

DESIGN OF STATE OBSERVERS USING LINEAR MATRIX INEQUALITIES (LMIS) APPROACH FOR FAULT DETECTION IN ROTATION SYSTEMS

Edson Hideki Koroishi, edson.koroishi@gmail.com[1]
Gilberto Pechoto de Melo, gilberto@dem.feis.unesp.br[2]
Valder Steffen Jr, vsteffen@mecanica.ufu.br[1]

[1] Federal University of Uberlândia - School of Mechanical Engineering - Campus Santa Mônica, Av. João Naves de Ávila 2121, Bloco 1M, Uberlândia, MG, Brazil, ZIP CODE 38400-902, www.femec.ufu.br
[2] GMSINT – Grupo de Materiais e Sistemas Inteligentes, Sao Paulo State University (UNESP) - Department of Mechanical Engineering -Faculdade de Engenharia de Ilha Solteira - FEIS, Av. Brasil 56, Ilha Solteira, SP, Brazil, ZIP CODE 15385-000, www.dem.feis.unesp.br/gmsint

Abstract. *Rotating systems have many applications in wide-ranging industrial contexts. The breakdown of this equipment results in economic loss. To avoid such problems is very important, and it can be done through tools that informs about the existence of faults, as well as, about their progress in time. A review of the modeling process used for rotor-support-structure shows that the finite element method is the major method employed. In this paper, with the aid of well defined theoretical models, obtained using the finite element technique, and the state observer method for the identification and location of faults it is possible to monitor the parameters of a rotor-support-structure system, including the foundation effects. In order to improve safety, these parameters must be supervised that the occurrence of failures or faults can. The state observers were designed using LMIs.*

Keywords*: State Observers, Rotation System, Faults, LMIs*

1. INTRODUCTION

Nowadays one of the most dominant concerns of the industry is to maintain its systems operating without sudden stops. Due to this constant concern new techniques of fault detection and location in mechanical systems submitted to dynamic loads have been developed.

Since the introduction of the state observer's theory by Luenberger (1964), many methodologies have been proposed for condition monitoring of the machines using the state observers' technique (Ge et al, 1987; Elmas et al, 1996). Even so, most of the methodologies using state observers are intended for solving control problems and detecting possible faults in sensors and instruments (Clark, 1978; Watanabe et al, 1982). Moreover, many works are theoretical, without any experimental verification of the developed methodologies (Park et al, 2001; Trinh et al, 1998; Frank et al, 1991).

The main focus of the literature revision has been the detection and location of mechanical system faults. Theories which are related to state observers, fault detections and state observers using LMIs have been taken into account. State of the art theories are presented in chronological order and the most significant works are selected.

Luenberger (1964). Luenberger states that the major part of the theory of modern control is based on the assumption that the state vector of the system to be controlled is available for direct measurement. However, in many practical situations, only few output databases is available. The author shows how the inputs and outputs that are available can be used to build an estimate observer, or just observer. This work states the state observer theory.

Luenberger (1966). He proposed that for a linear system, its state vector can be approximately reconstructed by means of a designed observer. The "n" order state vector with "m" independent outputs can be reconstructed, rebuilding the remaining states from differential conditions. He also proved that the design of an observer with "m" outputs can be reduced to a design "m" observer as if they were simple output subsystems simplifying the observer complexity.

Luenberger (1971). He used a methodology for reconstructing states using state observers and explained topics about identity and reduced order observers.

Clark (1978). He introduced the concept of robust observer, with designing state estimation filters for instruments default detection, robust enough to withstand the uncertainties. The base for the filters was the separation of the effects of faults from the uncertainties.

Watanabe and Himmelbleau (1982). The authors presented a method to detect instruments faults in nonlinear time dependent processes, including uncertainties such as modeling mistakes, parameters ambiguity and input and output noise. The main goal of their work has been the development of state estimate filters with minimum sensitivity to uncertainties and maximum sensitivity to instruments faults, as those corresponding to slight deterioration or gradual chances, instead or sudden or catastrophic faults.

Ge, W. and Fang, C. Z. (1987). The authors described a novel conception for the detection of components under failure by robust observation, considering a mathematical model corresponding to "m" components coupled by non-estimated states. They determined the design of devices to monitor the operation of those "n" components and faults detection. In the case of an observable system, some first or superior order components can be monitored for the purpose of diagnosis without information of possible faults modes. Due to the observer robustness, the authors analyzed some reactions such as linearization and measurement errors, noise presence, numerical errors, and so on.

Wauer (1990). It shapes shaft of a rotational system using beam elements for which the properties of stiffness and damping of one crack is considered previously with great precision.

Cheli et al (1992). They esteem the modal parameters of the foundation of a rotational machine using two mathematical approaches. These methods are an alternative for the identification of the modal parameters, since the transfer function of the foundation is almost always unavailable.

Stephenson et al (1992). The authors have established a procedure to include the effect of the foundation using the technique of the modal analysis and measures of the frequency response function of the structure. The matrices of the foundation finally are added to the model of finite elements of the beam of the rotor, getting finally the complete system.

Cavalca et al (1993). The authors have accomplished a numerical simulation of the dynamic behavior of a centrifugal bomb using the modal truncation method. They show an approach for the evaluation of the foundation in the global dynamic response of the system.

Choy et al. (1995). The authors showed a methodology based in the vibration theory which can be used for detection of faults in systems shaped by finite elements using beam elements supported by elastic foundation.

Cavalcante et al (1997). The authors have showed the methodology of mixed coordinates for the analysis of the behavior of rotor-support-foundation system. The complete system is shaped using the methodology finite elements.

Faitakis et al (1998). The authors proposed a new approach for selecting alarm thresholds in a simple fault detection system. Bounds were computed on the magnitudes of the minimum detectable fault and the maximum non-detectable fault. The 2-norm's use for this calculation results in a LMI problem. An example was presented and a filter design was proposed that enhances the ability to distinguish between a fault and a disturbance.

Mohiuddin et al (1999). They demonstrated formularization by finite elements of the dynamic model of rotor-bearings, considering the gyroscopic effect and the combination between the deformations of torsion and flexing. They established a modal transformation using complex models and then, obtained a model of reduced order, which was validated numerically.

Valer (1999). He established control systems design using state observers. Also, he accomplished a revision of the principal types of observers. In addition, he used the modern techniques of robust control to improve the robustness properties of the control system.

Bara et al (2001). They investigated the design of a parameter-dependent state observer that allows estimating the state of an affine linear parameter-varying (LPV) system. The observer has the property to be parameter-dependent since the corresponding state space matrices are scheduled using an interpolation method. Moreover, the stability of the estimation error is based on the existence of an affine parameter-dependent Lyapunov function. The main contribution of this paper is that the problem of the observer design and the existence of such a Lyapunov function are interpreted as a flexible LMI feasibility condition.

Jiang et al (2004). This work focused on the problem of robust stabilization for a class of linear systems with uncertain parameters and time varying delays in states, using LMIs to determine the gain of the state observers and controllers

Lemos et al (2004). He presented the state observer methodology for detection and location of faults in rotary systems, taking into account their foundations. According to him, the state observer methodology is able to reconstruct non measured states or estimate values coming from difficult access locations in the system. In fact, those faults can be detected without the need for a direct measurement.

Morais et al (2005). They used the Kalman filter as a stochastic state observer for the detection of faults in mechanical systems in the presence of aleatoric noises and non linear inputs.

Yaz et al (2006). They addressed the important problem of stochastic resilience of a discrete-time Luenberger observer, which is the maintenance of convergence and/or performance when the observer is erroneously implemented. A common LMI framework was presented to address the stochastic resilient design problem for various performance criteria in the implementation based on the knowledge of an upper bound on the variance of the random error in the observer gain.

Jiang et al (2006). The authors proposed a new approach for synchronization of complex dynamical networks based on state observer design. Some conditions for synchronization, in the form of an inequality, are established based on the Lyapunov stability theory

Fernandes et al (2007). They used state observers for diagnosis of faults in mechanical systems with dynamic vibration absorbers (DVAs). In these works, the authors use ADVs as one of the type vibrations blades and the technique of state observers for diagnosis of fault in mechanical systems.

Abbaszadeh et al (2008). They proposed a new approach for the design of robust H_∞ observers for a class of Lipchitz nonlinear systems with time-varying uncertainties based on LMIs.

Park et al (2008). They studied the design problem of state estimator for a class of discrete-time neural networks. A delay-independent LMI criterion for the existence of the estimator is derived by using the Lyapunov method.

The state observers' technique can reconstruct the non-measured states or can estimate the values of difficult access points in the system. Thus, the faults at these points can be detected without the knowledge of measured data, hence, monitoring them through the reconstructions of their states (Luenberger, 1964). This technique consists of developing a model for the system to be analyzed and to compare the output at the observer with the output of the system.

In order to supervise the process, a set of observers is mounted where each observer is dedicated only to an instrument or physical parameter of this system. For fault detection in the system, an observer of global state is projected first. The global observer has the role of verifying if the system is working properly without any indications of faults, because in this observer's assembly the same system matrix of the mechanical system is used in the analysis. Thus, the global observer can detect a possible fault or irregularity in the system if the system's response is not coincident with the global observer's response.

In detecting a possible fault, the next step would be to locate such fault, which is the reason why robust observers are used. Thus, a bank of observers is set up, where each observer is dedicated to a physical parameter of the system.

In practice, the mathematical models, representing the behavior of the systems, are not free of unknown disturbances and of variations in their own parameters. In most state observers' projects, the parameters of the system are known or can be identified through some specific methods found in the literature. In cases where the parameters are not accurately known or where they are subject to changes during the operation of the system, the observer's response can supply an incorrect estimate of the reconstructed states, therefore inducing certain permanent mistakes that assume the false alarms in the faults detection and location.

Therefore, this paper proposes the development of a methodology for faults detection and location in mechanical systems with relation to the structure of the mechanical system, i.e., with relation to the variation of the system parameters, as stiffness and damping and a rotor system using the state observers' technique in order to avoid false alarms and unnecessary stops of mechanical systems.

2. LINEAR MATRIX INEQUALITIES (LMIS)

The history of LMIs in the analysis of dynamical systems goes back more than 100 years. In 1890, when Aleksandr Mikhailovich Lyapunov presented his work, introducing the Lyapunov Theory (Boyd et al, 1994). He showed that the differential equation:

$$\dot{x}(t) = Ax(t) \tag{1}$$

is stable (all the trajectories converge to zero), if and only if there is a positive-definite matrix P such that:

$$A^T P + PA > 0 \tag{2}$$

The inequation given by Eq. (2) is known as the Lyapunov inequality.

Currently, LMIs have been the object of study by many important researchers around of the world: control of continuous and discrete systems in time (Ghaoui et al, 2000), optimal control and robust control (VanAntwerp et al, 2000; Silva et al, 2004), model reductions (Assunção, 2000), control of nonlinear systems, theory of robust filters (Palhares, 1998), systems identification, control with variable structures (Teixeira et al, 2000), control using Fuzzy model (Teixeira et al, 2000), detection, location and quantification of faults (Abdalla et al, 1999; Abdalla et al, 2000; Wang et al, 2007).

3. STATE OBSERVERS USING LMIS

A state observer is defined by:

$$\left\{\dot{\tilde{x}}(t)\right\} = [A]\left\{\tilde{x}(t)\right\} + [B]\left\{u(t)\right\} + [L]\left(\left\{y(t)\right\} - \left\{\tilde{y}(t)\right\}\right) \tag{3a}$$

$$\left\{\tilde{y}(t)\right\} = [C_{me}]\left\{\tilde{x}(t)\right\} \tag{3b}$$

Where:

$[A] \in R^{nxn}$ is the dynamical matrix;

$[B] \in R^{nxp}$ is the input matrix;

$[C_{me}] \in R^{kxn}$ is the measure matrix;

n is the order of the system, p the number of inputs $\{u(t)\}$, k the number of outputs $\{y(t)\}$.

$[L]$ is the observer matrix;

$\{\overleftarrow{y}(t)\}$ is the output of the observer;
$\{\overleftarrow{x}(t)\}$ is the state vector of the observer.

In this case, the study of stability of the state observer is attained by using the following LMIs:

$$\begin{aligned} &P(A - LC_{me}) + (A - LC_{me})^T P < 0 \\ &P > 0 \end{aligned} \tag{4}$$

Where:
$P=P^T$;
$[A - LC_{me}]$ is the observability matrix.

It is necessary to perform some manipulations of Eq. (5), after these manipulations, we get:

$$PA - PLC_{me} + A^T P - C_{me}{}^T L^T P < 0 \tag{5}$$

Multiplying both sides of Eq. (6) by P^{-1}, the following is obtained:

$$AP^{-1} - LC_{me}P^{-1} + P^{-1}A^T - P^{-1}C_{me}{}^T L^T < 0 \tag{6}$$

Calling $X=P^{-1}$ and $G=P^{-1}L=XL$, we arrive at:

$$\begin{aligned} &AX + XA^T - GC_{me} - C_{me}{}^T G^T < 0 \\ &X > 0 \end{aligned} \tag{7}$$

where $X=X^T$. Note that P^{-1} exists, because $P>0$, in other words every eigenvalues of P are different from zero or the best bigger that zero.

Considering the decay rate:

$$\begin{aligned} &AX + XA^T - GC_{me} - C_{me}{}^T G^T + 2\alpha X < 0 \\ &X > 0 \end{aligned} \tag{8}$$

The gain of state observer is given by:

$$L = X^{-1} G \tag{9}$$

4. STATE OBSERVERS' METHODOLOGY

Many control systems are based on the supposition that the full state vector is available for direct measurement, but in practice, all the variables are not always available, and these variables must be estimated.

Therefore, control systems using state observers can reconstruct the non-measured states or estimate the values of difficult access points in the system. However, the necessity condition for this reconstruction is that all the states should be observable (Luenberger, 1964; D'Azzo et al, 1988).

Figure 1 shows a logical diagram for faults detection and location in mechanical systems using the state observers' technique.

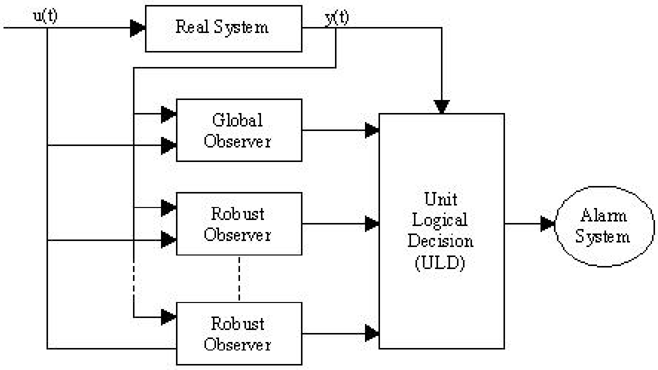

Figure 1. Observation System.

In the system of Fig. 1, when a certain component begins to fail, the state observer is capable of quickly detecting the influence of this fault, because the observer is quite sensitive to any incipient irregularity that appears in the system. The state observer is a group of ordinary first-order differential equations that represents the same response as that of the real system, when it is working property. Therefore, the idea is to use this effect for the state observer to detect and locate possible faults in a mechanical system.

In this set of observers, the role of the global observer is to verify if the system is working properly, without any indications of faults, because this observer uses the same system matrix of the mechanical system analysis. Thus, the global observer can detect a possible system fault or irregularity in the analysis if the system's response is not coincident with the global observer's response.

If a possible fault is detected, the next step would be to locate such fault, for this, the robust observers are used. The robust observers are projected by partly removing the parameters subject to faults in their dynamic matrix. Therefore, the robust observer's response that approaches the response of the faulty system will be the responsible observer for the location of this possible system fault. There still are possibilities for one or more parameters to fail at the same time. In this case, the solution would be to design robust state observers to all parameters subject to failures.

Finally, it is the Unit of Logical Decision (ULD) that collects and analyzes the difference between the real system and the designed state observers, in order to detect and locate faults or irregularities in the system. This unit also analyzes the progression of possible system faults, and activates, when necessary, an alarm system. This alarm system can be ready to be activated when a determined variation occurs in a certain parameter.

5. MODELING OF THE ROTOR-SUPPORT-STRUTURE

The mathematical model considers the whole system (rotor-supports-foundation) divided in two different subsystems: rotor-supports-subsystem and foundation subsystem. In Fig. 2 the interaction forces between the foundation and the oil film ($R_f(t)$) are shown (Cavalca, 1993).

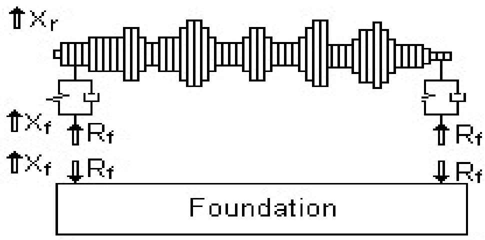

Figure 2. Rotor bearing subsystem and foundation subsystem.

Defining the vector containing both coordinates $\{\underline{x}_r(t)\}$ of the rotor and $\{\underline{x}_f(t)\}$ of the foundation connecting points, the equations of motions for the rotor-supports subsystem are:

$$[M]\{\underline{\ddot{x}}(t)\}+[C]\{\underline{\dot{x}}(t)\}+[K]\{\underline{x}(t)\}=\{F(t)\} \tag{10}$$

Where:

$$\{\underline{x}(t)\}=\begin{bmatrix}\underline{x}_r(t)\\ \underline{x}_f(t)\end{bmatrix} \text{ and } \{\underline{F}(t)\}=\begin{bmatrix}\underline{F}_r(t)\\ \underline{R}_f(t)\end{bmatrix}$$

[M], *[C]* and *[K]* are the mass, respectively, damping and stiffness matrices containing the mass matrix for each

finite element of the rotor and the equivalent damping and stiffness matrices of the oil film for each bearing.

In Eq. (3), $\{\underline{F}_r(t)\}$ represents the vector of the external generalized forces applied to the rotor and $\{\underline{R}_f(t)\}$contains the forces transmitted between rotor and foundation in each connecting node. The $\{\underline{R}_f(t)\}$ forces are unknown and depend on both rotor and foundation dynamic behavior. By matrix partitioning techniques Eq. (3) can be rewritten as:

$$\begin{bmatrix} [M_{rr}] & [M_{rf}] \\ [M_{fr}] & [M_{ff}] \end{bmatrix} \begin{Bmatrix} \ddot{\underline{x}}_r(t) \\ \ddot{\underline{x}}_f(t) \end{Bmatrix} + \begin{bmatrix} [C_{rr}] & [C_{rf}] \\ [C_{fr}] & [C_{ff}] \end{bmatrix} \begin{Bmatrix} \dot{\underline{x}}_r(t) \\ \dot{\underline{x}}_f(t) \end{Bmatrix} + \begin{bmatrix} [K_{rr}] & [K_{rf}] \\ [K_{fr}] & [K_{ff}] \end{bmatrix} \begin{Bmatrix} \underline{x}_r(t) \\ \underline{x}_f(t) \end{Bmatrix} = \begin{bmatrix} \underline{F}_r(t) \\ \underline{R}_f(t) \end{bmatrix} \tag{11}$$

The equations of motion of the foundation subsystem can be written by means of a modal approach:

$$[m_f]\{\ddot{\underline{q}}(t)\} + [c_f]\{\dot{\underline{q}}(t)\} + [m_f]\{\underline{q}(t)\} = -[\phi^t]\{\underline{R}_f(t)\} \tag{12}$$

Where, $\{\underline{q}(t)\}$ is the vector of the foundation modal coordinates corresponding to the coordinates $\underline{x}_f(t)$ of the connecting points:

$$\{\underline{x}_f(t)\} = [\phi]\{\underline{q}(t)\} \tag{13}$$

Where *[ϕ]* is the matrix containing the foundation normal modes which are evaluated corresponding to the connecting nodes. The *[ϕ]* matrix is rectangular with as many columns as the foundation vibration modes considered and as many rows as the degrees of freedom associated to the connecting nodes.

It is possible to define the connecting forces vector $\{\underline{R}_f(t)\}$ between rotor and supporting structure as a function of $\{\underline{x}_f(t)\}$ applying $\{\underline{q}(t)\} = [\phi]^{-1}\{\underline{x}_f(t)\}$, thus:

$$[M_f]\{\ddot{\underline{x}}_f(t)\} + [C_f]\{\dot{\underline{x}}_f(t)\} + [K_f]\{\underline{x}_f(t)\} = \{-\underline{R}_f(t)\} \tag{14}$$

Where:

$[M_f] = [\phi^t]^{-1}[m_f][\phi]^{-1}$

$[C_f] = [\phi^t]^{-1}[c_f][\phi]^{-1}$

$[K_f] = [\phi^t]^{-1}[k_f][\phi]^{-1}$

This transformation is possible only if the *[ϕ]* matrix is square. In this case, it is necessary to consider a number of natural frequencies equal to the number of degrees of freedom of the connecting nodes (Bonello et al, 2001). Substituting Eq. (14) into Eq. (11), gives the equation of motion of the complete system including foundation, when subject to an external excitation.

$$\begin{bmatrix} [M_{rr}] & [M_{rf}] \\ [M_{fr}] & [M_{ff}]+[M_f] \end{bmatrix} \begin{Bmatrix} \ddot{\underline{x}}_r(t) \\ \ddot{\underline{x}}_f(t) \end{Bmatrix} + \begin{bmatrix} [C_{rr}] & [C_{rf}] \\ [C_{fr}] & [C_{ff}]+[C_f] \end{bmatrix} \begin{Bmatrix} \dot{\underline{x}}_r(t) \\ \dot{\underline{x}}_f(t) \end{Bmatrix} + \begin{bmatrix} [K_{rr}] & [K_{rf}] \\ [K_{fr}] & [K_{ff}][K_f] \end{bmatrix} \begin{Bmatrix} \underline{x}_r(t) \\ \underline{x}_f(t) \end{Bmatrix} = \begin{bmatrix} \underline{F}_r(t) \\ \underline{R}_f(t) \end{bmatrix} \tag{15}$$

6. NUMERICAL RESULTS

A numerical example is given in this section starting from the developed methodology and considering a system of rotors, modeled by Finite Element Technique using beam elements. The disc is taken into account as additional masses, neglecting their stiffness and considering only the effect of inertial mass. The gyroscopic effect is considered (Lemos et al, 2004). Figure 4 shows a scheme of the rotor model with the location of bearings and foundation parts. The system was excited with only initial conditions of the displacement and velocity. The internal damping of the system was disregarded. The values of physical parameters adopted for the system are shown in Tab. 1.

For this example, the parameters subject to fail are the inertial moments of each finite element of the rotor (to simulate problems associated with the axis of the rotor) and the stiffness of the bearings. The interval of time used for the simulation went from 0 to 1 second, and the number of sampled points taken was equal to 1024.

Table 1. Physical parameters of the system.

Elements	Values
Bearings	K_{xx}= 3E9 N/m; C_{xx}= 5E4N.s/m
Disk	M_{d1}=0,04 kg, r_{d1}= 0,04m, , M_{d2}=0,05kg, r_{d2}= 0,06m,
Shaft	E=2E11 N/m^2, $d_1 = d_2 = d_3 = d_4 = d_5 = d_6 = d_7 = d_8$= 0,05m , $L_1 = L_2 = L_3 = L_4 = L_5 = L_6 = L_7 = L_8$= 0,15m, ρ=7850 kg/$m^3$
Foundation	$K_{f1} = K_{f2} = K_{f3}$= 3E9 N/m; $M_{f1} = M_{f2} = M_{f3}$=40 kg
Rotation	$\omega_1 = \omega_2$ = 1500 rpm.
Initial Condition	$x_1(0)$=0.005 m; $\dot{x}_4(0) = 0.25\, m/s$.

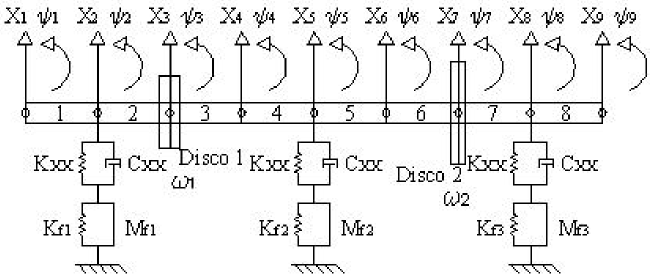

Figure 3. Representative scheme of rotor system to finite element mounted on lumped foundation.

In order to simulate possible faults in the system, some waste of the stiffness of the bearings were considered. Thus, these faults can be detected and located. The values obtained are shown in Tabs. 2, 3 and 4 illustrating the differences of the RMS values between the displacement $\{x_1(t)\}$ of real system and the values reconstructed $\{\hat{x}_1(t)\}$ for the global and the robust state observers.

Table 2. Difference RMS between the real system without and with faults the robust observers.

Set of Observers	System without fault	System with fault 2%K_{XX1}	System with fault 4%K_{XX1}
Global Observer	**2,8607X10^{-019}**	1,3299X10^{-010}	2,7046X10^{-010}
Rob. Obs. 2% K_{XX1}	1,3525X10^{-010}	**2,5992X10^{-019}**	1,3981X10^{-010}
Rob. Obs. 4% K_{XX1}	2,1342X10^{-010}	1,0853X10^{-010}	**1,8716X10^{-019}**
Rob. Obs. 6% K_{XX1}	3,3160X10^{-010}	2,2483X10^{-010}	1,1435X10^{-010}
Rob. Obs. 2% K_{XX2}	6,6675X10^{-012}	1,3291X10^{-010}	2,6964X10^{-010}
Rob. Obs. 4% K_{XX2}	1,4053X10^{-011}	1,3890X10^{-010}	2,8050X10^{-010}
Rob. Obs. 6% K_{XX2}	1,2613X10^{-012}	6,3752X10^{-012}	1,2833X10^{-011}
Rob. Obs. 2% K_{XX3}	1,7609X10^{-014}	1,3293X10^{-010}	2,7033X10^{-010}
Rob. Obs. 4% K_{XX3}	3,4743X10^{-014}	1,3282X10^{-010}	2,7012X10^{-010}
Rob. Obs. 6% K_{XX3}	5,1404X10^{-014}	1,3269X10^{-010}	2,6984X10^{-010}

Table 3. Difference RMS between the real system without and with faults the robust observers.

Set of Observers	System with fault 6%K_{XX1}	System with fault 2%K_{XX2}	System with fault 4%K_{XX2}
Global Observer	4,1258X10^{-010}	6,7219X10^{-012}	1,3671X10^{-011}
Rob. Obs. 2% K_{XX1}	2,8435X10^{-010}	1,3584X10^{-010}	1,3679X10^{-010}
Rob. Obs. 4% K_{XX1}	1,1233X10^{-010}	2,1373X10^{-010}	2,1420X10^{-010}
Rob. Obs. 6% K_{XX1}	**9,8036X10^{-020}**	3,3191X10^{-010}	3,3232X10^{-010}
Rob. Obs. 2% K_{XX2}	4,1105X10^{-010}	**4,6487X10^{-020}**	6,8994X10^{-012}
Rob. Obs. 4% K_{XX2}	4,2712X10^{-010}	7,1531X10^{-012}	**9,9561X10^{-020}**
Rob. Obs. 6% K_{XX2}	1,9588X10^{-011}	8,3679X10^{-013}	4,1681X10^{-013}
Rob. Obs. 2% K_{XX3}	4,1239X10^{-010}	6,7177X10^{-012}	1,3666X10^{-011}
Rob. Obs. 4% K_{XX3}	4,1206X10^{-010}	6,7111X10^{-012}	1,3656X10^{-011}
Rob. Obs. 6% K_{XX3}	4,1165X10^{-010}	6,7030X10^{-012}	1,3643X10^{-011}

Table 4. Difference RMS between the real system without and with faults the robust observers.

Set of Observers	System with fault 6%K_{XX2}	System with fault 2%K_{XX3}	System with fault 4%K_{XX3}
Global Observer	$2,0866X10^{-011}$	$1,7616X10^{-014}$	$3,4777X\ 10^{-014}$
Rob. Obs. 2% K_{XX1}	$1,3816X10^{-010}$	$1,3525X10^{-010}$	$1,3525X10^{-010}$
Rob. Obs. 4% K_{XX1}	$2,1420X10^{-010}$	$2,1341X\ 10^{-010}$	$2,1341X10^{-010}$
Rob. Obs. 6% K_{XX1}	$3,3285X10^{-010}$	$3,3160X10^{-010}$	$3,3160X10^{-010}$
Rob. Obs. 2% K_{XX2}	$1,4049X10^{-011}$	$6,6644X10^{-012}$	$6,6614X10^{-012}$
Rob. Obs. 4% K_{XX2}	$7,4242X10^{-012}$	$1,4051X10^{-011}$	$1,4048X10^{-011}$
Rob. Obs. 6% K_{XX2}	**$3,0809X10^{-020}$**	$1,2605X10^{-012}$	$1,2597X10^{-012}$
Rob. Obs. 2% K_{XX3}	$2,0860X10^{-011}$	**$1,7581X10^{-019}$**	$1,7159X10^{-014}$
Rob. Obs. 4% K_{XX3}	$2,0846X10^{-011}$	$1,7149X10^{-014}$	**$1,6902X10^{-019}$**
Rob. Obs. 6% K_{XX3}	$2,0827X10^{-011}$	$3,3832X10^{-014}$	$1,6701X10^{-014}$

In Tab. 2, 3 and 4, the fault can be detected and located by comparing the global system without fault with the global observer (second line in the second column of Tab. 2), verifying that the order of difference of *RMS* (Root mean square) is 10^{-019}, hence showing that the curves are practically coincident and indicating that there is no irregularity in the system (The observer's response is the same as the real system's response). The fault is detected when the real system with fault is compared to the global observer, the RMS difference, showing that the system has some irregularity (see second line in the third column of Tab. 2). To locate such irregularity, the response of the global fault system is compared with the robust observer dedicated to each fault parameter, and when this comparison is made, it is noticed that the differences of RMS increase for the order of 10^{-019} or 10^{-020}.

Analyzing Tab. 2, 3 and 4, it can be noticed that faults or stiffness variations can be detected in all the areas of the rotation system (axis and bearings).

7. CONCLUSIONS

With the results obtained, it was possible to conclude the efficiency of the methodology developed. The technique of fault detection using state observers consists in the capacity of state observer of reconstruction the states not measured or values proceeding from points of difficult access in the system. It was verified the necessity to choose the parameters subject to fault for the construction of robust observers, therefore, certain components require a constant accompaniment, then, for these parameters are mounted a bank of state observers with a system of alarms that generate a curve of trends. A restriction in the methodology developed is the fact that the system should be observable with the number of measures realized.

8. ACKNOWLEDGEMENTS

The authors acknowledge the financial support of FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo).

9. REFERENCES

Abbaszadeh, M., Marquez, H. J., 2008, "LMI optimization approach to robust H_∞ observer design and static output feedback stabilization for discrete-time nonlinear uncertain systems", International Journal of Robust and Nonlinear Control.

Abdalla, M.O.; Zimmerman, D.C.; Grigoriadis, K. M., 1999, "Structural Damage Detection using Strain Data via Linear Matrix Inequaly Based Methods", American Control Conference, San Diego, Califórnia, pp. 1114-1118.

Abdalla, M.O.; Zimmerman, D.C.; Grigoriadis, K.M., 2000, "Reduce Optimal Parameter Update in Structural Systems Using LMIs", American Control Conference, Chicago, Illinois, pp. 991-995.

Assunção, E., 2000, "Redução H2 e H∞ de Modelos através de Desigualdades Matriciais Lineares: Otimização Local e Global" , Tese de Doutorado, UNICAMP, Campinas, SP.

Bara, G.I., Daafouz, J., Kratz, F., Ragot, J., 2001, "Parameter-Dependent State Observer Design for Affine LPV Systems", International Journal of Control, vol. 74, n. 16, pp. 1601-1611.

Boyd, S.; Balakrishnan, V.; Feron, E.; El Ghaoui, L., 1994, "Linear Matrix Inequalities in Systems and Control Theory", Siam Studies in Applied Mathematics, USA, 193p.

Bonello P., Brennan, M. J., 2001, " Modelling The Dynamic Behaviour Of A Supercritical Rotor On A Flexible Foundation Using The Mechanical Impedance Technique", Journal of Sound and Vibration, vol. 239, nº 2, pp. 445-466.

Cavalca, K. L., Bachschmid, N., 1993, "Dynamical Behaviour Analysis of Multistage Centrifugal Pumps with Interstage Seals By a Modal Truncation Method" Journal of The Brazilian Society of Mechanical Sciences, Vol. XV, n.3, pp. 263-280.

Cavalcante, P. F.,Cavalca, K. L., 1997, “Representation of a Foundation in a Complete Rotating Machinery Through its Impedance Matrix”, DINAME.

Cheli, F., Cavalca, K. L., Dedine, F. G., Vania, A., 1992, “Suppoting structure effects on rotating machinery vibrations”, Imeche - Procedings of the institution of mechanical engineers.

Choy, F. K., Liang, R., Xu, P., 1995, “Fault Identfication of Beams on Elastic Foundation”, Computers and Geotechnics, vol. 17, pp. 157-176.

Clark, R. N., 1978, “Instrument fault detection”, IEEE Transactions on Aerospace and Eletronic Systems, Vol.AES-14, no3, pp.456-465.

D’azzo, J. J., Houpis, C. H., 1988, “Linear Control System Analysis and Design”, São Paulo, McGraw-Hill, Inc, 660p.Luenberger, D.G., 1964, “Observing the state of a Linear System”, IEEE Transactions on Military Eletronics, Vol MIL-8, pp. 74-80.

Elmas, C., De La Parra H. Z., 1996, “Application of a full-order extended Luenberger Observer for a position sensorless operation of a switched reluctance motor drive”, IEE Proceedings Control Theory and Applications, vol.143, n.5, pp.401-480.

Faitakis, Y. E., Tthapliyal, S., Kantor, J. C., 1998, “An LMI Approach to the Evaluation of Alarm Thresholds”, International Journal of Robust and Nonlinear Control, vol. 8, ed. 8, pp. 659-667.

Fernandes, F. V., Koroishi, E. K., Melo, G. P., 2007, “Diagnosis of Fault Using State Observers in with Dynamic Vibrations Absorbers”. In: Proceedings of 19th International Congress of Mechanical Engineering, Brasília-DF.

Frank, P.M., Seliger, R., 1991, “Fault Detection and Isolation in Automatic Processes”, Control and Dynamics Systems, Vol.49, pp. 241-287.

Ge, W., Fang, C. Z., 1987, “Detection of faulty components via robust observation”, Department of Automation, Tsinhua University, Beijing, China, pp.582-599.

Ghaoui, L, Niculescu, S., 2000, “Advances in Linear Matrix Inequalities Methods in Control”, Advances in design and Control, Siam, EUA.

Jiang, P., Li, C., 2004, “Observer-based robust stabilization for uncertain systems with unknown time-varying delay”, Journal of Control Theory and Applications, vol. 2, n. 2, pp. 155-160.

Jiang, G.-P., Tang, W. K.-S., 2006, “A State-Observer-Based Approach for Synchronization in Complex Dynamical Networks”, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 53, ed. 12, pp. 2739 – 2745.

Lemos, G.F., Melo, G.P.,2004, “Fault Diagnosis in Rotation System Using well Conditioned State Observer”, IMAC XXII-A Conference on Structural Dynamics, Dearborn, Michigans USA.

Luenberger, D.G., 1966, “Observers for Multivariable Systems”, IEEE Transactions on Automatic Control, Vol AC11, pp. 190-197.

Luenberger, D. G., 1971, “An Introduction to Observers”, IEEE Transactions on Automatic Control, Vol AC16, nº. 6, pp. 596-602.

Mohiuddin, M. A., Khulief, Y. A., 1999, “Coupled Bending Torsional Vibration of Rotors Using Finite Element”, Journal of Sound and Vibration, vol. 223, nº2, pp.297-316.

Morais, T. S., Koroishi, E. H., Daniel, G. B. e Melo, G. P., 2005, “Utilização do Filtro de Kalman para Detecção de Falhas em Sistemas Mecânicos Considerando Ruídos e entradas Não Lineares”, Anais do XXVI Iberian Latin American Congress Computational Methods Engineering, Guarapari - ES.

Palhares, R. M., 1998, “Filtragem Robusta: Uma Abordagem por Desigualdades Matriciais Lineares”, Tese de Doutorado, UNICAMP, Campinas, SP.

Park, J.K., Shin, D.R., Chung, T.M., 2001, “Dynamic observers for linear time-invariant systems”, Automatica, pp.1-5.

Park, J. H.,Jung, H.-Y., Park, C. H., 2008, “Design of State Estimator for Discrete-Time Neural Networks: Delay-Independent Approach”, Journal of the Physical Society of Japan.

Silva, S., Lopes Jr, V., Assunção, E., 2004, “Robust Control to Parametric Uncertainties in Smart Structures Using Linear Matrix Inequalities”, Journal of the Brazilian Society of Mechanical Science and Engineering, vol. XXVI, Nº 4, pp. 430-437.

Stephenson, R. W. & Rouch, K. E., 1992, “Generating Matrices of the Foundation Structure of a Rotor System From Test Data”, Journal of Sound and Vibration, vol. 154, nº3, pp.467-484.

Teixeira, M. C. M., Pietrobom, H. C., Assunção, E., 2000, “Novos Resultados sobre a Estabilidade e Controle de Sistemas Não-Lineares Utilizando Modelos Fuzzy e Lmi”, Controle & Automação. vol. 11, n. 1, pp.37–48.

Teixeira, M. C. M., Lordelo, A. D. S., Assunção, E., 2000, “On LMI Based of SPR Systems and Output Variable Structure Controllers”, 6th International Workshop on Variable Structure Systems (VSS’2000), Coolangatta Gold Coast, Australia, pp. 199–208.

Trinh, H., Aldeen, M.,1998, “A reduced-order state observer for large-scale discrete-time systems”, Pergamon, Computers Eletric. Engng, pp. 301-309.

Van Antwerp, J. G., Braatz, R. D., 2000, “A Tutorial on Linear and Bilinear Matriz Inequalities”, Journal of Process Control, vol. 10, pp. 363–385.

Yaz, E. E., Jeong, J. S., Yaz, Y. I., 2006, "An LMI approach to discrete-time observer design with stochastic resilience", Journal of Computational and Applied Mathematics, vol. 188, ed. 2, pp. 246-255.

Wang., H. B., Wang, J. L., Lam J., 2007, "Robust Fault Detection Observer Design: Iterative LMI Approaches", Journal of Dynamic Systems, Measurement, and Control, vol. 129, ed. 1, pp. 77-82.

Watanabe, K., Himmelblau D. M., 1982, "Instrument Fault Detection in Systems with Uncertainties", International Journal of Systems Science, vol.13, n.2, pp. 137-158.

Wauer, J., 1990, "Modeling and Formulation of Equations of Motion for Cracked Rotating Shafts", Int. Journal Solids Structures, vol. 26, nº 8, pp. 901-914.

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.