MULTIVARIABLE OPTIMAL PID CONTROL OF A HEAT EXCHANGER WITH BYPASSES

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Abstract. *Nowadays, there are a lot of different control methodologies that could be used within industrial processes. Some of these methodologies have a complex design and also demands an extra engineering effort to design the controller with a superior performance. Some other controllers may not lead to a desirable performance although they are too easy to design. The most common controller used on chemical plants is the PID that presents a simple design (such as the heuristic tuning methods) and easy implementation. If the chemical process presents a lot of inputs/outputs, the PID design will demand an supplementary engineering attempt to tune and design the controller. In this situation, the DMC (Dynamic Matrix Controller) controller, which is the most suitable control strategy to be used within industrial processes that involve a lot of inputs/outputs, complex dynamics, dead-time or inverse output response. The drawback of the DMC is that it needs too much engineering contribution to be designed, which is normally true for all Predictive Controllers. Midway between the simplest and the most complex controller designs, there is an intermediate solution based on optimal control theory. This work intends to present this intermediate solution, i.e., a relative simplicity in design combined with a superior performance, applied to a heat exchanger with bypasses. An optimal control strategy, called Linear Quadratic Regulator (LQR), was developed successfully for the heat exchanger and the LQR controller law is implemented in conjunction with a state observer, since the states are not all accessible on the heat exchanger model. Simulations results obtained show that even being so straightforward to design, it presents a reasonable performance, i.e., the variables become almost totally decoupled and the associated settling-time can be adjusted by means of a simple tuning procedure. The main contribution of this work is to demonstrate how a PID controller may be optimally designed by using the LQR controller design as its support. The LQR control law is mapped to an equivalent control law in the form of a PID controller, using the A, B and C constant matrices from the state-space form of the heat exchanger model, in order to obtain the P, I and D gains of the controller. To illustrate, the performance obtained with the optimal PID controller is compared with the LQR controller.*

Keywords: *Optimal Control, LQR, PID, Heat Exchanger*

1. INTRODUCTION

A heat exchanger is a device built to transfer energy from a hot stream to a cold stream and it is widely used in chemical and power plants. This equipment promotes energy recovery among process streams, reducing utility consumption. In terms of control, hot stream and cold stream outlet temperatures TH_{OUT} and TC_{OUT} are the controlled variables while bypass fractions f_H and f_C represent manipulated variables.

The most common controller used on chemical plants is the PID controller. Surely, if the chemical process has a lot of inputs/outputs, the design of these PID controllers becomes too complex. In these cases, some other controllers may be used to be more suitable to particular cases, such as the DMC (*Dynamic Matrix Controller*) controller. This is a control strategy to be used on industrial processes that involves a lot of inputs/outputs, complex dynamics, dead time or inverse output response, for example. The drawback of the DMC is that it needs an engineering effort to be designed, what is normally true for all Predictive Controllers, as the MPC controller (Doyle and Stain, 1981).

Between the simplest and the most complex controller design, there are some other techniques, like RGA array and decoupling system that could be used in combination with the PID, in order to eliminate the interactions between the variables that are often presented in multivariable systems (Ogunnaike, 1994).

These two techniques, the RGA array and the decoupling system, are broadly used on industrial processes, such as the PID controller. However, there are some drawbacks related to them that will be also presented in this work.

The main objective of this work is to show that there is a midway solution based on optimal control theory. This solution presents an extraordinarily ease to design, and, at the same time, it can also result in a superior performance when applied to the design of a heat exchanger controller.

This control methodology is called LQR – *Linear Quadratic Regulator* and the control law has the form of a state feedback control gain $u = -Kx$ (Doyle and Stain, 1981). It is based on the minimization of the quadratic cost function

$$J = \int_0^\infty \left(x^T.Qx + u^T.Ru \right) dt\,, \tag{1}$$

where Q and R are constant matrices with appropriate dimensions, x is the state vector and u is the input vector.

The LQR is a control technique developed on the time domain. During its design, the time domain response is not taken into account explicitly and the closed-loop response is checked a posteriori, as it is usually done with the PID control design. If the time response does not present an acceptable performance, a new controller can be tried by changing the Q and R matrices.

To illustrate the performance of the LQR controller, it was applied to a shell and tube heat exchanger non-linear model and the simulations were successfully performed.

Although there is nothing new in this paper on the LQR design methodology, there is an advantage on the utilization of it. The LQR controller can be mapped to an equivalent control law in the form of PID controllers which is the most common ones used on industrial applications. The main contribution of this work is to demonstrate how a PID controller may be optimally designed by using the LQR controller design as its support. It is known that the PID controller could be designed by root locus analysis, frequency domain or by using some heuristics methodologies, as *Cohen and Coon* or *Ziegler and Nichols*. Another contribution of this work is that the PID design could also be done with optimal characteristics, inherited from the LQR control. The LQR control law mapping to an equivalent control law based on PID controllers uses the A, B and C constant matrices from the state-space form of the heat exchanger model, in order to obtain the P, I and D gains of the controller. To illustrate the performance obtained with this "optimal PID controller", simulations were performed and its performance is compared with the one of the LQR controller.

This article is divided into 5 sections. On section 2, a basic introduction of the heat exchanger non-linear model, including its dynamics characteristics, is presented and it brings about the main characteristics of the variable interactions frequently exhibited by MIMO (*Multiple Input, Multiple Output*) systems. On section 3, the LQR was designed and its performance is demonstrated by simulations analysis. Finally, section 4 brings the main positive contribution of this work, which is the procedure to obtain the optimal PID control. Simulations were also performed in order to demonstrate the closed-loop control system response. Section 5 presents the conclusions obtained from this work and the next steps.

2. THE SHELL AND TUBE HEAT EXCHANGER MODEL

Shell and tube heat exchangers are the most common exchangers used in chemical plants, due to their particular characteristics such as low cost and easy maintenance (Hewitt, Shires and Bott, 1993). This kind of exchanger can be built in a large range of heat exchange area and constructed with a lot of different materials, denoting its versatility and robustness. Figure 1 represents a schema of a heat exchanger with bypasses, considered in this work.

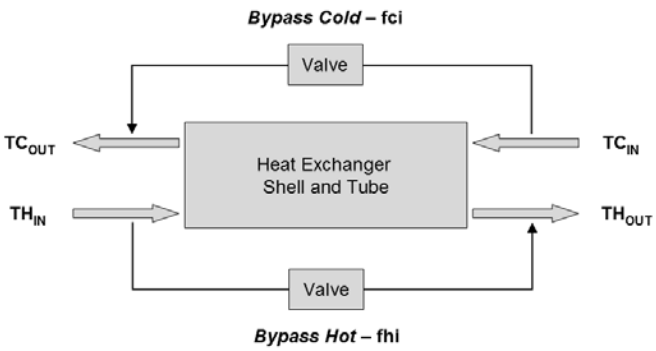

Figure 1 – Shell and tube heat exchanger representation.

The mathematical model of the heat exchanger used on this work was obtained through stream energy balance, as represented in equations (2) and (3) (Novazzi, 2006):

$$\frac{\partial T_H}{\partial t} = -\frac{\dot{m}_H}{\rho_H.v_H}.\frac{\partial T_H}{\partial z} - \frac{U.A}{\rho_H.V_H.Cp_H}\left(T_H - T_C\right) \tag{2}$$

$$\frac{\partial T_C}{\partial t} = \frac{\dot{m}_C}{\rho_C . v_C} . \frac{\partial T_C}{\partial z} + \frac{U.A}{\rho_C . V_C . Cp_C}\left(T_H - T_C\right) \tag{3}$$

where: T is the stream temperature, m is the mass flow rate, ρ is the density, v is the relationship between volume and length of the exchanger, t is the time, z is the axial position, A is the heat transfer area, C_P is the specific heat, V is the volume and U is the global heat transfer coefficient. The indices c and h refers to the variables related to the *cold* and to the *hot* stream. Equations (2) and (3) were discretized and solved by finite difference method, in *Matlab* [5].

Although bypasses positions f_H and f_C are not explicitly written in equations (2) and (3), they are the manipulated variables and directly affect the controlled variables TH_{OUT} and TC_{OUT}. The nominal values for all variables of the model, including the physical dimensions of the exchanger, are described by the Table 1 as follows:

The physical dimensions and nominal values of the variables of the exchanger are shown on Table 1. The mathematical model of this exchanger is described by partial differential equations, and they were discretized to be solved by the finite difference method (Novazzi, 2006).

Table 1. Physical dimension and nominal process values of the heat exchanger.

Items	**Value**
Heat exchanger area ***A***	220 m^2
The global coefficient ***U***	190 $W \cdot m^{-2} \cdot {}^oC^{-1}$
Cold input temperature $\boldsymbol{TC_{IN}}$	133,1 ºC
Hot input temperature $\boldsymbol{TH_{IN}}$	204,4 ºC
Cold output temperature $\boldsymbol{TC_{OUT}}$	188,1 ºC
Hot output temperature $\boldsymbol{TH_{OUT}}$	150,8 ºC
Cold input stream $\boldsymbol{m_C}$	4 kg/s
Hot input stream $\boldsymbol{m_H}$	6 kg/s
Bypass cold position ***fci***	50%
Bypass hot position ***fhi***	50%

In order to analyze the dynamic characteristics of this exchanger, some simulations were performed with the model by applying disturbances on nominal inputs. These disturbances were taken as a step input signal on nominal values of the temperatures, flow and bypass position, respectively. By modifying nominal input temperatures values, the output temperatures (TH_{OUT} and TC_{OUT}) exhibit a first-order dynamics in association with a dead-time, as shown on Figure 3.

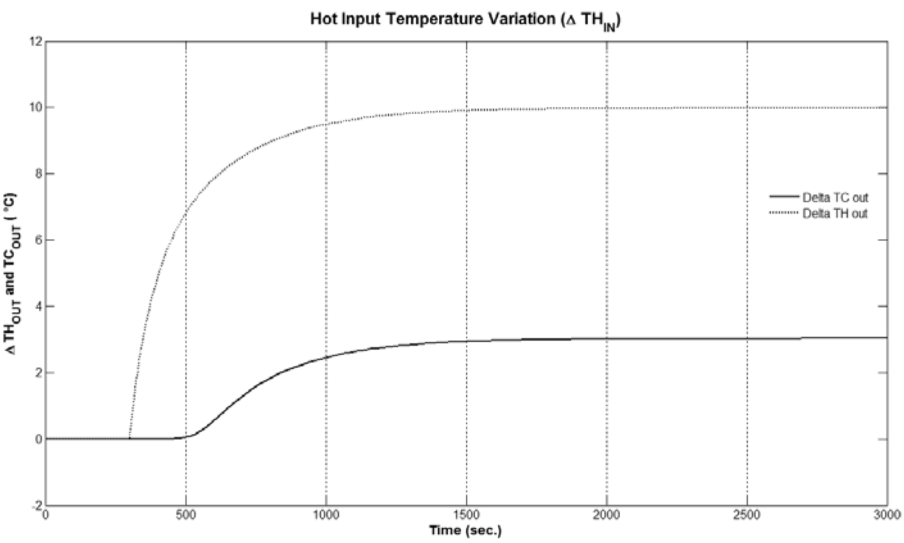

Figure 3 – Positive hot input temperature disturbance.

Moreover, the disturbance is applied on the nominal input flow values. It is easy to notice that the output responses are stronger on the streams directly disturbed, and also they exhibit a first-order dynamics. Figure 4 restates the previous explanation and shows a disturbance on the hot flow (m_H) as well as the resulting effects on both output temperatures (TH_{OUT} and TC_{OUT}).

To conclude this dynamic analysis, the bypasses values were changed. The output temperature responses could then be approximated by a transfer function with one zero and one pole. Figure 5 shows a positive step disturbance on the bypass hot and the effects on both output temperatures (TH_{OUT} and TC_{OUT}).

With all of the output responses presented on Figures 3, 4 and 5, it can be noticed that the dynamics involved on the exchanger are basically represented by first-order transfer functions. These analyses were also performed to show that the heat exchanger dynamics present important characteristics, such as small dead-time, no complex dynamics and non-inverse response. Thus, although presenting simple dynamics, it still has the unavoidable interactions.

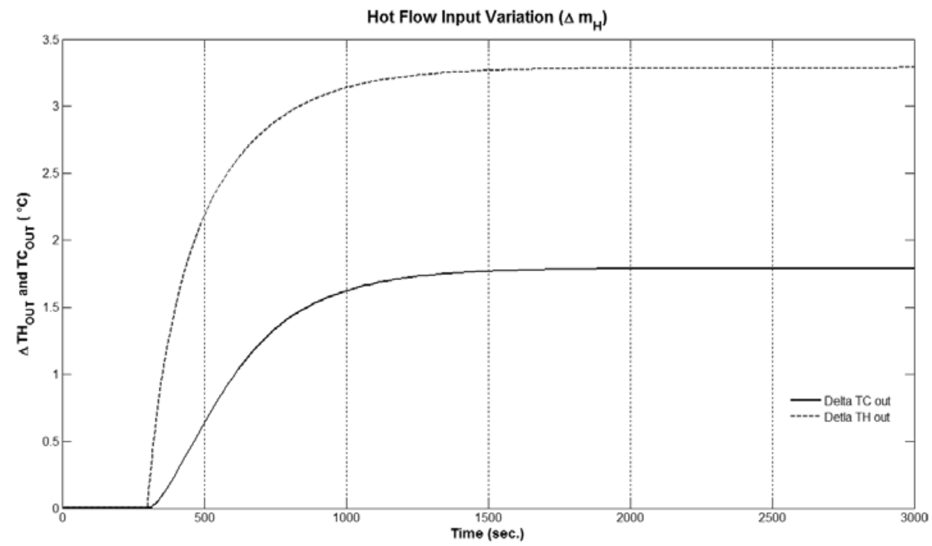

Figure 4 – Positive hot input flow step disturbance.

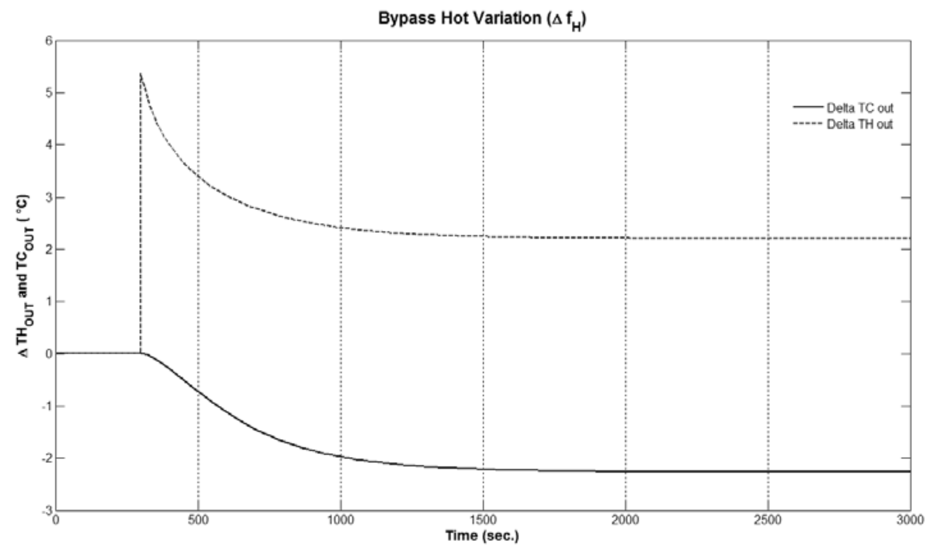

Figure 5 – Positive bypass hot step disturbance.

3. THE LQR CONTROLLER

This section will present the application of the LQR state feedback technique to the shell and tube heat exchanger with bypasses. The LQR design was done based on a linear model of the heat exchanger properly linearized around its specific operation point.

The heat exchanger considered in this work is a MIMO system, containing two inputs and two outputs, with variable interactions. These interactions could be observed by analysing Figures 3, 4 and 5 from the previous section, where both output variables are affected by the disturbances applied on the nominal inputs. Figure 6 represents these interactions between the variables in terms of the system transfer functions.

These interactions are also the reason why the multivariable system is difficult to control. If these interactions did not exist, the control design would have reduced to two SISO (*Single Input, Single Output*) loops control.

To eliminate these variables interactions, the most common techniques are the decoupling system and the RGA array. These techniques will return an appropriate transfer function combination in order to eliminate the G_{12} and the G_{21} interactions (decoupler).

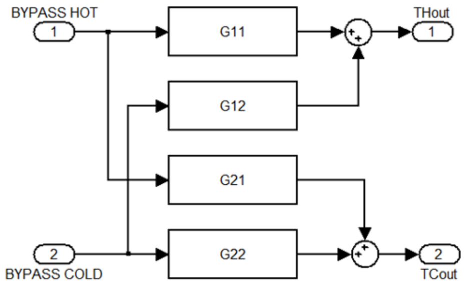

Figure 6 – Loop interactions for the heat exchanger

Both techniques demand a PID controller to implement a closed-loop control. Nevertheless, another design, based on the output response analysis of the system plus decoupler, is required in order to find the *P*, *I* and *D* gains. Unquestionably, it will demand an extra effort and some drawbacks could be related to these techniques:

- ✓ If a poor mathematical model to perform the variables interactions elimination is used in order to find a satisfactory relationship (GI_1 and GI_2) to minimize the effects of G_{12} and the G_{21} transfer functions, new unexpected dynamics will be added to the system. This weak mathematical model might produce an unknown output response, which can cause an inability of the controller to execute the control loop;
- ✓ A loop pairing study is necessary on the RGA array in order to obtain the minimum interactions between the variables;
- ✓ If the model can represent appropriately the system, and the loop paring can be done, the output performance is mainly determined by the PID controller, and in general it will require several trial-and-error steps to tune the PID in order to achieve a superior output performance.

The advantage of the use of the LQR controller structure is that it can yield both the decoupling of the output variables and a desired performance for the closed-loop control system. The simulation results shown in Figure 7 used unitary steps as input signals on both inputs, and the variable decoupling could be easily checked. One step has been applied at t=300 seconds for the first input and another at t=600 seconds for the second input. Step disturbances were also simulated and it is possible to notice that they affect almost exclusively the respective output channel where they were directly applied.

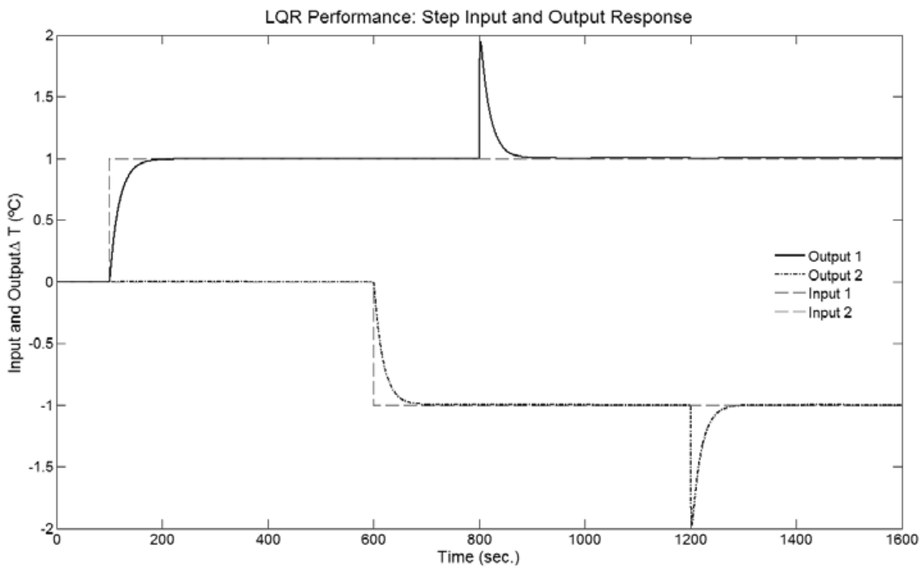

Figure 7 – LQR step input signal performance with disturbance

In the LQR technique, if the Q and R matrices are adequately weighted, it could return a satisfactory time response. Observing an analyzing Figure 7, the outputs presents a first order response with a specific time constant. If this time constant is not too short/small or it needs to become shorter or greater, a new controller could be designed by changing the Q and R matrices in the cost function presented by the equation (1).

4. THE PID OPTIMAL CONTROLLER

This section presents an alternative way of tuning PID controllers based on the LQR. Consider the linear system model in state-space structure, with a PID controller to close the loop control as shown by Figure 8, and consider ***U*** as the PID output. The PID equation follows immediately.

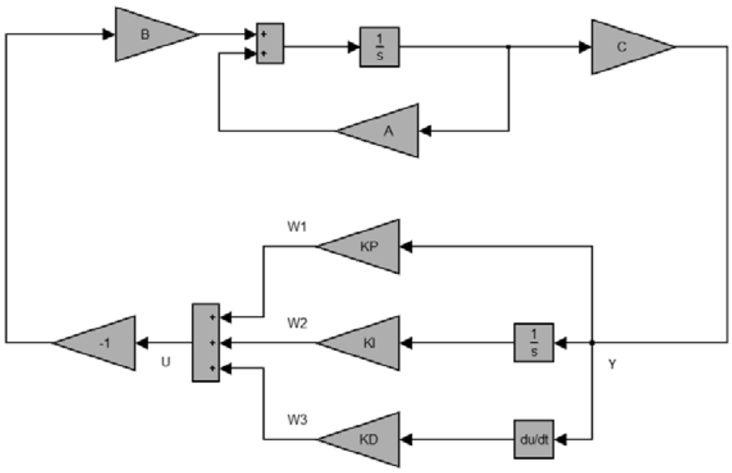

Figure 8 – State Space Structure with PID Feedback Control

$$
\begin{aligned}
u &= w_1 + w_2 + w_3 \\
u &= -K_P.y - K_I.\int_0^t ydt - K_D.\dot{y}
\end{aligned}
\tag{4}
$$

The LQR controller was implemented as shown in Figure 9. An alternative form of the state feedback control gain was implemented, adding extra states by using an integral term. The "new" control law is shown in equation 5.

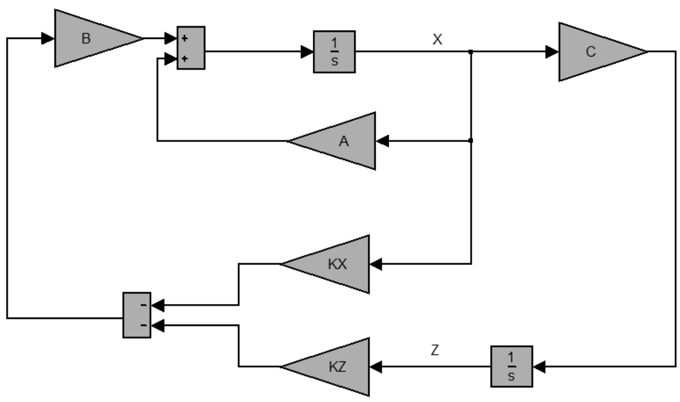

Figure 9 – State Space Structure with LQR Feedback Control

$$
u = -KX.x - KZ.\int_0^t ydt \tag{5}
$$

The main idea here is to make equations 4 and 5 equivalent showing thus that a PID controller can be tuned by using an optimal control approach. The individual gain matrices of the PID controller (K_P, K_D, K_I) can thus be computed by equations (6) and (7).

$$\begin{bmatrix} K_D & K_P \end{bmatrix} = K_X . \begin{bmatrix} C \\ C.A - C.B.K_X \end{bmatrix}^{-1} \tag{6}$$

$$\begin{bmatrix} K_I \end{bmatrix} = \left(I + K_D.C.B \right).K_Z \tag{7}$$

A simulation of the heat exchanger model was performed in order to check if the PID tuned by using the optimal design presented a satisfactory response. The same signals from the LQR simulation were used, namely, unit steps were applied at the first and second inputs respectively at times t=300 seconds and t=900 seconds. A disturbance was also simulated. It can be noticed that it affected mainly the output channel where it was directly applied. Figure 10 shows the results obtained.

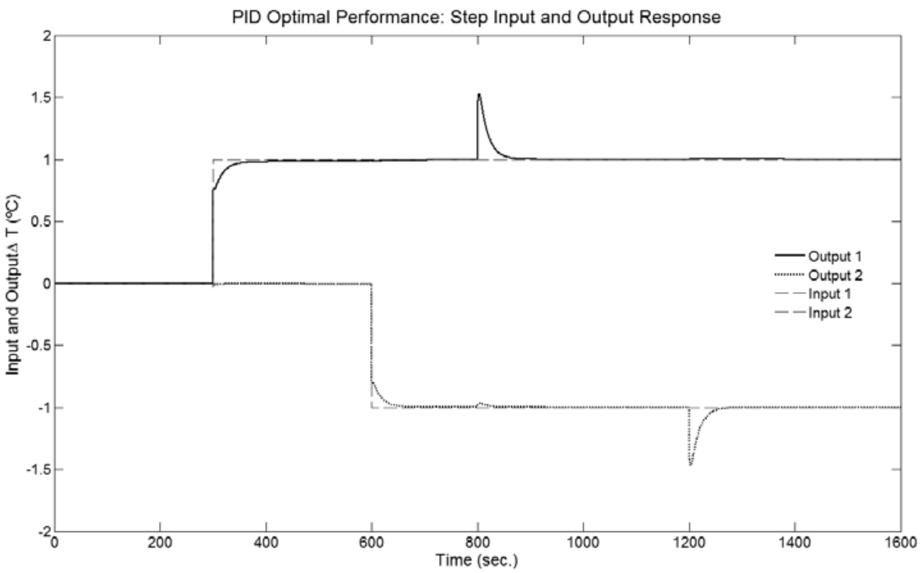

Figure 10 – PID optimal step input signal performance with disturbance

5. CONCLUSIONS

Observing the LQR and PID controlled heat exchanger simulation results (Figures 7 and 10) the following conclusions can be drawn:

- ✓ In spite of LQR not being a new control methodology, the results obtained suggest that it can be an interesting alternative to heat exchanger control when compared to other techniques like the RGA array and decoupling;
- ✓ The PID optimal controller exhibited a better performance in the simulations carried out;
- ✓ In the simulations run the LQR gave rise to bigger disturbance amplification when compared to the PID optimal. In addition, observing the singular values shown in Figure 11, it can be noticed that the LQR controller has a larger gain in small frequencies than the PID optimal. For frequencies higher than 10 rad/sec, both gains have approximately the same values;
- ✓ Although the integers were included in the system in the LQR design, the equivalent optimal PID exhibits a small gain at low frequencies (Figure 11). As a consequence there will be a steady-state error for step inputs. If necessary this could be reduced by adding an extra gain in the loop;
- ✓ All the simulations reported in this work carried out were successful. The next step of this research will be to test the controllers on a real system. Figure 12 shows the experimental apparatus that is under development in the Chemical Engineering Laboratory of Centro Universitário da FEI. It contains two shell and tube heat exchangers, two hot streams and one cold stream. In the future, energy interchanging network studies will be carried out using this system.

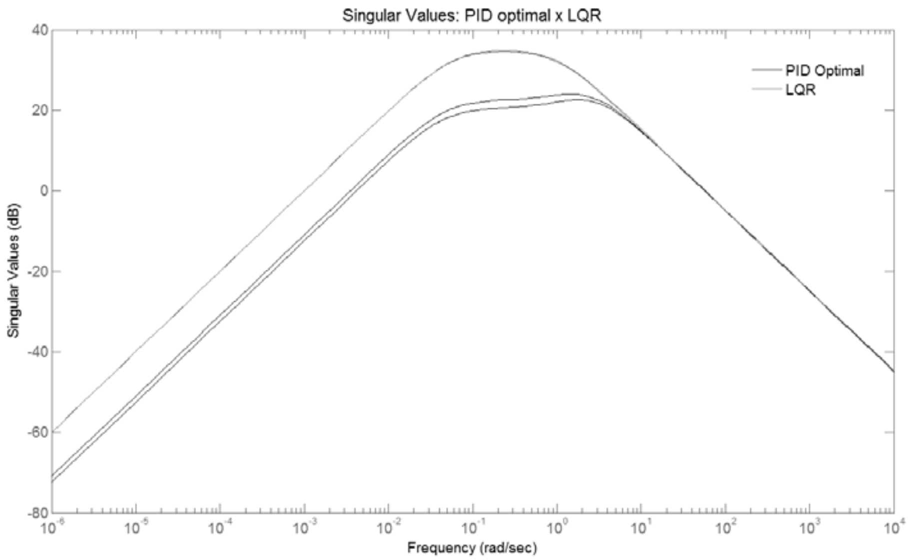

Figure 11 – LQR and PID optimal singular values

Figure 12 – Experimental apparatus: The heat exchanger network

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