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STATIC ANALYSIS OF THE RISERS-SOIL INTERACTION USING POSITIONAL FORMULATION

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Abstract. *One of the critical stages of offshore oil production process into deep water refers to the risers installation and operation. Particularly, for deep water with water layer higher than* $2000\ m$ *where steel catenary riser may be a good choice. In this case, two nonlinear effects are specialy importants: large geometric non-linearity behavior from the line riser and soil-structure interaction in the seabed contact region. In this way, problems as behavior of risers and contact between the Steel Catenary Risers (SCR) and seabed, in marine studies, still require correct understanding. Thus, soil-fluid-structure interaction studies using a simple and effective tools is important to risers dynamics area. The proposal of this paper is to present one mechanical model to describe initially the static behaviour of SCR and one computational tool capable of representing the SCR contact with the seabed. For this way, a recent formulation named Positional Formulation was used to model a line riser. Positional Formulation is different from commonly used by workers because it is based on positions and not the displacement. It uses Lagrangian reference that is based on starting position of the body. This formulation is relatively simpler than those of custom and its implementation follows the same line. Newton-Raphson was used to solve the nonlinear system and Penalty Method was used to solve the contact problem. It was developed a comparative study about soil-structure interaction where were considered two specific conditions for displacement, linear and nonlinear, and two different soils, linear and bilinear. Always as possible the results obtained with the developed simulator were compared with existing literature.*

Keywords: *risers, finite elements, positional formulation, static, soil.*

1. INTRODUCTION

Recently, Brazil has discovered a important oil reserve in the pre-salt layer. This discovery suggests large investments to exploit these reserves, among others because the oil reserve is under a water layer higher than $2000\ m$. The explotation in ultra high depth water layer demands high technology and efforts that allow reducing the cost of operation making it viable.

To reduce operations costs is common to use computacional simulation in the design stage. Thus, there are softwares able to simulate some general cases, what is not the best thing to do in some cases like this that are very specifics. Most of then has been implemented using Finite Element Method [Bathe, 1982, Cook et al., 2002]. So, is common to find some dificulty to simulate this kind of problem, where is present non-linearity effect. In this way, this paper propose to use an alternative formulation to describe geometric nonlinear behavior and apply it for risers analysis. Nowadays, there are some formulations used to describe nonlinear problems but they usually are complex. For this reason, one new formulation named Positional Formulation [Coda, 2003a] based Finite Element Method propose to be less complex and more didatic.

This work is the first stage of the project that the biggest aim is construct a simple and effective tool able to realize static and dynamic analysis of SCR with or without contact with the ground and including or not others effects. This paper presents a brief description of the Positional Formulation, which can be best assessed through references. In the sequence, a verification of the positional element of the beam is showed. Finally, a comparative study of simplified model proposed to evaluate the soil-structure interaction were done. In this study, the model was validated for a linear case, because it gives analytical solution and, subsequently, was evaluated for situations with large nonlinearities and then ground with linear and bilinear.

2. POSITIONAL FORMULATION

The Positional Formulation is a new formulation developed recently aiming atempt difficultys found with geometric non-linearity [Coda, 2003a]. It is a relative simple and didatic formulation used to model engineering problems using finite element method (FEM). In this work, this formulation was used to describe beam elements with four nodes and three degrees of freedom per one. This formulation is basead in the principle of minimum potential energy that can be written using position considerations even displacements. In this way, the conservation law applied to an elastic medium results in

$$\Pi = U_e - P \tag{1}$$

where Π is the total potential energy, U_e is the strain energy and P is the potential energy of the applied forces.

In this case, will be used a linear constitutive relation with conjugate Green-Lagrange finite strain tensor and 2^{nd} Piola-Krchhof stress tensor. The strain energy can be written for the reference volume V_o as

$$U_e = \int_{V_0} u_e \, dV_0 \tag{2}$$

where u_e is the especific strain energy.

The potencial energy of the applied forces can be calculed as, following:

$$P = F_i^T X_i \tag{3}$$

where F_i is the force applied in each node and X_i is the set of positions independent of each other, it is important to note that X may be occupied by a point of the body. Other point interisting is the fact that the potencial may not be zero in the reference.

In this way, is possible to write the equation of total potential energy as following:

$$\Pi = \int_{V_0} u_e \, dV_0 - F_i^T X_i \tag{4}$$

Applying the equilibrium condition in the expression of total potential energy, Eq. (4), we can write:

$$\frac{\partial \Pi}{\partial X_i} = \int_{V_0} \frac{\partial u_e}{\partial X_i} \, dV_0 - F_i = 0 \tag{5}$$

To solve this non linear problem the Newton-Raphson procedure was used. Thus, the Eq. (5) may be written in a compact notation

$$\frac{\partial \Pi}{\partial X_i} = g_i(X_i) = f_i(X_i) - F_i = 0 \tag{6}$$

where X_i is a generealized parameter and indices are related to nodal positions by $(1, 2, 3, 4, 5, 6, ...) = (X_1, Y_1, \Theta_1, X_2, Y_2, \Theta_2, ...)$.

The next stage is solve the Eq. (6) using the Newton-Raphson procedure using Taylor expantion truncated in the first term, thus, we can write

$$g_i(X_i) = 0 \cong g_i(^0X_i) + \nabla g_i(^0X_i)\Delta X_i \tag{7}$$

or

$$\Delta X_i = -[\nabla g_i(^0X_i)]^{-1} g_i(^0X_i) \tag{8}$$

where $[\nabla g(^0X_i)]$ is the Hessian and $g(^0X_i)$ is the nodal force vector.

In order to calculate the Hessian and the Nodal Force Vector the equations (9) and (10), respectively, are used.

$$\nabla g_i(^0X) = \int_{V_0} \frac{\partial^2 u_e}{\partial X_k \partial X_j} dV_0 \tag{9}$$

and

$$g_i(^0X) = \int_{V_0} \frac{\partial u_e}{\partial X_i} dV_0 - F_i \tag{10}$$

where X_i, X_j, X_k are nodal variables.

More details of the Positional Formulation can be found in [Coda, 2003a, Coda, 2003b, Maciel, 2008].

2.1 Mapping the Beam

To map a beam using positional formulation is necessary to understand that should have a initial and final reference. It's necessary too, to know that the method is basead in Lagragian Referencial, i.e., the initial reference never changes. An auxiliary non-dimensional space is used to map the beam. The Fig. 1 shows the auxiliary non-dimensional space used to the mapping the beam.

We define three reference spaces: B_0 represent the initial position, (ξ, η) is the isoparametric referencial and B_1 represent the actual position. To transforme the quantities from the B_0 to the B_1 spaces is necessary to use a non-dimensional isoparametric space (ξ, η). So, is necessary to define the gradient operators A_0 e A_1, [Coda, 2003a].

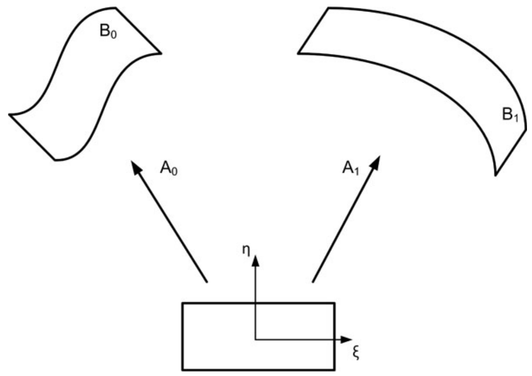

Figure 1. Auxiliary Non-Dimensional Space

A mathematical combination for the Jacobian operator, $A = A_1 A_0^{-1}$, allow relate the initial and final states.

Now, is possible to describe the beam basead in position in this paper, Reissner Beam Model was used. A_i is the gradient matrix. The Fig. 2 shows the mapping of the beam.

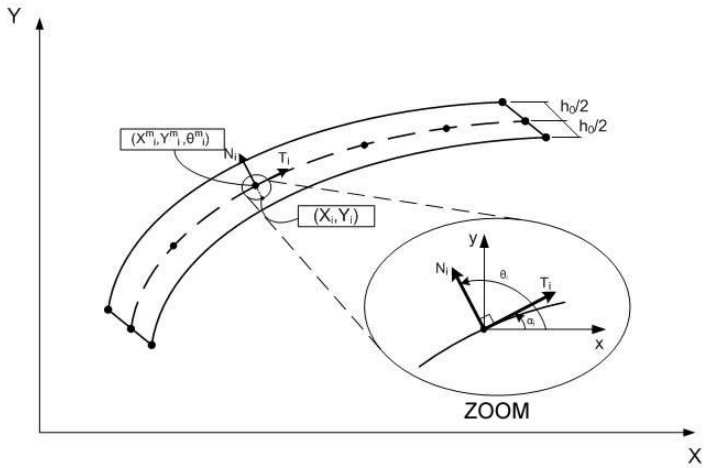

Figure 2. Mapped of the beam.

The beam geometry may be represented, in the initial reference, by the follow equations

$$X_1 = \phi_i X_i^1 + \frac{h_0}{2}\eta\cos(\phi_i \theta_i^0) \tag{11}$$

$$Y_1 = \phi_i Y_i^1 + \frac{h_0}{2}\eta\cos(\phi_i \theta_i^0) \tag{12}$$

and, in the final reference, by the follow equations

$$X_2 = \phi_i X_i^2 + \frac{h_0}{2}\eta\cos(\phi_i \theta_i) \tag{13}$$

$$Y_2 = \phi_i Y_i^2 + \frac{h_0}{2}\eta\cos(\phi_i \theta_i) \tag{14}$$

where ϕ_i is the shape function, h_0 is the height of the beam, θ_i is the rotation. In the initial and final position will be the angle aproximation given by

$$\theta_j = \phi_i \theta_i^j \tag{15}$$

where i is the number of the shape function and j represents initial and final position. Again, more detail can be found in [Coda, 2003a, Coda, 2003b, Maciel, 2008].

3. VERIFICATION AND VALIDATIION

First of all, the main idea was verify if the element would be able to ensure the nonlinear behaviour which some riser could be submitted. Thus, the beam element was analyzed to assess their performance. The Fig. 3 shows one example which was applied a bending moment at the free end of the beam and clamped another end. The beam was fixed in the left extremity and the moment was imposed in the right extremity. This was a qualitative test where the beam had $1.5\ m$ lenght, inertia moment equal to $1.08x10^{-6}\ m^4$, cross sectional area equal to $3.06x10^{-4}\ m^2$ and the bending moment applied was $50\ KNm$. In this example, 19 load steps and a regular mesh with 5 elements where used.

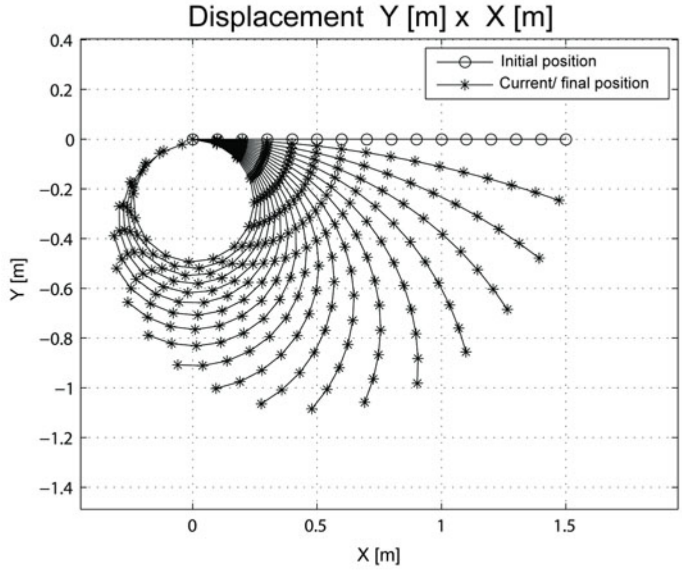

Figure 3. Beam subjected to a bending moment in the end.

The bending moment was imposed until the body had completed one loop. The Fig. 3 shows the ability of the element to solve this problem and demonstrated the capability to solve problems with large geometric nonlinear effects.

This element was evaluted with some works presented in literature. So, this is not the proposal of the work and more informations can be found in [Maciel, 2008, Morini, 2009].

4. RESULTS

This work propose a study the interaction between SCR and seabed, i.e, soil-structure interaction. The typical and general configuration for 2D-SCR with soil interaction is showed by the Fig. 4. This figure present a suspended catenary region of the riser and a region of the pipe interacting with seabed for the global analysis, the seabed is normaly represented by simplified winkler model [Pesce & Martins, 2004, Barros et al., 2009, Aubeny & Biscontin, 2009]

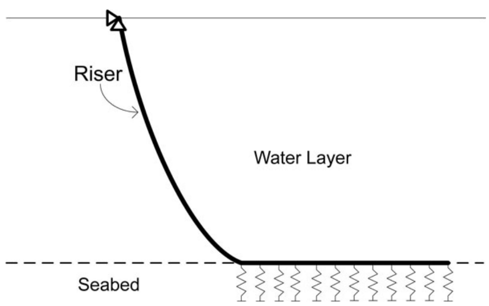

Figure 4. General model for 2D-SCR with soil interaction.

To study the contact region between risers and soil it is possible to adapted a simple model with a straight pipe. In this simplified model, was considered a local model that could be compared with analitic results. Thus, half of this pipe is supported by springs that represent the soil behavior and another half was considered free with the end simply supported. To ensure the weight force was considered a force uniformly distributed along the pipe. The Fig. 5 show the model assumed.

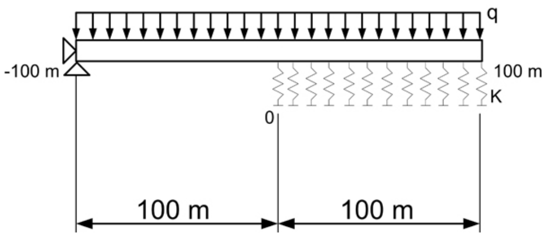

Figure 5. Model proposed for interaction riser-soil.

It was considered two soil models, linear and bilinear, where the first one once turned on it would not be turned off while the second could turn on or off depending on whether the pipe was penetrating the soil. Both cases were solved using Penalty Method [Bazaraa & Shetty, 1979, Serpa, 1996, Morini, 2009] which considered known stiffness of the soil. Figure 6 shows both soils, at the left is presented a linear soil where is possible to percieve that springs continue active after pipe detachment and at the right the bilinear soil is showed where springs do not continue active after pipe detachment.

The properties used to solve this problem are showed in the Tab. 1.

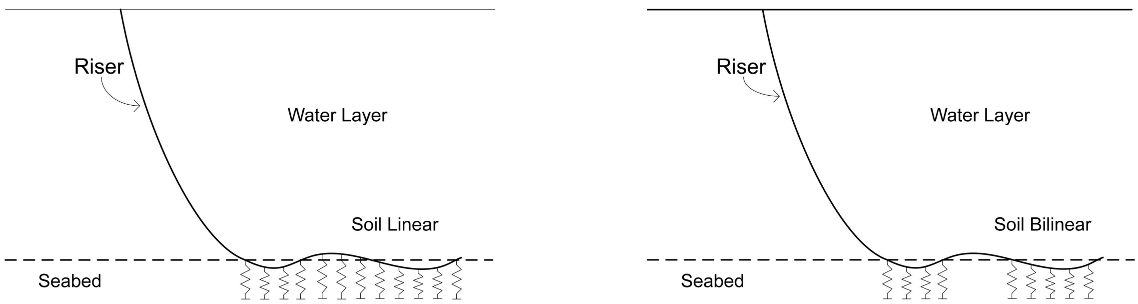

Figure 6. Soil Models - Linear(left) and Bilinear(right)

Table 1. Especific Risers and Soil properties

Risers Properties	Values
External Diameter (m)	0.22860
Internal Diameter (m)	0.20325
Flexural Modulus of Steel (GPa)	208
Density of Steel (Kg/m^3)	7850
Soil Properties	Values
Rigidity of the Soil N/m	2e7

4.1 Linear Conditions - lower loads conditions

Initially, in order to validate the problem of contact with the ground, the first simulated example considered small displacements and small rotations so that the results could be compared with results obtained by the finite element method for linear problems and analytical results. For both, was considered a lower load uniformly distributed load of $13.245\ Nm$ and 56 elements. Figure 7 and Fig.8 show the results for the displacement and bending moment, respectively. Three models are compared. The first model is a tridimensional continum finite element model to the soil with linear conditions in the contact region, i.e, the riser and soil nodes are showed (FEM). The second model is a winkler analitical solution [Barros et al., 2009]. The third model is the actual solution based in Positional Formulation (PF).

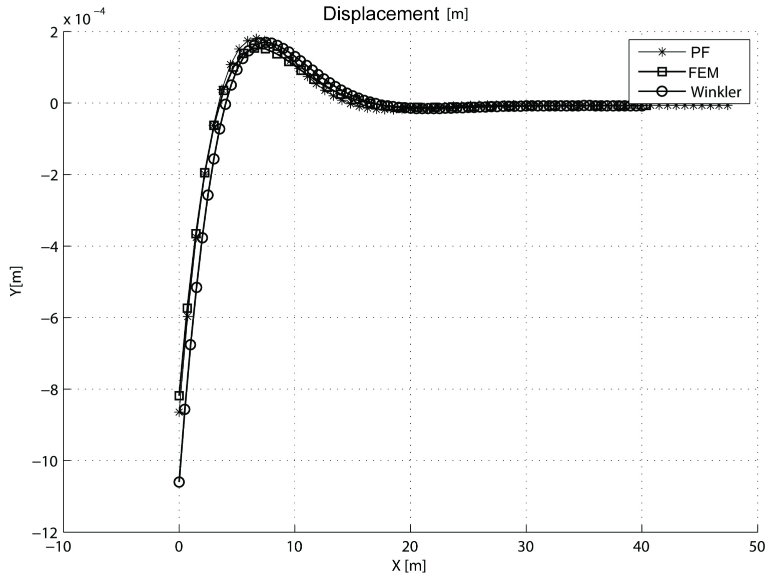

Figure 7. Interaction Riser-Soil: displacement - linear conditions

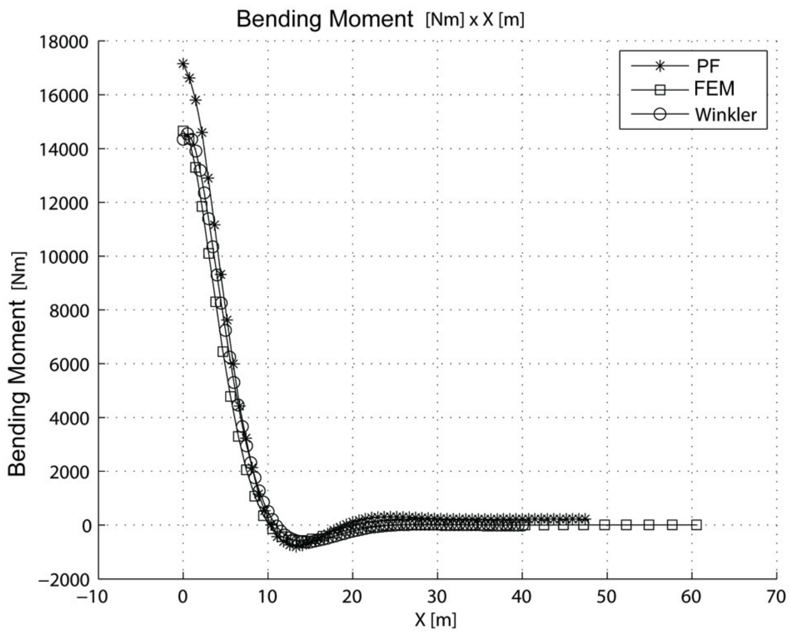

Figure 8. Interaction Riser-Soil: bending moment - linear conditions

Through the results for linear conditions can be observed that there is a detachment of the pipe in the region close to zero, i.e, the region in which the touch of riser with the ground. Also, we find a strong gradient of the bending moment in this region, due to sudden variation of conditions imposed on the riser.

It appears that the models show similar results, and thus, the positional formulation can be used to evaluate linear problems.

4.2 NonLinear Conditions - high loads conditions

In the next study developed was considered the same soil-structure model, but now with nonlinear conditions, i.e, the loading was changed allowing geometric nonlinear condition, thus, was considered a distributed load of $2649\ N/m$. The Fig. 9 and Fig.10 show displacement and bending moment obtained, respectively. In this case, we consider a linear behavior of the riser-soil coupling.

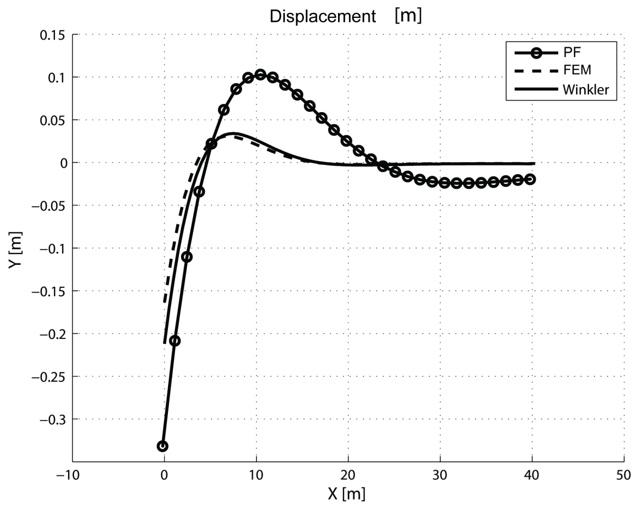

Figure 9. Displacement - nonlinear conditions, with linear riser-soil coupling.

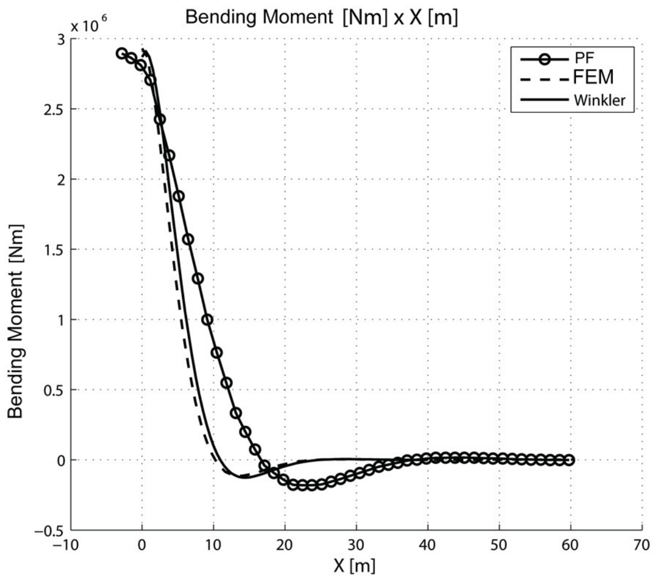

Figure 10. Bending Moment - nonlinear conditions, with linear riser-soil coupling.

In this case, it is possible to verify a larger displacement of the pipe in the touch down region and the bending moment becomes softer and moves to the right, this should occur because the gradient of stiffness that represents the presence of soil and because the big loading impose to the pipe.

4.3 NonLinear Conditions - Bilinear Soil

In the latter instance, the soil was considered bilinear, with nonlinear behavior too, and the same conditions of the previous example. The figures 11 and 12 show displacement and bending moment obtained for this case, respectively.

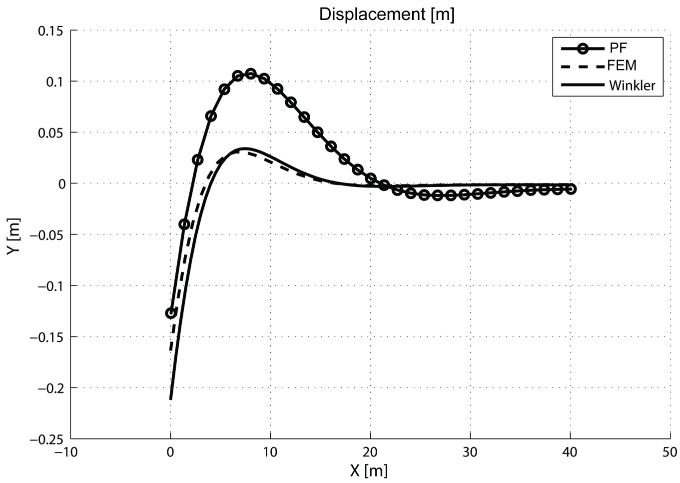

Figure 11. Displacement - nonlinear conditions and bilinear riser-soil coupling.

In this case, the displacement increased compared with the last example and the pipe sank approximately 1.5 times the diameter of.

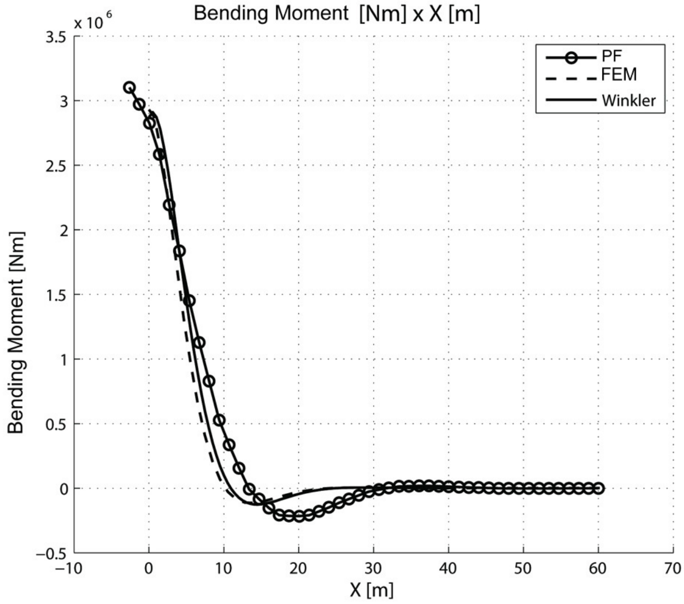

Figure 12. Bending Moment - nonlinear conditions and bilinear riser-soil coupling.

The magnitude of the bending moment does not change, but it was observed that both the detachment of the riser as the peak moving a little to the left as compared to the last example and that both become a little softer.

5. CONCLUSION

As mentioned in the introduction, the Positional Formulation provides a model relatively simple and intuitive, and can be implemented with relative facility.

Based on the results obtained in the validation tests, we can conclude that the Positional Formulation can be applied to the problems involving large geometric non-linearity effects. Furthermore, as expected, it solves linear problems as seen in the example linear.

The Positional Formulation was able to attend nonlinear problems, and has showed consistent results. The same conclusion can be taken for problems involving effects of soil nonlinear, where the last example showed to be consistent.

Thus, the Positional Formulation is promising for application in the modeling of risers. May become an important tool for analysis of risers.

Once completed this work, the next steps are: implement a 3D model, considering effects of Vortex Induced Vibrations (VIV), implementing new models of soil, among others.

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7. REFERENCES

Aubeny, C. P. & Biscontin, G. (2009). Seafloor-riser interaction model. *International Journal of Geomechanics - ASCE*.

Barros, P. L. A., Morooka, C. K., Neto, E. M., & Pavanello, R. (2009). Scr-seafloor interaction modeling with winkler, pasternak and kerr beam-on-elastic-foundation theories. *Proceedings of OMAE09 28th- International Conference on Offshore Mechanics and Arctic Engineering*.

Bathe, K. J. (1982). *Finite Elements Procedures in Engineering Analysis*. Englewood Cliffs - New Jersey: Prentice-Hall.

Bazaraa, M. S. & Shetty, C. M. (1979). *Nonlinear Programming - Theory and Algorithms*. John Wile & Sons.

Coda, H. B. (2003a). Análise não linear geométrica de sólidos e estruturas: Uma formulação posicional baseada no mef. volume ii.

Coda, H. B. (2003b). Notas de aula. Não Linearidade Geométrica.

Cook, R. D., Malkus, D. S., & Plesha, M. E. (2002). *Concepts and Applications of Finite Element Analysis*. New York: John Wile & Sons.

Maciel, D. N. (2008). *Análise de Problemas Elásticos Não Lineares Geométricos Empregando Método dos Elementos Finitos Posicional*. PhD thesis, Escola de Engenharia de São Carlos.

Morini, R. G. (2009). Método dos elementos finitos posicional aplicado à análise estática de *Risers*. Master's thesis, Universidade Estadual de Campinas - Campinas.

Pesce, C. P. & Martins, C. A. (2004). Riser-soil interaction: Local dynamics at tdp and a discussion on the eigenvalue and the viv problems. *ASME 23rd Inernational Conference on Offshore Mechanics and Arctic Engineering*, (pp. ASME Paper No. OMAE2004–51268).

Serpa, A. L. (1996). Problema de contato com atrito utilizando o método do lagrangiano aumentado. Master's thesis, Universidade Estadual de Campinas - Campinas.