m$	18.03
Error	1.10%	-28.62%	3.18%	-29.94%

The result and time used in the optimization process using GA-FEM is show in Tab. 15.

Table 15. Results using GA-FEM - Example 2

Results	
Laminate	$[(90_4, \pm 45)_2, 90_2]_S$
(NC_{max})	1.000
(NC_{crit})	0.560
(U_{max})	$27.2 \times 10^{-3} m$
Fitness	20.60
GA Generations	40
Time	239.72 min.

To verify the quality of the optimization process using GA-ANN, parameters values for both designs, obtained with ANN and FEM are compared in Tab. 16.

A graphical comparison of both designs is shown in Fig. 4, where NC_{crit} and U_{max} are indicated.

The processing time comparison using GA-ANN and GA-FEM is shown in Tab.17.

In this example the time saved using GA-ANN is 50.79%; to evaluate te quality of the optimization, the whole design space analysis was used, and in this space the design obtained by the GA-ANN is the 9th. among the near optimum designs (this set includes the optimum design).

4.3 Fundamental natural frequency maximization of a laminated plate

The objective of this optimization is to maximize the natural frequency of vibration (ω) of a simply supported composite laminated plate. No constraints are imposed to the problem. To obtain (ω) the eingenvalues problem is solved,

Table 16. Real differences of optimums obtained by GA-ANN and GA-FEM - Example 2

	GA-ANN	GA-FEM	Differences
Laminate	$[90_4, \pm 45, (90_2, 0_2)_2]_S$	$[(90_4, \pm 45)_2, 90_2]_S$	
(NC_{max})	1.000	1.000	0.00%
(NC_{crit})	0.495	0.560	-13.03%
(U_{max})	$27.4 \times 10^{-3} m$	$27.2 \times 10^{-3} m$	1.08%
Fitness	18.03	20.60	-14.26%

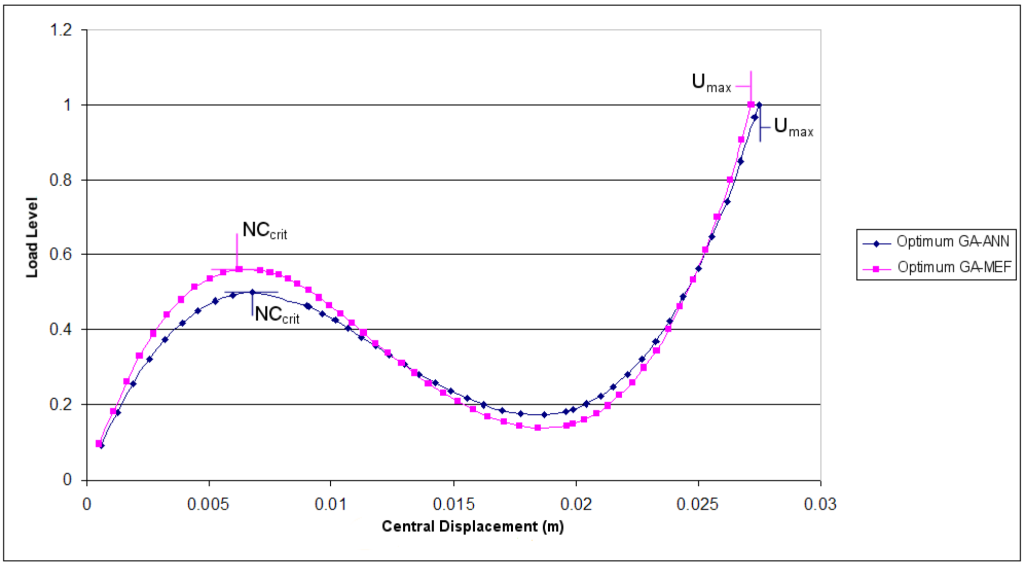

Figure 4. Curve load level - central displacement

Table 17. Time comparison (minutes) Example 2

	GA-ANN	GA-MEF
Training set generation	117.31	-
Neural Network training	0.65	-
GA execution	0.01	239.72
Total time	117.97	239.72

which is given by $(K - \omega^2 M)\Phi = 0$, where K is the stiffness matrix and M is the mass matrix, ω^2 and Φ represents the eingenvalues and eigenvectors respectively. The GA modifies only the reinforcement fibers orientation in each ply. The material and number of plies are fixed. The number of plies is fixed in 8, with thickness equal to 2 mm each one. Then the total height is 16 mm. Figure 5 shows the plate, boundary conditions and the mesh (with 256 elements).

Graphite-epoxy is used to build this plate, and the material properties are given in Tab. 18.

Table 18. Properties of Graphite-epoxy

Properties	Values	Properties	Values
E_1	181 GPa	F_{1t}	1500.0 MPa
E_2	10.3 GPa	F_{1c}	1500.0 MPa
G_{12}	7.17 GPa	F_{2t}	40.0 MPa
ν_{12}	0.28	F_{2c}	246.0 MPa
ρ	15.7 KN/m^3	F_6	68.0 MPa

The genetic alphabet used in this example is shown in Tab. 19. In this example only the orientation of fiber reinforce-

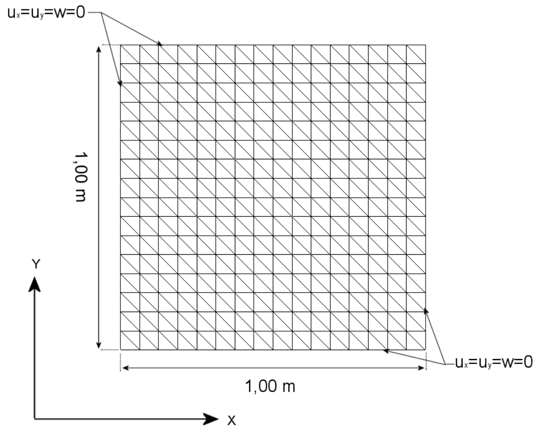

Figure 5. Composite laminated plate with its boundary conditions and the FE mesh

ment are the design variables. The number of layers, the thickness and the material of each layer are fixed. The total number of layer is 8. The chromosome length is 8. Double break point is used in the crossover operation. The size of the design space (SDS) is 6561.

Table 19. Genetic codification alphabet - Example 3

Angle genes	
code	angle
1	1 ply at 0^o
2	1 ply at 45^o
3	1 ply at 90^o

The GA parameters used here are $P = 30$, $N_e = 3$, $p_{ma} = 5\%$, $p_{mm}0\%$, $p_{gs} = 80\%$, $p_{pa} = 0\%$, $p_{pd} = 0\%$, $N_{LG} = 200$, $N_{SD} = 100$, with the same meaning as in the previous example.

The neural network architecture used in this example is 8-17-17-1. The training time and error are shown in Tab. 20. The training set has 420 designs, the time to make it was 26.75 minutes.

Table 20. Neural Network training time and error for Example 3

Neural network to approximate ω	
Time	1.25 min.
Error	0.01

The design obtained here as well as ω are shown in Tab. 21

Table 21. Result using GA-ANN - Example 3

Result	
Laminate	$[90, 45, 0_3, 90, 45_2]$
ω	6.24 rad/s.
Generations	182
Time	0.03 min.

The difference of ω obtained using ANN and FEM is shown in Tab. 22.

Making a GA-FEM optimization, the same design is found. A comparison of the processing time required for each method is shown in Tab. 23.

In this example the processing time saved is 59%. The quality of the optimization is high, this occurs because the function that has to be replaced by the neural network is very simple.

Table 22. Difference between ANN and FEM - Example 3

	ω
ANN	6.24
FEM	6.27
Error	0.48%

Table 23. Time comparison (minutes) for Example 3

	GA-ANN	GA-FEM
Training set generation	26.75	-
Neural networks training	1.25	-
GA execution	0.03	68.44
Total time	28.03	68.44

5. FINAL REMARKS

As it was mentioned in the three examples, an important amount of processing time can be saved using the GA-ANN scheme. The quality of the optimum design is very little affected by the use of ANN instead of complete FE analyses. Accuracy can be improved with a larger and more elitist training set as well as increasing the number of GA applications to generate the samples of the training set. In cases where the approximated structural function is simple, as in the last example, the quality is almost not affected by the use of GA-ANN instead of GA-FEM.

6. ACKNOWLEDGEMENTS

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