MODELING AND CINEMATIC ANALYSIS OF TOROIDAL CVTs: INFLUENCE OF THE GEOMETRIC PARAMETERS IN PERFORMANCE.

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Abstract. *This study aims to evaluate that impact of variations in the geometric parameters has on the ability of torque and power transmission on Continuously Variable Transmissions (CVTs). Changes in the numerical values of these parameters allows a verification of their influence on the entire system life-time for any type of cycle, as defined in the theory of life provided the causes of failure, fatigue and subsurface damage. The geometric modeling of toroidal type transmissions (Half-Toroidal to Full-Toroidal) also allows an evaluation of the system transmission ratio maximum range, considering restrictions due limitation to spin and slip effects. Then, the geometric model of the traction based toroidal CVT was further explored by adding new geometric parameters related to the system control mechanism (the roller tilting angle). The influence of those parameters variations was detected and discussed in this work. Additionally, an Elasto-Hydrodynamic (EHD) contact theory is applied by using a numerical method in an example system, to evaluate the capability of transmission fluid in transmitting torque through the system contacts.*

Keywords*: Continuously Variable Transmission, Toroidal CVT, Parametric optimization, transmission life-time*

1. INTRODUCTION

Between all the known types of power transmittion systems, the Continuously Variable one (CVTs) have the capability of maximizing the efficiency of Internal Combustion Engines, allowing them to work close to their optimal operation condition, regulating the load conditions, adjusting them for the levels of torque and speed of the engine. This is possible once that CVTs can change steplessly through an infinite number of effective Transmission Ratio (*TR*) between maximum and minimum values by changing its operation geometry. Some kinds of CVT are also being used to unite different sources of power, like eletrical and combustion engines, allowing them to work together and with synergy into hybrid vehicles. One of the latest applications of CVTs can be observed in Formula One vehicles, with mechanical KERS (Kinetic Energy Recovery Systems, as observed in Fig.1), regulating the charging and discharging of a flywheel stored power into the car powertrain system (SAE, 2008).

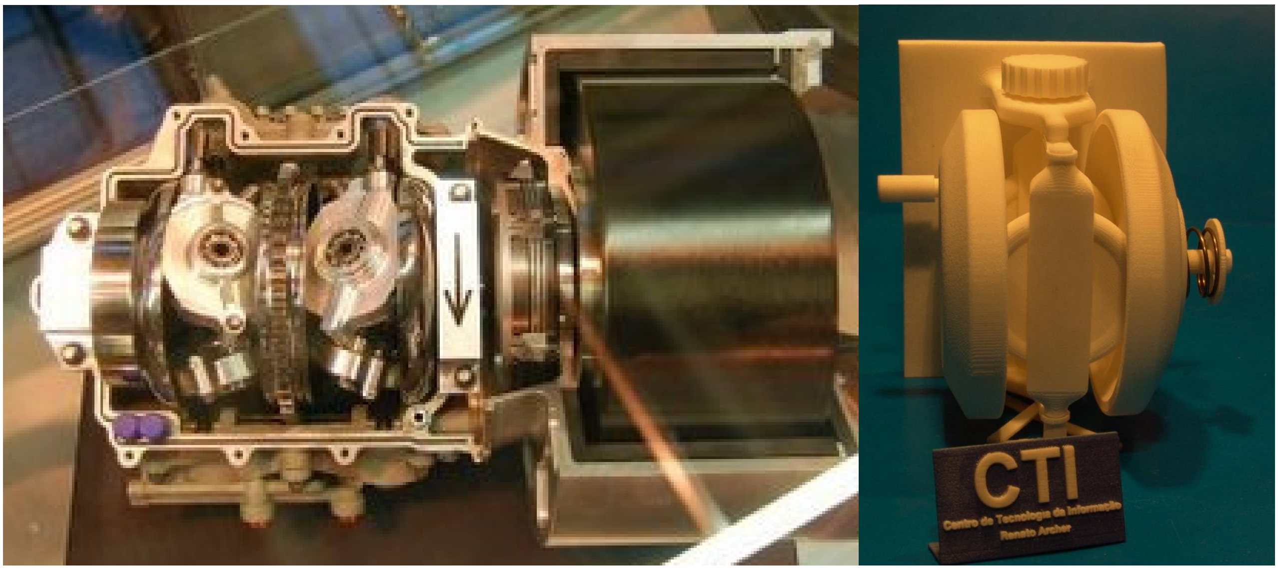

Figure 1. Application of a Full Toroidal CVT transmission in a KERS system (Chris Hayes, 2009) on the left, and a didactic prototype made with a rapid prototipation process in CTI, on the right.

There are two main groups of CVTs: Traction based, and Friction based (belt-driven). Traction based CVTs work by changing the position of contact point in its structure. Among other kinds of CVTs, Y. Zhang considered Traction based CVTs as the most promising one, since it allows a higher transmission of torque with simplier mechanisms (Y. Zhang, 2000).

The torque transmission capacity is directly linked to the contact regions conditions. To avoid losses, it is necessary for the interface region dimension to be negligible in relation to the system size. Therefore, for a reasonable amount of torque, the contact must operate under huge pressure loads. Under those pressures the lubrication fluid changes its properties, solidifying itself (Nabil A. Attia, 2005). Is very important for the fluid, on in this environment, to have a high traction constant. In other words, it should be able to transmit a high tangential strength in relation to the normal strength applied. But it also must be able to lubricate and, with that, preserve the set. On these conditions, the Elasto-Hydrodynamic (EHD) theory is necessary to predict and simulate the contact properties. But, usually, the classical Hertzian contact theory is applied to the Toroidal drive contact analysis (Y. Zhang, 2000).

The position of the contact points in the mechanism determines the system *TR*. The overall system size, compared to the amount of power transmitted, has direct impact on the system lifetime and the mechanism final cost. The design of the control system is linked to the geometry variation mechanism. The contact loading mechanism is often created with cam elements on the power input. Additionally, some of the system losses (like the spin effect), related to the contact efficiency on the torque transmission capacity, are generated by geometrical factors. Therefore, the system geometry and the system geometry variation are key technical issues in traction drives design.

This work is focused on geometric analysis for performance optimization of Toroidal CVTs (Half-Toroidal and Full-Toroidal), as the one shown in Fig.2. The KERS mechanism shown on Fig.1 has a Full Toroidal CVT.

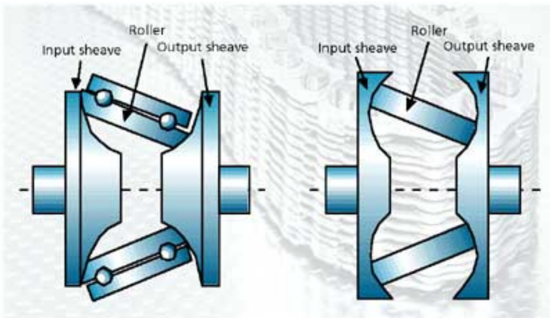

Figure 2. The two kinds of Toroidal CVT: the Half-Toroidal (on left) and the Full-Toroidal (on right) (SAE International, 2009).

A toroidal transmission works by changing the roller inclination angle, called tilt-angle (θ, see Fig. 3). The roller must always be in contact with two discs (system input and output). The distances between the contact points and main axe of the toroidal geometry (y_e and y_s) define the theoretical *TR*. In these transmissions, the distance between a contact point and the center of tilting (*O*) must be constant, so the roller can be in contact with the toroids for any tilt-angle configuration. There is an axial gap in the system, but its purpose is to allow the application of force that is responsible for the final pressure at the contact points, and therefore, it must not change the trajectory of the contact points. A simple way of applying this axial force is through a cam mechanism (Y. Zhang, 2000 & Nabil A. Attia, 2005).

2. METHODOLOGY

As commented in Sec.1, the basic geometry of a Toroidal CVT has a huge impact on the system performance. Most of the parameters that define the geometric model of a HT-CVT, can be seen on Fig.3. The roller contact radius is omitted in this image. FT-CVT will be defined by making specific restrictions in the contact possible positions and *h* (see Fig.3) equal to zero.

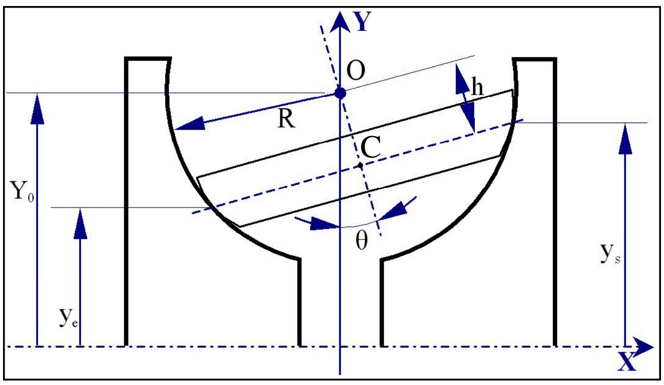

Figure 3. Basic Geometric Model of Half Toroidal CVT.

This geometric model is representing a 2D mechanism, and the equations that give the contact positions as a function of the tilt angle θ can be easily found. But on the following modeling, two new parameters were simultaneously added to the basic geometric model. Those parameters can change the distance between the contact points without changing the roller main radius, so the angle formed by the contact points and the toroid circumference center is not constant. Additionally, the representation of the roller center is important in the following formulation. The new parameters added to the geometric model change the inclination of the plane that contain the roller center (C) and the contact points. This property permits changes in the tangent velocity vector of the roller at the contact point. This is directly related to the contact interface conditions, and therefore, to the torque transmission capacity and the system lifetime. They, also, do not change the distance between the center O and each contact point, and so, not interfering in the axial gap. And since those distances are not altered, and the discs axis are aligned, the surfaces that represent the possible contact interfaces also do not change. A geometric model was created to show those surfaces (see Fig.4).

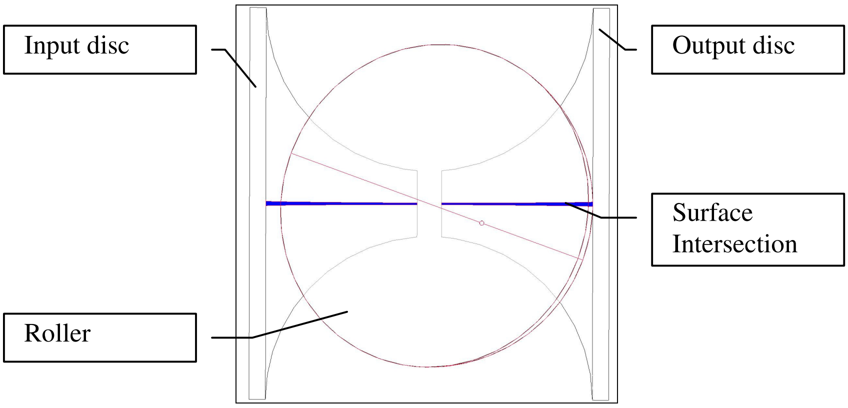

Figure 4. Surface revealing the contacts trajetory.

The new parameters chosen are angles that alters the roller center final position: α and β. α is a parameter designed to change the tilt plane (the plane that contain C and O), as seen on Fig.5. Traditionally, the tilting plane is always aligned with the discs rotation axis (in the coordinate reference chosen, is the X axe, see Fig.3). The angle α is a rotation of the tilting plane around the Y axe.

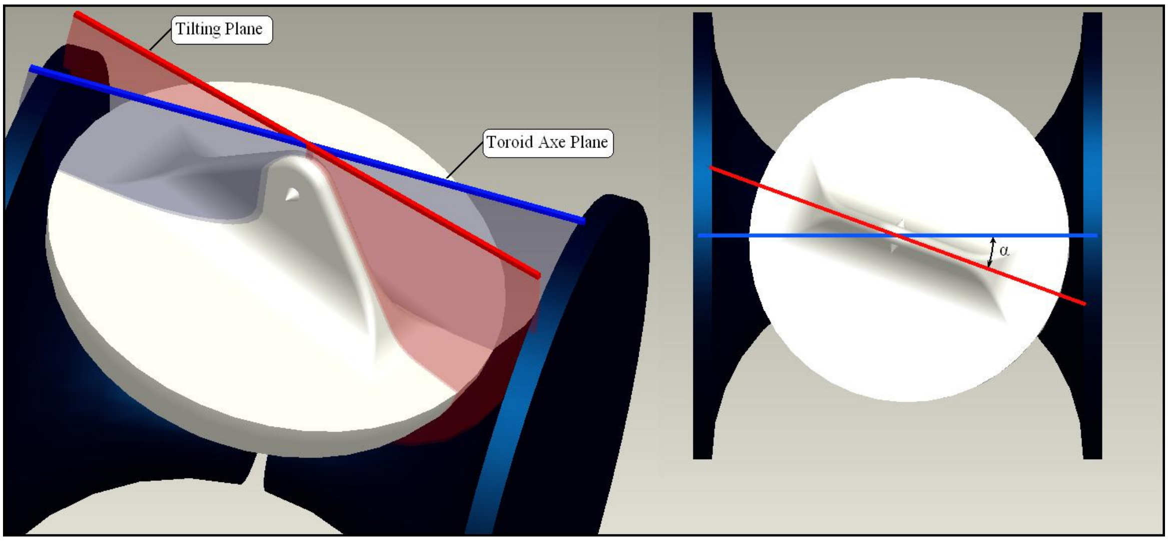

Figure 5. Effect of parameter α on the tilting plane.

The parameter β does not interfere with on tilting plane, but modifies the roller center position. That way, the tilting plane does not contain the roller center. Figure 6 shows the geometry modified by β.

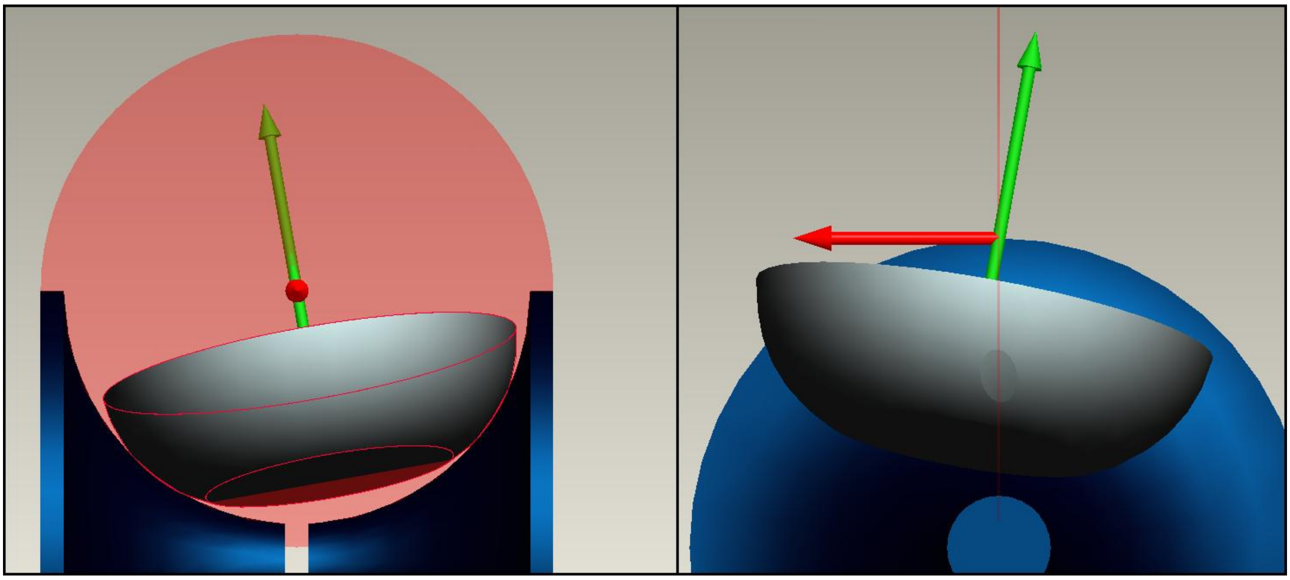

Figure 6. Effect of parameter α on the tilting plane β.

At this point, an equation that describes y_e and y_s as functions of the design parameters (R, h, Y_0), the tilting angle (θ) and the new parameters (α and β) is necessary. A simple procedure for that is by arranging the equations for the contact points possible locations (at the discs, Eq. (1)) with the equation that defines the plane that contain the roller center, the contact points and is normal to the vector that links C with O (vector $\vec{n}$), written as Eq. (2).

$$\vec{P}_C = \begin{bmatrix} \pm\sqrt{R^2-(y-Y_0)^2} \\ y \\ 0 \end{bmatrix} \tag{1}$$

$$\left(\vec{P}_{xyz}-\vec{C}\right)^T \cdot \vec{n} = 0 \tag{2}$$

Equation (2) can be expanded as Eq. (3), and the Eq. (4) gives the intersection:

$$\left(\vec{P}_{xyz}-\begin{bmatrix}0\\Y_0\\0\end{bmatrix}\right)^T \cdot \begin{bmatrix}\cos(\alpha) & 0 & \mathrm{sen}(\alpha)\\ 0 & 1 & 0\\ -\mathrm{sen}(\alpha) & 0 & \cos(\alpha)\end{bmatrix} \cdot \begin{bmatrix}\cos(\theta) & -\mathrm{sen}(\theta) & 0\\ \mathrm{sen}(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\beta) & -\mathrm{sen}(\beta)\\ 0 & \mathrm{sen}(\beta) & \cos(\beta)\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\end{bmatrix} + h = 0 \tag{3}$$

$$\vec{P}_{xyz} = \vec{P}_C \tag{4}$$

By ysolating y from these equations generates equations for the position of the input contact point height (y_e), seen in Eq. (5), and for the output contact point height (y_s), seen in the Eq. (6).

$$y_e = Y_0 - \frac{h\cdot\cos(\beta)\cdot\cos(\theta) + \left(\cos(\alpha)\cdot\sin(\theta)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta)\right)\cdot\sqrt{G\cdot R^2 - h^2}}{G} \tag{5}$$

$$y_s = Y_0 - \frac{h\cdot\cos(\beta)\cdot\cos(\theta) - \left(\cos(\alpha)\cdot\sin(\theta)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta)\right)\cdot\sqrt{G\cdot R^2 - h^2}}{G} \tag{6}$$

Where G is given by Eq. (7):

$$G = 1 - \cos^2(\alpha)\cdot\sin^2(\beta) - \sin^2(\alpha)\cdot\cos^2(\beta)\cdot\sin^2(\theta) - \frac{\sin(2\cdot\alpha)\cdot\sin(2\cdot\beta)\cdot\sin(\theta)}{2} \tag{7}$$

The horizontal coordinates for the equation points can be found by using Eq. (1), or Eq. (8), where the straight line that contain the contact points is expressed.

$$y = x\cdot\frac{(\cos(\alpha)\cdot\cos(\beta)\cdot\sin(\theta) - \sin(\alpha)\cdot\sin(\beta))}{\cos(\beta)\cdot\cos(\theta)} - \frac{h}{\cos(\beta)\cdot\cos(\theta)} + Y_0 \tag{8}$$

For the initial analysis, the parameters in Eq. (5) and Eq. (6) can be made dimensionless, using Eq. (9), Eq. (10) and Eq. (11).

$$y'_{e,s} = {}^{y_{e,s}}\!/_{R} \tag{9}$$

$$h' = {}^{h}\!/_{R} \tag{10}$$

$$Y'_0 = {}^{Y_0}\!/_{R} \tag{11}$$

The influence of those parameters at the *TR* was used for a first comparison. In theory, with no losses, no sliping, no creeping and punctual contacts, we can assume that the tangential velocity of the roller and discs at the contact points should be the same. And, therefore:

$$TR = \frac{\omega_e}{\omega_s} = \frac{y_s}{y_e} = \frac{Y'_0\cdot G - h'\cdot\cos(\beta)\cdot\cos(\theta) + (\cos(\alpha)\cdot\sin(\theta)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta))\cdot\sqrt{G - h'^2}}{Y'_0\cdot G - h'\cdot\cos(\beta)\cdot\cos(\theta) - (\cos(\alpha)\cdot\sin(\theta)\cdot\cos(\beta) - \sin(\alpha)\cdot\sin(\beta))\cdot\sqrt{G - h'^2}} \tag{12}$$

The difference between FT-CVT and HT-CVT is basically the range of the toroidal geometry adopted by the mechanism. Normally, a HT-CVT uses the lower side from the toroidal only. This makes different types of roller suspension and control possible, as well inserting the parameter *h* with fewer complications.

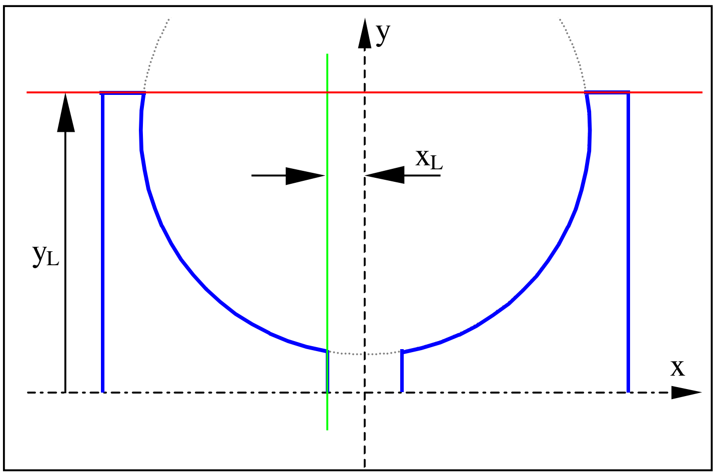

Figure 7. The discs limits defined by horizontal and vertical restrictions.

A FT-CVT is normally limited only by the horizontal restrictions. This enables a *TR* range larger than the one found usually in HT-CVT, but the horizontal limits must be larger now to permit the access of the roller control system inside the toroidal geometry. The current work will also contain in further steps the analysis of the impact of these restrictions (vertical and horizontal, restricting the toroidal geometry used) in the system efficiency. But it is considered that the roller should work close to the 1:1 *TR* point, because that reduces the losses by spin (A. W. Forti, 2002). The Fig. 7 is an example of the horizontal and vertical limitations that were introduced to the System.

3. RESULTS

Numerical data were taken from Zhang's (2000) work and used in the following example. The initial values for the system parameters and the range of analysis are displayed in Table 1. At first, X'_L is fixed and Y'_L is a function of Y_0'.

Table 1. Geometric parameters for dimensionless analysis.

Parameter	Initial Value	Analysis Range
θ (rad)	0,0000	±1,5708
h' ( )	0,4617	±0,4000
Y_0' ( )	1,5876	±0,3000
α (rad)	0,0000	±0,7854
β (rad)	0,0000	±0,7854
X_L/R ( )	0,3000	-
Y_L/R ( )	$1{,}1213 * Y_0'$	-

Figure 8 and Fig. 9 shows the behavior of the *TR* function when changing the tilting angle (θ) and some other design parameter. Analyzing Fig.8, one can note some tendency on the surfaces behavior. As expected, the smaller that Y_0' is, the larger the maximum *TR* possible is. There is a point of maximum on the surface that express *TR* as a function of θ and *h'*.

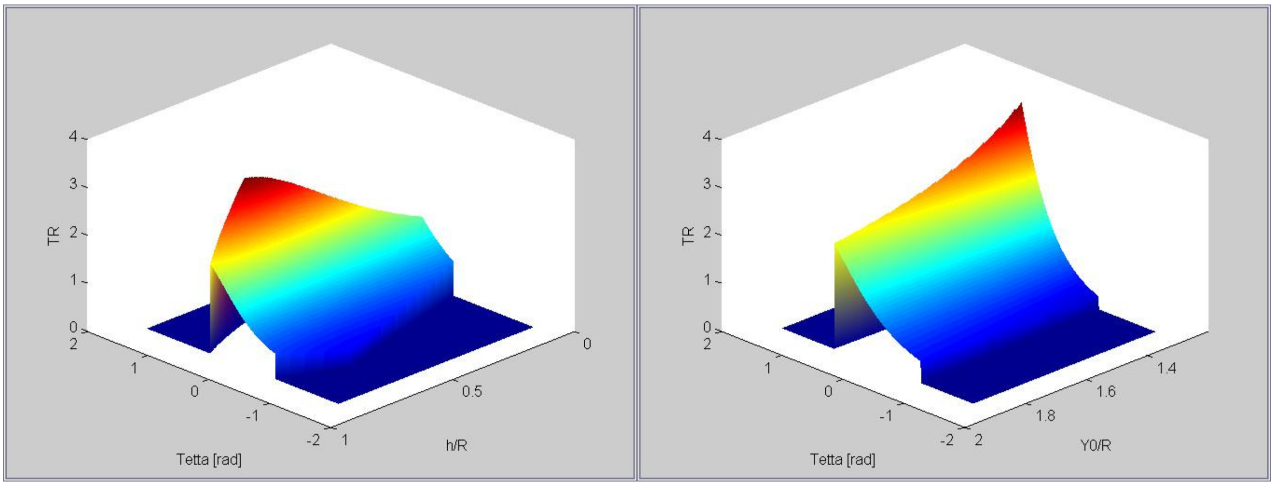

Figure 8. Effect of *h'*, on the left, and the effect of Y_0', on the right, in *TR* function.

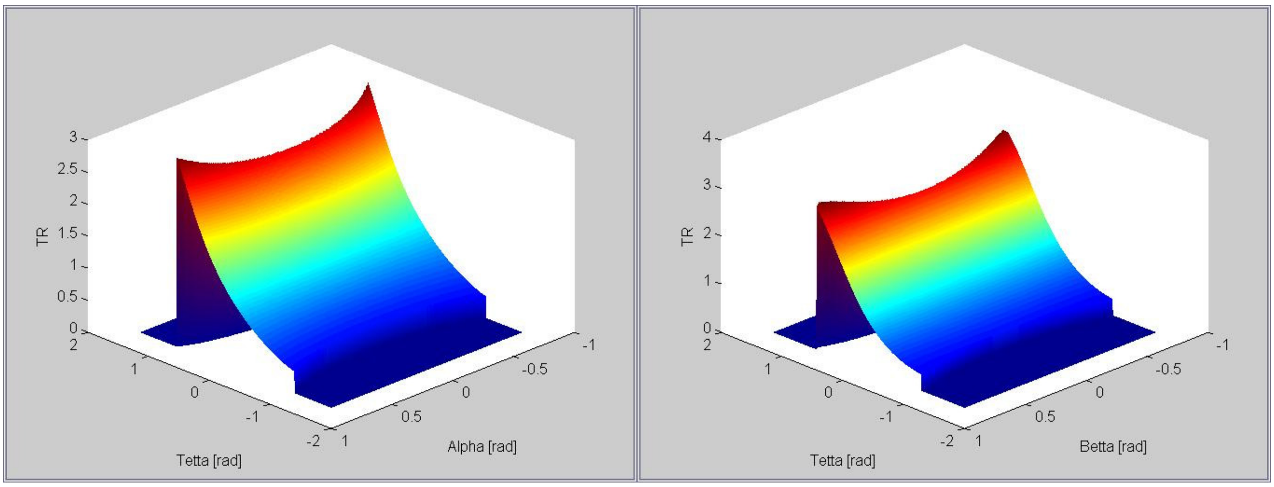

Figure 9. Effect of α, on the left, and the effect of β, on the right, in *TR* function.

The effect of the new geometric parameters, α and β, are shown in Fig.9. Small variations on those parameters have little impact on the final *TR* function, as observed. However, since those parameters have direct influence on the contact conditions (the tangential velocity vector of the surfaces, for example), they must also have an impact on the system efficiency in transmitting torque and the effective transmission ratio. Therefore, a more sophisticated analysis is required.

For a further analysis of the dimensionless parameter h', an algorithm was developed to find only the optimal point in *TR* as function of θ and h', as the one seen in Fig. 8, inside a range of values of Y_0' and Y_L'. The numerical values used in this analysis are given in Tab. 2.

Table 2. Geometric parameters for h' optimization.

Parameter	Initial Value	Analysis Range
θ (rad)	0,0000	±1,5708
h' ( )	0,4800	±0,4800
Y_0' ( )	1,600	±0,4000
Y_L/R ( )	1,800	±0,4000
α (rad)	0,0000	-
β (rad)	0,0000	-
X_L/R ( )	0,3000	-

The graphical results from h' optimization are given in Fig. 10. Not considering X_L, is possible to say that when Equation (13) is satisfied, the system becomes a Full Toroidal CVT. So, as expected, the optimal h' value for FT-CVT is 0.

$$Y_L > Y_0 + R \tag{13}$$

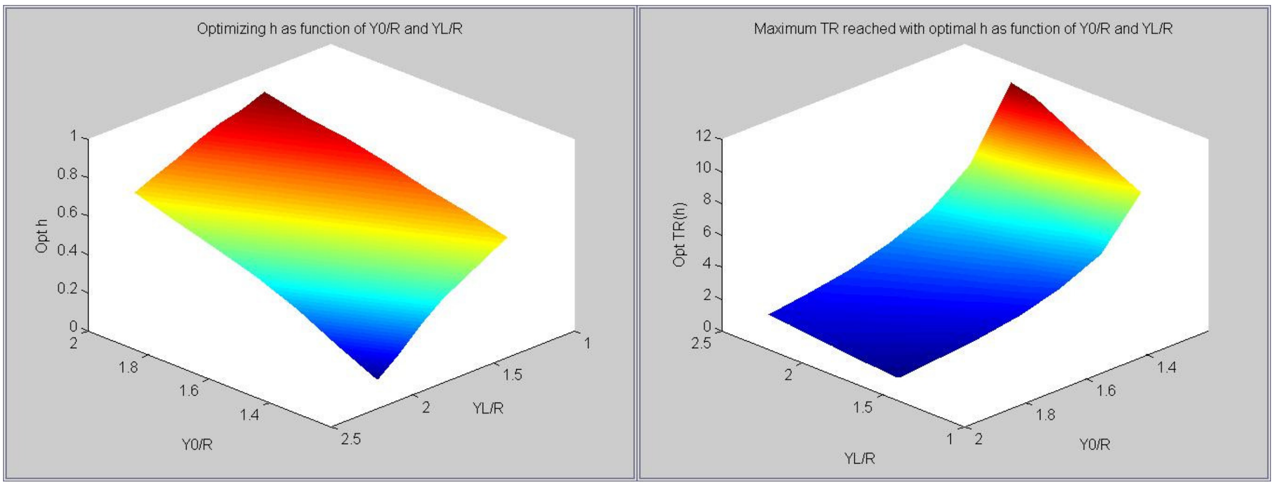

Figure 10. The results of h' optimization. On the left, the surface giving the optimal h' as function of Y_0' and Y_L'. On the right, the maximum *TR* allowed using the optimal h'.

4. CONCLUSION

The maximum and minimum *TR* reached in the transmission has a direct impact on its application, provided that a wider range of *TR* allows the CVT to work in broader range of circumstances and requirements. And therefore, optimizing the transmission *TR* function is one of the fundamental aspects of its design.

There is a need of a contact theory able to evaluate the efficiency of the mechanical energy transmission through the system contacts. These contacts operate under Elasto-Hydrodynamic lubrication conditions, and therefore, the flow of lubrication fluid, system temperature (and its impact over the material from the discs and the roller and over the properties of the fluid) and the lubrication fluid properties should also be considered. Effects that generate losses on the power transmission, like slip, side-slip, creep, and spin, must also be modeled. All these considerations are needed to estimate as effective *TR* function, the system torque transmission capacity, and a more reliable system lifetime expectancy.

A more accurate contact model will also allow a more consistent the estimation for the horizontal and vertical limits, that will give a better balance between the *TR* range and the losses in the contact.

The new parameters were added with the purpose of modeling possible deviations in the design of Toroidal CVTs. In any mechanical design, tolerances must be added to create a balance between set cost and system performance. So, one of the initial purposes of this work was to model possible geometric “flaws” and evaluate their impact on the system life-time and efficiency. But a theorical model that can fully evaluate those impacts has not been found yet.

If the further steps of this study show that the parameters studied have little impact, or positive impact, on the system performance, they can be used as real project parameters. The angle α, as an example, was shown to be able to enlarge the range of the *TR* in a system with horizontal and vertical restrictions. Probably, there are optimal values to α and β that give a better balance between system efficiency, lifetime and *TR* range.

5. ACKNOWLEDGEMENTS

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