CHARACTERISTIC VALUES AND COMPATIBILITY CONDITIONS FOR THE NO-PRESSURE-WAVE MODEL APPLIED TO PETROLEUM SYSTEMS

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Abstract. *In this paper, the characteristic values and the compatibility conditions (resulting equations in the propagation directions) are shown for a one-dimensional multiphase oil, water and gas flow model. Continuity equation for each phase and a simplified momentum equation without inertia terms (no-presssure-wave -NPW- approximation) for the phases flowing together are considered. Oil and water phases are considered to have the same velocity and are homogenized. Slip between the liquid and gas phases are taken into account by using a drift flux model. Mass transfer between the oil and gas phases are calculated using the black oil model. It is shown that a mixed hyperbolic-parabolic model is obtained and that two characteristic finite velocities exist, related to the gas and liquid velocities. A procedure for the numerical solution of the resulting equations, based on the method of characteristics, is discussed.*

Keywords: *multiphase flow, petroleum production systems, characteristic values, no-pressure-wave model, black oil model.*

1. INTRODUCTION

Transient simulation of multiphase oil and gas flows in pipes requires a considerable computational effort. There are many models used in the literature, varying the number of conservation equations and/or the closure laws (Masella *et al.*, 1998). Among these, it can be cited the two-fluid model (based in a momentum equation for each phase) and the drift flux model (based on one momentum conservation equation and an algebraic slip relation).

In most of the transients occurred in oil and gas transport, for instance in severe slugging, the response of the system proves to be relatively slow, showing that pressure waves do not have a strong effect on the initiation and transport of void waves. In the no-pressure-wave (NPW) model, acoustic waves are ruled out by neglecting inertia terms from the momentum equation, resulting an algebraic relation for the pressure gradient.

Due to its simplicity, experimental studies on severe slugging were usually realized in air-water systems (Jansen *et al.*, 1996; Mokhatab, 2007; Taitel *et al.*, 1990; Wordsworth *et al.*, 1998). Although basic mechanisms of severe slugging can be investigated using air-water systems, there are many limitations when trying to extrapolate these results to petroleum production systems:

- Pipeline lengths and riser heights in petroleum production systems are much greater (order of kilometers long) than the values for air-water experimental facilities. The high pressure ratios between the bottom and top of the riser give rise to important expansion effects in the gas phase, invalidating models based on the assumption of a mean void fraction.

- Petroleum is a multicomponent system in which both liquid and gas phases coexist at operating conditions (McCain, 1990). Mass transfer between the phases are dependent on pressure and temperature through the PVT curve. With the high pressure variations in the riser, mass transfer effects cannot be ignored. Besides, the fluid coming from the reservoir has a water content, so three phases can coexist in the general case.

- Most of the experiments in air-water systems were realized keeping a constant separation pressure as a boundary condition. A few experiments investigated the effect of a choking valve at the top of the riser. Because of the low pressures involved, the valve operated in subcritical conditions. In petroleum production systems, a choke valve operating in critical conditions is located at the top of the riser.

For a model to describe physical phenomena correctly it must be well-posed, this is, the solution must exist, must be uniquely determined and must depend in a continuous fashion on the initial and boundary conditions (Drew and Passman, 1999). This property is particularly important in multiphase flows, where partial differential equations of hyperbolic nature can be found; in this case, well-posedness implies that the characteristic values (eigenvalues or characteristic wave velocities) must be real.

In this paper a one-dimensional model for multiphase flow (oil, gas and water) is developed and its characteristic values and compatibility conditions are studied. This model will be used in severe slugging simulations and stability studies, extending a previous model developed for air-water systems (Baliño *et al.*, 2007; Baliño, 2008) for petroleum production conditions.

2. MULTIPHASE FLOW MODEL

2.1 Conservation equations

The model considers one-dimensional, isothermal flow. Solubility of gas and vaporization are neglected for water. Considering continuity equations for the phases oil, gas and water and a mixture momentum equation in which inertia terms are neglected, we get:

$$\frac{\partial}{\partial t}\left(\rho_g\,\alpha\right)+\frac{\partial j_g}{\partial s}=\Gamma \tag{1}$$

$$\frac{\partial}{\partial t}\left(\rho_o\,\alpha_o\right)+\frac{\partial j_o}{\partial s}=-\Gamma \tag{2}$$

$$\frac{\partial}{\partial t}\left(\rho_w\,\alpha_w\right)+\frac{\partial j_w}{\partial s}=0 \tag{3}$$

$$\frac{\partial P}{\partial s}=-\frac{4\,\tau_w}{D}+\rho_m\,g_s \tag{4}$$

$$\rho_m=\rho_g\,\alpha+\rho_o\,\alpha_o+\rho_w\,\alpha_w \tag{5}$$

where D is the pipe diameter, P is pressure, s is the coordinate along the flow direction, t is time, ρ_m is the density of the mixture, ρ_g, ρ_o and ρ_w are the densities of the phases (correspondingly gas, oil and water), j_g, j_o and j_w are the superficial velocities, α, α_o and α_w are the volume fractions, g_s is the gravity component in the s-direction, Γ is the vaporization source term and τ_w is the mean shear stress at the pipe wall. The volume fractions are related by:

$$\alpha_o+\alpha_w+\alpha=1 \tag{6}$$

2.2 Closure laws

In order to close mathematically the problem, some simplifications must be made.

2.2.1 Homogenization of liquid phases

Assuming equal velocities for oil and water, we obtain:

$$j_o=j_l\,\frac{\alpha_o}{1-\alpha} \tag{7}$$

$$j_w=j_l\,\frac{\alpha_w}{1-\alpha} \tag{8}$$

$$j_l=j_o+j_w=u_l\,(1-\alpha) \tag{9}$$

where j_l and u_l are correspondingly the superficial velocity and the velocity of the liquid (oil plus water) phase.

2.2.2 Shear stress at the wall

The shear stress at the wall is estimated using a homogeneous two-phase model and a correlation (Chen, 1979) for the Fanning friction factor f, resulting the following relations:

$$\tau_w=\frac{1}{2}\,f_m\,\rho_m\,j\,\left|j\right| \tag{10}$$

$$f_m = f\left(Re_m,\, \frac{\epsilon}{D}\right) \tag{11}$$

$$f\left(Re,\, \frac{\epsilon}{D}\right) = \left\langle -4\log_{10}\left\{\frac{1}{3,7065}\frac{\epsilon}{D} \;-\frac{5,0452}{Re}\log_{10}\left[\frac{1}{2,8257}\left(\frac{\epsilon}{D}\right)^{1,1098} + \frac{5,8506}{Re^{0,8981}}\right]\right\}\right\rangle^{-2} \tag{12}$$

$$Re_m = \frac{\rho_m\, D\, |j|}{\mu_m} \tag{13}$$

$$\mu_m = \mu_o\,\alpha_o + \mu_w\,\alpha_w + \mu_g\,\alpha \tag{14}$$

$$j = j_o + j_w + j_g \tag{15}$$

where Re_m and μ_m are correspondingly the Reynolds number and dynamic viscosity of the mixture, μ_o, μ_w and μ_g are the viscosities of the phases, ϵ is the pipe roughness and j is the total superficial velocity.

2.2.3 Real gas

Because of the high pressures involved, the constitutive relation for the gas phase is considered as:

$$\rho_g = \frac{\gamma_g\, M_a}{\Lambda\, T}\,\frac{P}{Z} \tag{16}$$

where $\gamma_g = \dfrac{M_g}{M_a}$ is the gas specific gravity, M_g and $M_a = 28.966$ are respectively the molar masses of gas and air, Z is the gas compressibility factor (dependent on pressure, temperature and gas composition) and $\Lambda = 8.314\, m^2 s^{-2} K^{-1}$ is the gas universal constant.

2.2.4 Drift flux model

The superficial velocities for the liquid and gas phases are determined by using a drift flux model (Zuber and Findlay, 1965):

$$j_g = \alpha\,(C_d\, j + U_d) \tag{17}$$

$$j_l = (1 - \alpha\, C_d)\, j - \alpha\, U_d \tag{18}$$

$$j = j_l + j_g \tag{19}$$

where the parameters C_d and U_d depend on the local geometric and flow conditions (Bendiksen, 1984; Chexal *et al.*, 1992). In a general form, it will be assumed that $C_d = C_d\,(\alpha,\, P,\, j,\, \theta)$ and $U_d = U_d\,(\alpha,\, P,\, j,\, \theta)$, where θ is the local inclination angle of the pipe.

2.2.5 Black oil model

The vaporization term can be calculated by using the black oil model (McCain, 1990). According to this approximation, composition of the hydrocarbons is considered as constant. In this way, many properties corresponding to the phases at operating conditions can be estimated based on parameters at standard condition and a set of correlations depending on pressure, temperature and composition, which will be considered as locally and instantaneously valid.

The vaporization term can be expressed as:

$$\Gamma = -\frac{\rho_{g\,0}\,\alpha_o}{B_o}\left(\frac{\partial R_s}{\partial t} + \frac{j_o}{\alpha_o}\,\frac{\partial R_s}{\partial s}\right) \tag{20}$$

where $\rho_{g\,0}$ is the gas density at standard condition, B_o is the oil formation volume factor and R_s is the solution gas-oil ratio. It is worth noting that for $\Gamma > 0$ must be $\alpha_o > 0$, while for $\Gamma < 0$ must be $\alpha > 0$.

The densities of the oil and gas phases ρ_o, ρ_g and ρ_w can be calculated as:

$$\rho_o = \frac{\rho_{o\,0} + \rho_{g\,0}\, R_s}{B_o} \tag{21}$$

$$\rho_g = \frac{\rho_{g\,0}}{B_g} \tag{22}$$

where $\rho_{o\,0}$ is the oil density at standard condition and B_g is the gas formation volume factor. It can be shown that Eq. (16) reduces to Eq. (22) for constant gas specific gravity.

2.3 Conservation equations in terms of the state variables

The state variables of the multiphase flow model are α, α_o, P and j. The conservation equations (1), (2), (3) and (4) can be expressed in terms of the state variables and their derivatives assuming that R_s, ρ_g, ρ_o, ρ_w depend on pressure and temperature and that $j_g = j_g(\alpha, P, j, \theta)$:

$$\begin{aligned}&\rho_g \frac{\partial \alpha}{\partial t} + \left(\alpha \frac{\partial \rho_g}{\partial P} + \frac{\rho_{dg\,0}\, \alpha_o}{B_o} \frac{\partial R_s}{\partial P}\right) \frac{\partial P}{\partial t} + \rho_g \frac{\partial j_g}{\partial \alpha} \frac{\partial \alpha}{\partial s} + \left(j_g \frac{\partial \rho_g}{\partial P} + \rho_g \frac{\partial j_g}{\partial P} + \frac{\rho_{dg\,0}\, j_o}{B_o} \frac{\partial R_s}{\partial P}\right) \frac{\partial P}{\partial s}\\ &+\rho_g \frac{\partial j_g}{\partial j} \frac{\partial j}{\partial s} + \rho_g \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} = 0\end{aligned} \tag{23}$$

$$\begin{aligned}&\rho_o \frac{\partial \alpha_o}{\partial t} + \left(\alpha_o \frac{\partial \rho_o}{\partial P} - \frac{\rho_{dg\,0}\, \alpha_o}{B_o} \frac{\partial R_s}{\partial P}\right) \frac{\partial P}{\partial t} + \frac{\rho_o}{\alpha_o + \alpha_w} \left(j_o - \alpha_o \frac{\partial j_g}{\partial \alpha}\right) \frac{\partial \alpha}{\partial s} + \frac{\rho_o\, j_o}{\alpha_o} \frac{\partial \alpha_o}{\partial s}\\ &+ \left(j_o \frac{\partial \rho_o}{\partial P} - \frac{\alpha_o\, \rho_o}{\alpha_o + \alpha_w} \frac{\partial j_g}{\partial P} - \frac{\rho_{dg\,0}\, j_o}{B_o} \frac{\partial R_s}{\partial P}\right) \frac{\partial P}{\partial s} + \frac{\alpha_o\, \rho_o}{\alpha_w + \alpha_o} \left(1 - \frac{\partial j_g}{\partial j}\right) \frac{\partial j}{\partial s} - \frac{\alpha_o\, \rho_o}{\alpha_w + \alpha_o} \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} = 0\end{aligned} \tag{24}$$

$$\begin{aligned}&\rho_w \frac{\partial \alpha}{\partial t} + \rho_w \frac{\partial \alpha_o}{\partial t} - \alpha_w \frac{\partial \rho_w}{\partial P} \frac{\partial P}{\partial t} + \frac{\rho_w}{\alpha_o + \alpha_w} \left(j_o + \alpha_w \frac{\partial j_g}{\partial \alpha}\right) \frac{\partial \alpha}{\partial s} + \frac{\rho_w\, j_w}{\alpha_w} \frac{\partial \alpha_o}{\partial s}\\ &+ \left(-j_w \frac{\partial \rho_w}{\partial P} + \frac{\alpha_w\, \rho_w}{\alpha_o + \alpha_w} \frac{\partial j_g}{\partial P}\right) \frac{\partial P}{\partial s} - \frac{\alpha_w\, \rho_w}{\alpha_o + \alpha_w} \left(1 - \frac{\partial j_g}{\partial j}\right) \frac{\partial j}{\partial s} + \frac{\alpha_w\, \rho_w}{\alpha_o + \alpha_w} \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} = 0\end{aligned} \tag{25}$$

$$\frac{\partial P}{\partial s} = -\frac{4\, \tau_w}{D} + \rho_m\, g_s \tag{26}$$

3. METHOD OF CHARACTERISTICS

The method of characteristics is the natural numerical procedure for first-order hyperbolic systems. By an appropriate choice of coordinates, the original system of hyperbolic partial differential equations can be replaced by a system of ordinary differential equations expressed in the characteristic coordinates. Characteristic coordinates are the natural coordinates of the system in the sense that, in terms of these coordinates, differentiation is simpler (Ames, 1992).

3.1 Mathematical procedure

Consider the vector of state variables:

$$\mathbf{u} = [\alpha \;\; \alpha_o \;\; P \;\; j]^T \tag{27}$$

The partial derivative of **u** with respect to time and position are given by:

$$\frac{\partial \mathbf{u}}{\partial t} = \left[\frac{\partial \alpha}{\partial t} \;\; \frac{\partial \alpha_o}{\partial t} \;\; \frac{\partial P}{\partial t} \;\; \frac{\partial j}{\partial t}\right]^T \tag{28}$$

$$\frac{\partial \mathbf{u}}{\partial s} = \left[\frac{\partial \alpha}{\partial s} \;\; \frac{\partial \alpha_o}{\partial s} \;\; \frac{\partial P}{\partial s} \;\; \frac{\partial j}{\partial s}\right]^T \tag{29}$$

Consider also the quasilinear hyperbolic system of four equations and four unknowns:

$$\sum_{i=1}^{4} \left(a_{ji} \frac{\partial u_i}{\partial s} + b_{ji} \frac{\partial u_i}{\partial t}\right) + d_j = 0, \;\; j = 1, 2, 3, 4 \tag{30}$$

We now adopt the matrix notation:

$$\mathbf{A} = [a_{ji}]\,, \;\; \mathbf{B} = [b_{ji}]\,, \;\; \mathbf{d} = [d_j]$$

The system in matrix form becomes:

$$\mathbf{A}\,\frac{\partial \mathbf{u}}{\partial s} + \mathbf{B}\,\frac{\partial \mathbf{u}}{\partial t} + \mathbf{d} = 0 \tag{31}$$

Let us consider the matrix $\mathbf{T}$, defining a linear transformation. The system of equations, subjected to the linear transformation, has the form:

$$\mathbf{TA}\,\frac{\partial \mathbf{u}}{\partial s} + \mathbf{TB}\,\frac{\partial \mathbf{u}}{\partial t} + \mathbf{Td} = 0 \tag{32}$$

The new system of equations is equivalent to the original, in the sense that every solution of one is also a solution of the other. The linear transformation is used to develop a canonical (or normal) form. A convenient one is such that:

$$\mathbf{TA} = \mathbf{ETB} \tag{33}$$

where $\mathbf{E}$ is a diagonal matrix containing the eigenvalues $e_j, j = 1, 2, 3, 4$:

$$\mathbf{E} = \begin{pmatrix} e_1 & 0 & 0 & 0 \\ 0 & e_2 & 0 & 0 \\ 0 & 0 & e_3 & 0 \\ 0 & 0 & 0 & e_4 \end{pmatrix} \tag{34}$$

Under the assumption of Eq. (33), we may rewrite Eq. (32) as:

$$\mathbf{ETB}\,\frac{\partial \mathbf{u}}{\partial s} + \mathbf{TB}\,\frac{\partial \mathbf{u}}{\partial t} + \mathbf{Td} = 0 \tag{35}$$

To determine the elements of the matrix $\mathbf{T}$, we should determine the elements e_j of the matrix $\mathbf{E}$. For this purpose, Eq. (33) is analyzed in more detail:

$$\sum_{k=1}^{4} t_{j\,k}\, a_{k\,i} = \sum_{k=1}^{4} e_j\, t_{j\,k}\, b_{k\,i} \tag{36}$$

This equation can be modified to:

$$\sum_{k=1}^{4} \left(a_{k\,i} - e_j\, b_{k\,i}\right)\, t_{j\,k} = 0 \tag{37}$$

which is a system of homogeneous equations for $t_{j\,k}$. For a non-trivial solution to exist, the necessary and sufficient condition is:

$$det\left(\mathbf{A} - e_j\,\mathbf{B}\right) = 0 \tag{38}$$

The eigenvalues e_j are the roots of Eq. (38).

Considering a cartesian coordinate system in which the unit vector $\mathbf{i}$ represents the position s and the unit vector $\mathbf{j}$ represents the time t, the direction $ds_k\,\mathbf{i} + dt_k\,\mathbf{j}$, for which $ds_k/dt_k = e_k$, is known as a characteristic direction and the e_k are called the characteristics of the system.

By setting $\mathbf{ETB} = \mathbf{A}^*$, $\mathbf{TB} = \mathbf{B}^*$ and $\mathbf{Td} = \mathbf{d}^*$, the equation system (35) becomes:

$$\mathbf{A}^*\,\frac{\partial \mathbf{u}}{\partial s} + \mathbf{B}^*\,\frac{\partial \mathbf{u}}{\partial t} + \mathbf{d}^* = 0 \tag{39}$$

that can also be represented as:

$$\sum_{i=1}^{4} b^*_{ji}\left(e_j\,\frac{\partial u_i}{\partial s} + \frac{\partial u_i}{\partial t}\right) + d^*_j = 0, \quad j = 1, 2, 3, 4 \tag{40}$$

By inspection we can conclude that e_j has units of velocity and the directional derivative of the component u_i is defined as:

$$\frac{D_{e_j} u_i}{Dt} = e_j\,\frac{\partial u_i}{\partial s} + \frac{\partial u_i}{\partial t} \tag{41}$$

in which the index e_j of the directional derivatives refers to the characteristic direction defined by e_j.

Therefore, the system in normal form allows to represent the partial derivatives with respect to s and t as directional derivatives in the characteristic directions, thus simplifying the integration of the system of equations.

3.2 Characteristics and compatibility conditions

The matrixes **A** and **B** and the vector **d** are assembled from the system of equations (23) to (26):

$$\mathbf{A} = \begin{pmatrix} \rho_g \dfrac{\partial j_g}{\partial \alpha} & 0 & j_g \dfrac{\partial \rho_g}{\partial P} + \rho_g \dfrac{\partial j_g}{\partial P} + \dfrac{\rho_{dg\,0}\, j_o}{B_o} \dfrac{\partial R_s}{\partial P} & \rho_g \dfrac{\partial j_g}{\partial j} \\ \dfrac{\rho_o}{\alpha_o + \alpha_w} \left(j_o - \alpha_o \dfrac{\partial j_g}{\partial \alpha} \right) & \dfrac{\rho_o\, j_o}{\alpha_o} & j_o \dfrac{\partial \rho_o}{\partial P} - \dfrac{\alpha_o\, \rho_o}{\alpha_o + \alpha_w} \dfrac{\partial j_g}{\partial P} - \dfrac{\rho_{dg\,0}\, j_o}{B_o} \dfrac{\partial R_s}{\partial P} & \dfrac{\alpha_o\, \rho_o}{\alpha_w + \alpha_o} \left(1 - \dfrac{\partial j_g}{\partial j} \right) \\ \dfrac{\rho_w}{\alpha_o + \alpha_w} \left(j_o + \alpha_w \dfrac{\partial j_g}{\partial \alpha} \right) & \dfrac{\rho_w\, j_w}{\alpha_w} & -j_w \dfrac{\partial \rho_w}{\partial P} + \dfrac{\alpha_w\, \rho_w}{\alpha_o + \alpha_w} \dfrac{\partial j_g}{\partial P} & -\dfrac{\alpha_w\, \rho_w}{\alpha_w + \alpha_o} \left(1 - \dfrac{\partial j_g}{\partial j} \right) \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{42}$$

$$\mathbf{B} = \begin{pmatrix} \rho_g & 0 & \alpha \dfrac{\partial \rho_g}{\partial P} + \dfrac{\rho_{dg\,0}\, \alpha_o}{B_o} \dfrac{\partial R_s}{\partial P} & 0 \\ 0 & \rho_o & \alpha_o \dfrac{\partial \rho_o}{\partial P} - \dfrac{\rho_{dg\,0}\, \alpha_o}{B_o} \dfrac{\partial R_s}{\partial P} & 0 \\ \rho_w & \rho_w & -\alpha_w \dfrac{\partial \rho_w}{\partial P} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{43}$$

$$\mathbf{d} = \left(\rho_g \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} \quad -\frac{\alpha_o\, \rho_o}{\alpha_w + \alpha_o} \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} \quad \frac{\alpha_w\, \rho_w}{\alpha_o + \alpha_w} \frac{\partial j_g}{\partial \theta} \frac{d\theta}{ds} \quad \tau_w \frac{\mathcal{P}_m}{A} - \rho_m\, g_s \right)^T \tag{44}$$

From Eq. (38), the characteristics are given by:

$$e_1 = \frac{\partial j_g}{\partial \alpha}, \quad e_2 = \frac{j_o}{\alpha_o} = u_l, \quad e_3 = \infty, \quad e_4 = \infty \tag{45}$$

where u_l is the liquid velocity. If the parameters C_d and U_d are not dependent of α, i.e. $C_d = C_d(P, j, \theta)$ and $U_d = U_d(P, j, \theta)$ (as in the correlation developed by Bendiksen (1984)) we have, from Eq. (17):

$$\frac{\partial j_g}{\partial \alpha} = \frac{j_g}{\alpha} = u_g \tag{46}$$

where u_g is the gas velocity. For this particular case, the compatibility conditions take the form:

$$b^*_{11} \frac{D_g \alpha}{Dt} + b^*_{13} \frac{D_g P}{Dt} + d^*_1 = 0 \tag{47}$$

$$b^*_{21} \frac{D_l \alpha}{Dt} + b^*_{22} \frac{D_l \alpha_o}{Dt} + b^*_{23} \frac{D_l P}{Dt} = 0 \tag{48}$$

where the subscripts g and l are associated correspondingly to the gas and liquid characteristics and the coefficients b^*_{11}, b^*_{13}, b^*_{21}, b^*_{22}, b^*_{23} and d^*_1 are function of the state variables and dependent variables.

From Eq. (45), there exists an algebraically-double eigenvalue equal to ∞, resulting from the fact that the superficial velocities are related through algebraic relations and the pressure along the pipe can be calculated directly by integrating the mixture momentum equation. On this ground, the model is qualified as *mixed hyperbolic/parabolic*.

4. CONCLUSIONS AND PERSPECTIVES

A multiphase flow model suitable for petroleum production systems was presented and its characteristic values and compatibility conditions were calculated. The characteristics obtained are real, so that the formulation proves to be well posed.

The existence of two finite eigenvalues (velocities of the gas and liquid phases) allows to use the method of characteristics to solve the dynamic equations. The main advantage of the method of characteristics is the optimal determination of the time step, since the Courant-Friedrichs-Lewy (CFL) stability criterion is automatically satisfied.

In (Baliño *et al.*, 2007; Baliño, 2008) a single finite eigenvalue (gas velocity) was found for air-water systems and a numerical procedure based on a moving grid with the gas velocity was implemented. Based on the results obtained in this paper, it is possible to use a similar procedure based on a moving grid and, since $u_l < u_g$, propagate the characteristic with the liquid velocity from interpolated values at the solution at time t.

The model developed in this paper can be used to describe the flow in a riser of slowly varying inclination angle (vertical, catenary or lazy-wave configuration). In order to describe a petroleum production system, a pipeline connected to the bottom of the riser and a critical choke valve connected to the top of the riser must be added. As pressure variations with position in the pipeline are small, the pipeline subsystem can be modeled as a constant pressure cavity (Baliño *et al.*, 2007; Baliño, 2008), resulting a lumped parameter model for the gas pressure; this model, originally developed for air-water systems, can be extended for oil-water-gas systems. This is a work in progress.

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