NUMERICAL ASSESSMENT OF A HEURISTIC CONSTITUTIVE MODEL FOR ELASTOMERS

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Abstract. *The relationship between stress and strain in rubberlike materials is generally based on a hyperelastic model through a strain energy density function. The most common way to write the strain energy density functions is in terms of the principal strain invariants, but it can also be done in terms of the principal stretches. During the last years, many hyperelastic models have been proposed in the literature for both, compressible and incompressible rubbers. The aim of this paper is to implement a recently developed family of models in a commercial finite element software in order to compare its results against those obtained with some classical constitutive models, such as Mooney-Rivlin and Yeoh.*

Keywords*: Constitutive models, Hyperelasticity, Elastomers, Nonlinear Finite Elements*

1. INTRODUCTION

The increasing application of elastomeric materials in industry during the last decades made necessary to engineers and analysts to better understand the non-linear mechanical behavior of these materials. Since the first model proposed by Mooney (1940), several authors have proposed their own models for rubber-like materials based on phenomenological or molecular structure approach.

On the other hand, the inherently non-linear stress vs. strain relation of elastomers makes their characterization more complex, and virtually eliminates the possibility of analytical solutions, except for very simple cases. Because of this, the use of numerical methods such as the finite element method (FEM) becomes the best choice to perform complex structural analysis with rubberlike materials.

In this paper, two hyperelastic models developed by Hoss (2009) will be presented. In his work these models had shown to represent successfully the response of rubberlike materials under uniaxial extension, biaxial extension and pure shear; however no case considering uniaxial compression or non-homogeneous deformation were taken into account. With this in mind, the aim of this paper is to verify the performance of the models proposed by Hoss (2009) under circumstances other than those they were already tested for.

Data from three different experiments done with three different types of rubber samples will be used to compare these two proposed models with other classical ones such as the 5-terms Mooney-Rivlin (Rivlin and Saunders, 1951), 5-terms Yeoh (Yeoh, 1990) and 3-terms Yeoh (Yeoh, 1990). The tool used to perform the comparisons was the finite element software Abaqus 6.7, in which the studied models were implemented through FORTRAN subroutines.

In order to verify the validity of the implementation, experiments will be repeated numerically. Two non-homogeneous deformation cases will later be performed and their stress results will be compared between different models.

2. PROPOSED MODELS

Based on observations and performance of more than 40 distinct hyperelastic models, Hoss (2009) has developed a family of heuristic models aiming to accurately represent not only the stress vs. strain curve used in the calibration of the constitutive constants, but also provide good theoretical predictions for other deformation modes. The later is where many of the hyperelastic models reported elsewhere fail. The proposed models were built by combining the necessary functions to avoid such issues. These terms were identified in the various hyperelastic models studied after observing the common functions present in the models clearly performing well in fitting and showing good predicting capabilities.

2.1. The HMLSI (HM Low Strain Incompressible) model

The HMLSI model has a hybrid formulation, since it consists in the addition of an exponential term to the basic power-law model of Knowles (1977), responsible for improving the quality of fits and predictions at small strains (Yeoh, 1993). Its strain energy expression is based on the first strain invariant, only, and is fairly general since it allows particularization to simpler models (Hoss, 2009):

$$\Sigma = \frac{\alpha}{\beta}\left(1 - e^{-\beta(I_1-3)}\right) + \frac{\mu}{2b}\left[\left(1 + \frac{b(I_1-3)}{n}\right)^n - 1\right] \tag{1}$$

where α, β, μ, b, n and C_2 are the material constants.

2.2. The HMHSI (HM High Strain Incompressible) model

This is basically an improved version of the HMLSI model of Eq. (1), considering the influence of the second strain invariant, aiming to better capture the effect of stiffening at higher strain ranges. This is the term found responsible for providing a better sensitivity to the rapid stiffening at moderate and large stretches. The exponential term of Eq. (2) was not dropped, though, in order to keep the good prediction capabilities of the HMLSI model under small strains. The final expression for $\sum$ therefore considers both strain invariants:

$$\Sigma = \frac{\alpha}{\beta}\left(1 - e^{-\beta(I_1-3)}\right) + \frac{\mu}{2b}\left[\left(1 + \frac{b(I_1-3)}{n}\right)^n - 1\right] + C_2 \ln\left(\frac{I_2}{3}\right) \tag{2}$$

where C_2 is the additional constitutive constant.

2.3. Reference models

In order to analyze the performance of the HM models, it is interesting to compare the results obtained by them with those obtained when using some classical models. In this work, the results of the models in Eqs.(1) and (2) will be compared with the 5-terms Yeoh model and the 5-terms Mooney-Rivlin model. Their strain energy functions are given, respectively, by:

$$\Sigma = C_{10}(I_1-3) + C_{20}(I_1-3)^2 + C_{30}(I_1-3)^3 + C_{40}(I_1-3)^4 + C_{50}(I_1-3)^5 \tag{3}$$

$$\Sigma = C_{10}(I_1-3) + C_{01}(I_2-3) + C_{11}(I_1-3)(I_2-3) + C_{20}(I_1-3)^2 \tag{4}$$

3. NUMERICAL RESULTS

The four hyperelastic models given by Eqs.(1-4) were implemented in a commercial finite element software (Abaqus, 2007) through FORTRAN subroutines. Experimental data from different types of rubber were used to fit the constants of all models and, in order to guarantee the validity of the implementations, these experiments were repeated numerically. Once good agreement was verified, non-homogeneous deformation cases were analyzed and the results compared between different models.

3.1. Homogeneous deformation cases

Data from three types of homogeneous deformation experiments were used: uniaxial extension, uniaxial compression and pure shear (Marczak *et al.*, 2006). Uniaxial extension data were those obtained by Treloar (Jones *et al.*, 1960), uniaxial compression data were obtained by Amin *et al.* (2006) for samples of high dumping rubber, and pure shear data were obtained for Natural Rubber (Marczak *et al.*, 2006).

Figures 1, 2 and 3 show the numerical results obtained by simulating each test. In all cases it is observed a good agreement of the proposed models with the experimental data.

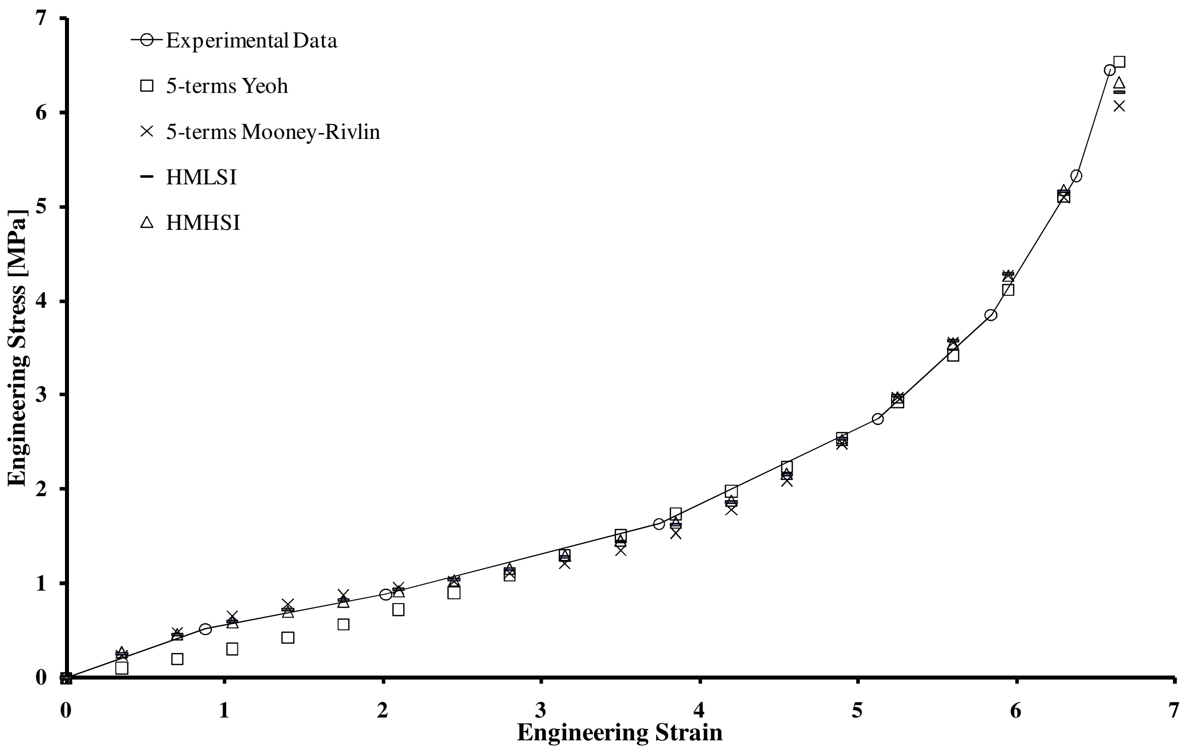


Figure 1. Results for numerical simulation of uniaxial extension testing.

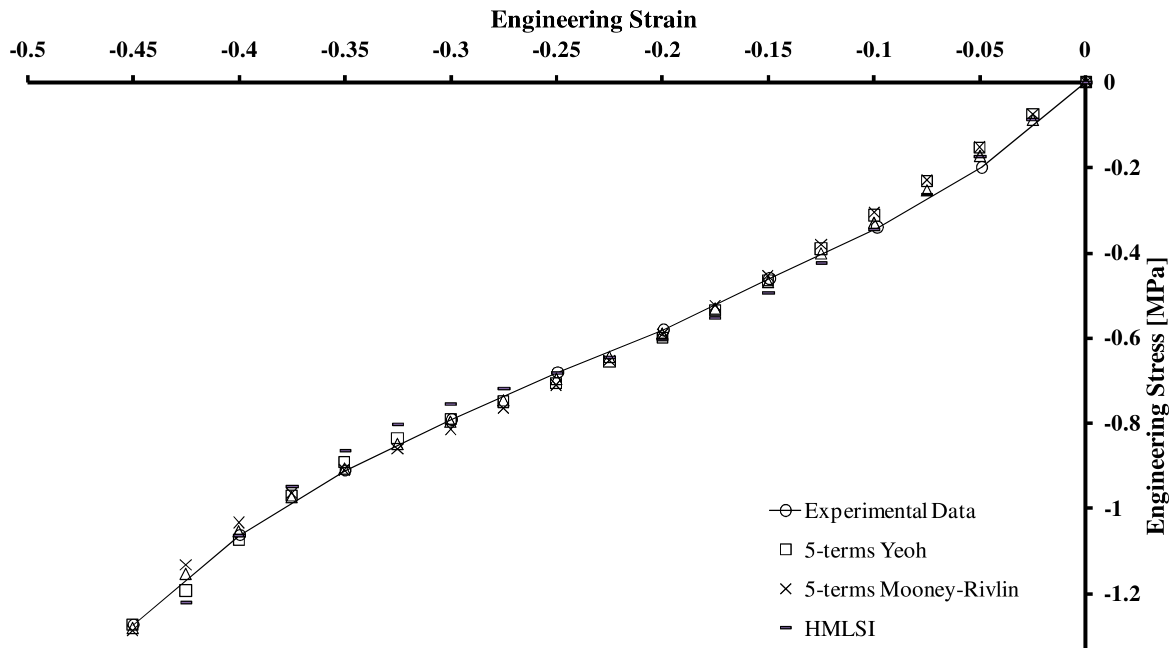


Figure 2. Results for numerical simulation of uniaxial compression testing.

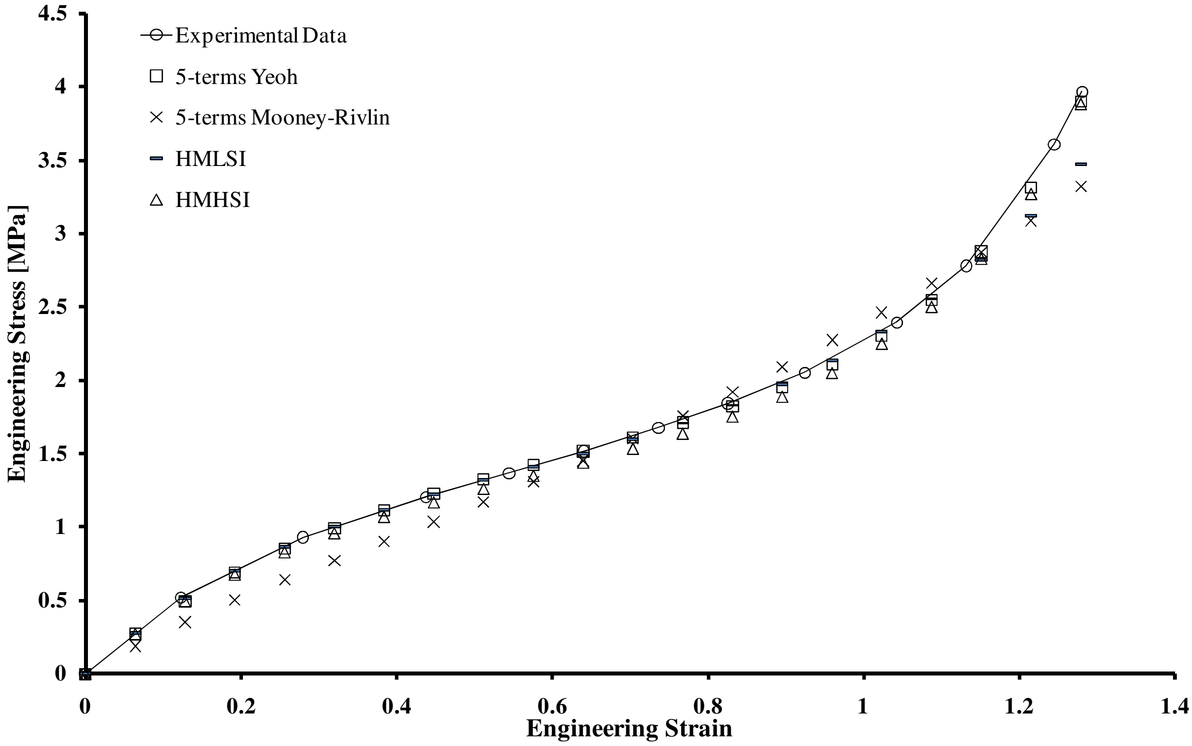


Figure 3. Results for numerical simulation of pure shear testing.

It can be noted that all models fitted well to the experimental data, from where one can conclude that the subroutine implementations were performed correctly.

Although referring to simple cases, these numerical tests already showed that not all model can reproduce accurately the softening behavior at moderate strains and stiffening characteristic of large strains as well.

3.2. Non-homogeneous deformation cases

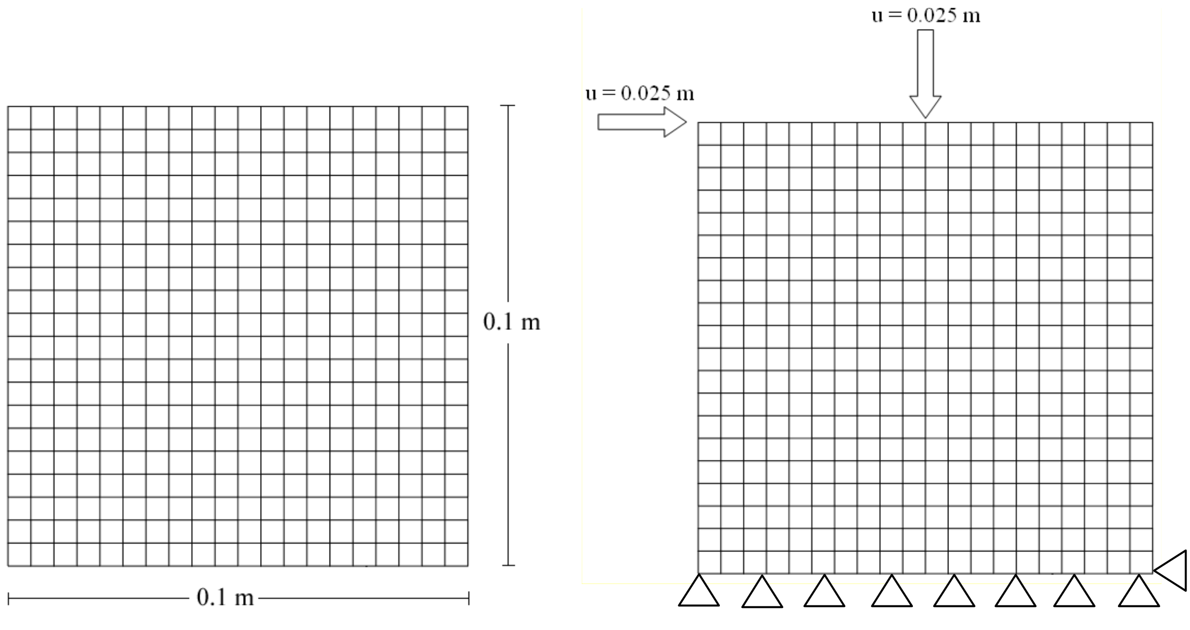
Once verified good accuracy of the HM models under homogeneous deformation cases, their performance under non-homogeneous deformations were also checked. Two different geometries subjected to more complex boundary conditions were analyzed.

The first case refers to a plane stress analysis of a square block subjected to simple shear combined to lateral compression. Figure 4 shows both the geometry used and boundary conditions applied. Essentially, this case depicts an engine mount subject to displacements imposed to its upper side. Shear and compression displacements were applied simultaneously along 20 load steps. The material constants used were the same of the uniaxial compression test in Fig.2.

None of the HM models presented any kind of convergence problems during the simulations. Surprisingly, however, both the 5-terms Yeoh (Yeoh, 1990) and the 5-terms Mooney-Rivlin (Rivlin and Saunders, 1951) models have failed to achieve convergence during the final steps. This made necessary to select a third hyperelastic model to be used as reference. The 3-terms Yeoh (Yeoh, 1990) model was chosen for its well known stability, in spite of the lower number of constants than in Eq. (3).

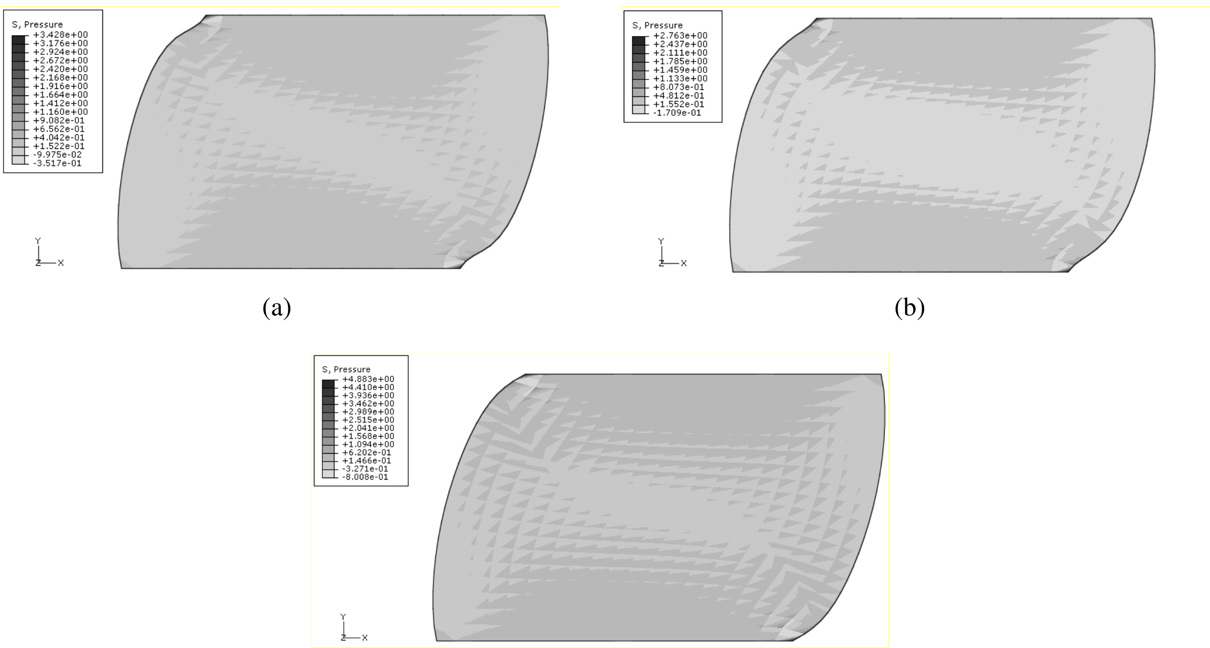
$$\Sigma = C_{10}(I_1 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3 \tag{5}$$

Figure 5 shows the fringe plots for hydrostatic pressure obtained by the three models. The good qualitative and quantitative agreement between the three models is evident. Figure 6 illustrates the isolines of shear stress for this case. Once again, good agreement is verified between all models.



(a) (b)

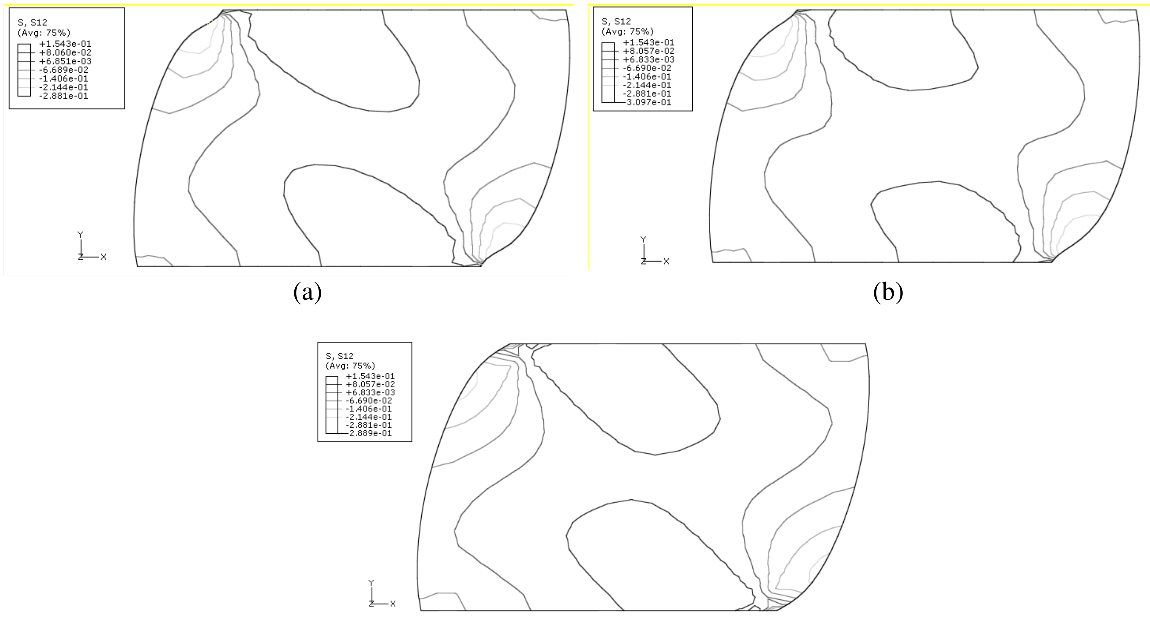
Figure 4. Square block under shear and compression. (a) block dimensions and (b) boundary conditions.



(a) (b) (c)

Figure 5. Hydrostatic pressure plots for the shear/compression test.
(a) 3-terms Yeoh model; (b) HMLSI model; (c) HMHSI model.

An interesting point to highlight is that, in his work, Hoss (2009) has not tested the HM models under compressive loads, and the results of Figs.5-6 confirm the good performance of these models under this type of deformation.



(a) (b)

(c)

Figure 6. Shear stress bands for the shear/compression test.
(a) 3-terms Yeoh model; (b) HMLSI model; (c) HMHSI model.

The other non-homogeneous deformation case analyzed is illustrated in Fig. 7. It refers to a folded rubber band subjected to very large displacements, something impossible to be simulated with the previous case due to geometry limitations. Plane stress was assumed, and the prescribed displacement conditions were applied in 40 equally spaced steps.

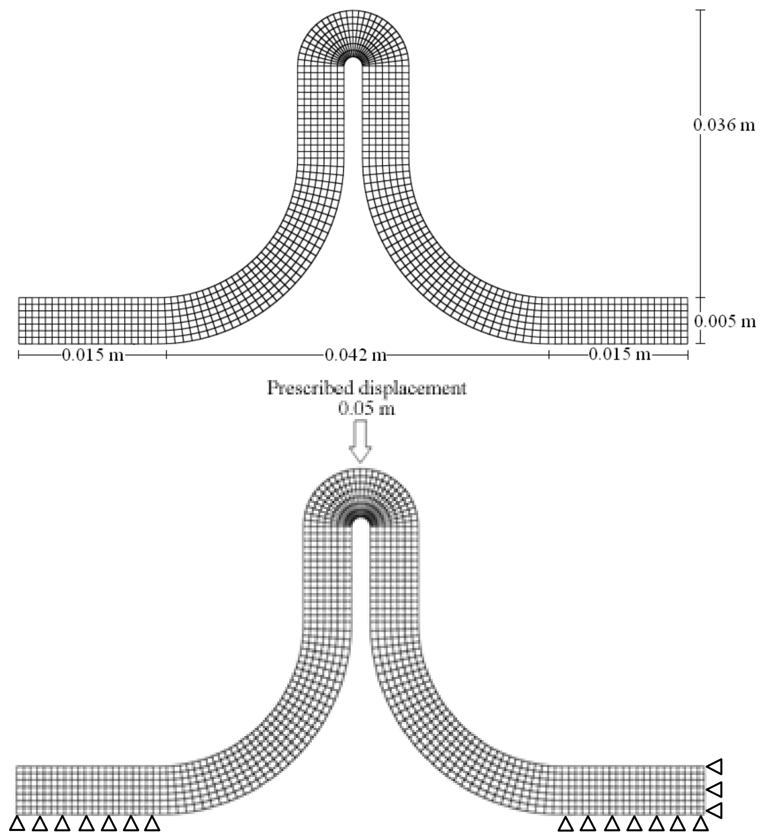
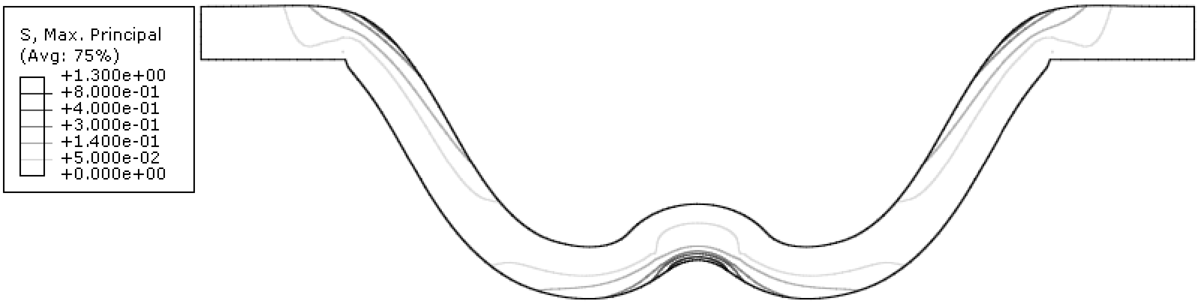
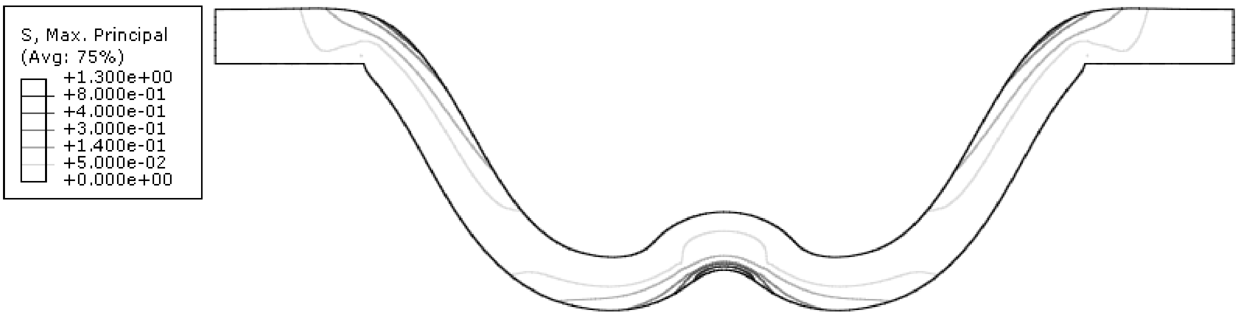


Figure 7. Rubber part tested. (a) part dimensions and (b) boundary conditions.

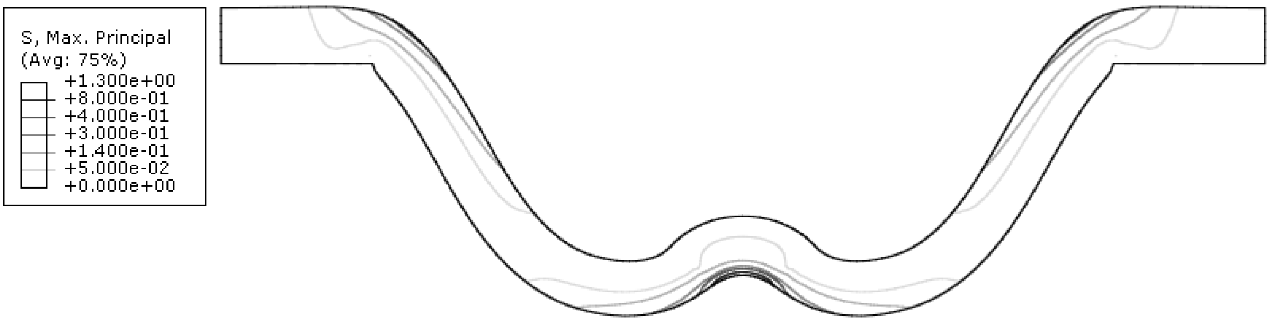
The analysis was performed considering the following models: 3-terms Yeoh (Yeoh, 1990), 5-terms Yeoh (Yeoh, 1990), 5-terms Mooney-Rivlin (Rivlin and Saunders, 1951), HMLSI (Hoss, 2009) and HMHSI (Hoss, 2009). The constitutive constants used were calibrated for uniaxial tensile testing (see Fig. 1). Figure 8 shows the deformed structure with the isolines of maximum principal stress. Once again, the 5-terms Mooney-Rivlin model failed to converge (Fig.8b) in the last steps. All other simulations generated similar values for the peak stress, while the stress distribution patterns obtained agree very well to each other.



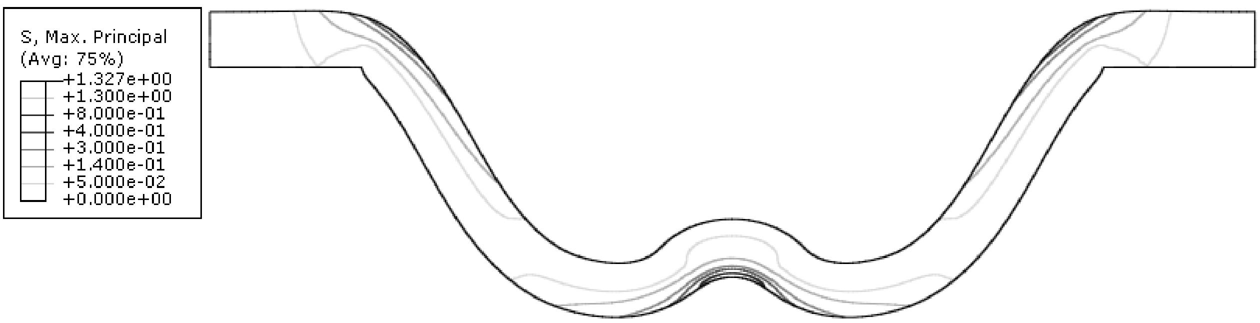
(a)



(b)



(c)



(d)

Figure 8. Isolines of maximum principal stress.
(a) 3-terms Yeoh; (b) 5-terms Yeoh; (c) HMLSI; (d) HMHSI.

4. CONCLUSIONS

The two new hyperelastic models proposed by Hoss (2009) to represent behavior of incompressible isotropic rubberlike materials were tested and compared to other well known models. The strain energy equations for both, the HMLSI and the HMHSI models were implemented in a finite element software and three different experiments were numerically simulated in order to verify the validity of the implementation. In all cases analyzed (uniaxial extension, uniaxial compression and pure shear) numerical results agreed satisfactorily with the experimental data. In particular, the performance of the new models was tested for the first time under uniaxial compression regime, and the results are very convincing.

Two cases of non-homogeneous deformation were also analyzed to check the performance of the new models in more complex cases. Stress and hydrostatic pressure bands obtained were compared with Yeoh and Mooney-Rivlin models. In the first case, a block subjected to simultaneous shear and compression, the results for hydrostatic pressure of the proposed models very good agreement with results obtained with the 3-terms Yeoh model, the same occurring for the in-plane shear stress results. In the second case, referring to a rubber component under rather large displacements, the new models generated maximum principal stress in excellent agreement to the Yeoh model.

The results presented herein confirm that the proposed models are very promising and, although further investigations are still needed, the HM models have already proved their potential as constitutive models for incompressible rubber-like and biologic materials.

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