A GEOMETRY SMOOTHING PROCEDURE FOR OPTIMAL DESIGNS IN DIFFUSIVE PROBLEMS

Anflor, C.T.M.
Universidade Nacional de Brasília
Área Especial 02 lote 14 – Setor Central, Caixa Postal 8114
Gama – DF – Brasil – 72405-210
anflorgoulart@yahoo.com.br

Marczak, R.J.
Universidade Federal do Rio Grande do Sul
Mechanical Engineer Department - GMAP-PROMEC
Rua Sarmento Leite 425
Porto Alegre – RS – Brasil – 90430-131
rato@mecanica.ufrgs.br

Abstract. *This paper aims to compare the final designs obtained through topology optimization methods with and without smoothing procedures. These procedures are introduced between intermediary iterations in a topological optimization scheme, in order to simplify and smooth the irregular boundaries commonly generated in such cases. Jagged edges occur when the material is removed from the original domain, laying holes over the design space. When these holes intercept the original geometry, indentations may appear, rendering the edge a "map-like" aspect. The boundaries are regularized using Bézier interpolation. An automatic procedure is responsible for detecting which edges need smoothing. In the present approach, the boundary element method is used to solve the governing equations, but the proposed scheme can be used in other numerical methods. Results for several cases of linear heat transfer are presented and discussed.*

Keywords*: shape optimization, boundary element method, topological derivative, Bézier smoothing.*

1. INTRODUCTION

Classical topology optimization based on density (SIMP) approaches (Dyck et al. 1994, Byun et al. 1999), often used for elasticity problems presents some well known drawbacks. The boundaries of final and intermediary topology generally results a see-saw aspect, which requires post-processing. Similar problems are found in evolutionary structural optimization (ESO) schemes like topological derivatives based on the boundary element method (BEM). If by one side BEM optimization methods does not deal with intermediary material densities (Marczak, 2007), on the other hand severely jagged boundaries occur due to the way the material is removed from the domain (Anflor and Marczak, 2009). In order to overcome these issues, a methodology for regularization of the resulting geometries is implemented in the present work. The idea consists in smoothing the boundary during or at the end of the process optimization, regardless shape and/or topology optimization are simultaneously performed. The boundary regularization is accomplished by Bézier interpolation on those portions of the boundary which were modified during the process. Segments belonging to the original design area are not modified. Figure 1 illustrates a BEM mesh smoothed by Bézier interpolation.

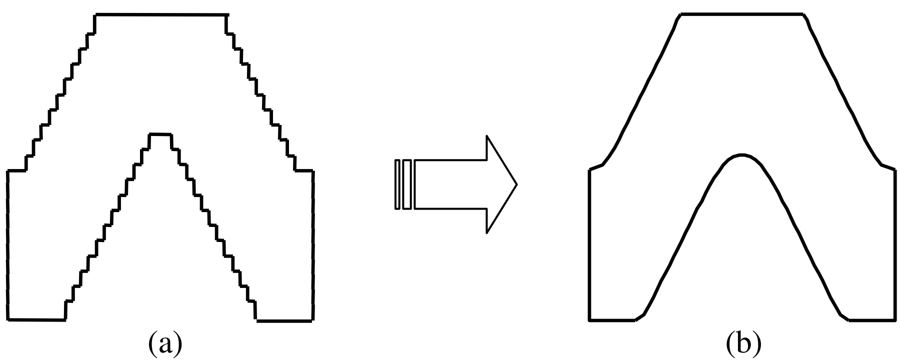


Figure 1 – Example of boundary smoothing through regularization. (a) Original BEM result. (b) Beziér-smoothed result.

The present work outlines the computational scheme used to extract those segments of the boundary which will be smoothed and illustrates the feasibility of the proposal by a single heat transfer problem. Results obtained are compared with the ones obtained by more traditional optimization methods.

2. OPTIMIZATION METHOD

The main idea of an optimization method is to identify unnecessary portions of a structure and eliminate them progressively using a suitable numerical method. Once a numerical solution of the problem is available, fluxes and potentials can be used to estimate the sensitivities of the problem to changes in a set of design variables. The material is removed from the domain in those areas showing the lowest sensitivities. The material removal procedure may differ significantly from SIMP and ESO methods. The present work compares results from ESO methods only, solved using finite elements (FEM) and BEM as numerical methods.

2.1. Evolutionary structural optimization using FEM

Xie and Steven (1993) developed a method based on the idea of elemental removal from the original mesh. Essentially, this is a hard-kill ESO scheme. This method was also implemented by Chu et al. (1996 e 1997), Li et al. (1999) and Li et al. (2004). Figure 2 depicts the results obtained using the method, where the initial design encompasses the whole FEM mesh. After a given stopping criteria is achieved, the final result is interpreted by the boundary of the remaining active elements.

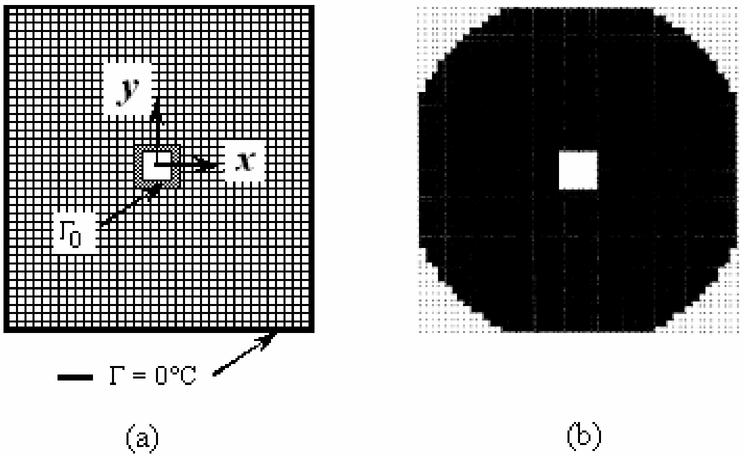


Figure 2 – Example of Shape optimization using ESO+FEM (Li et al. 1999): (a) initial mesh; (b) final result.

2.2. Topological derivative optimization using BEM

A topological derivative (DT) formulation for Poisson equation is used throughout this work (Céa et al, 2000, Feijóo et al. 2003, Novotny et al., 2003). The total potential energy is used to estimate the sensitivity of a cost function when the topology of the original analysis domain is changed. In this case the objective function is implicit. In this work, the method is implemented under a BEM structure, therefore requiring boundary-only discretization. Figure 3 illustrates the idea of the topological derivative optimization method as implemented by Marczak (2007) and Anflor and Marczak (2009), and used herein to test the smoothing procedures.

3 BÉZIER CURVES

As aforementioned, an aspect which generally displace topology optimization results from ready-made manufacturing drawings is the irregular aspect of the geometry obtained. This requires re-designing and re-analysis of the re-designed geometry, and adds a further step in the development cycle. Among the popular techniques to deal with irregular geometries one can find Bezier/spline interpolation, Douglas-Peucker regularization, polygon simplification, to cite a few (Cardoso and Marczak, 2006). This work uses the Bézier curves to smooth the topology resulting from the optimization process.

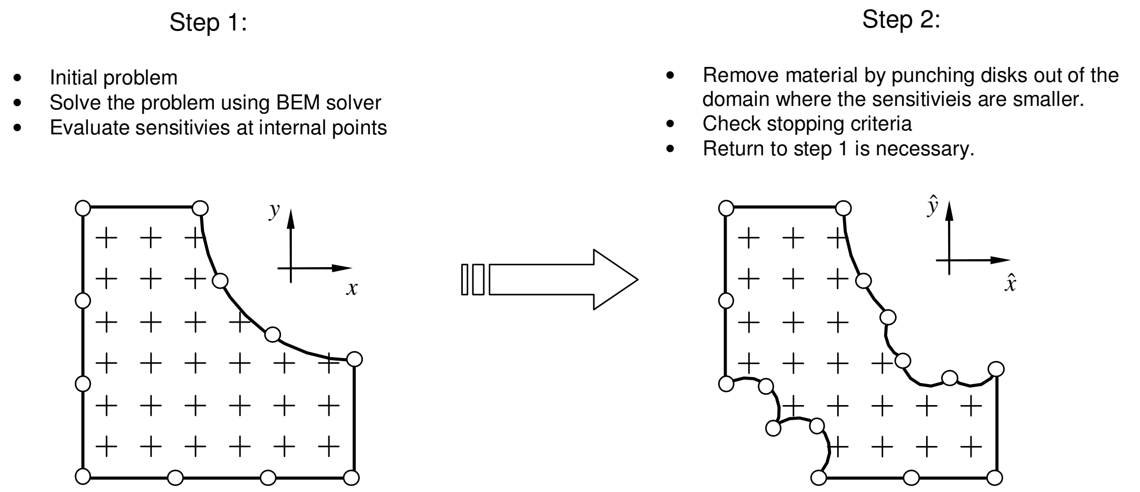


Figure 3 - BEM iterative scheme for material removal.

Bézier curves belong to a very well known family of interpolation curves (Newman and Sproul, (1982). The interpolation function $P(u)$ is defined by a n+1 control points p_i:

$$P(u)=\sum_{i=0}^{n} p_i B_{i,n}(u) \tag{1}$$

where $B_{i,n}(u)$ are blending functions (Bernstein polynomials) given by

$$B_{i,n}(u)=C(n,i)\,u^i(1-u)^{n-i} \tag{2}$$

and

$$C(n,i)=\frac{n!}{i!(n-i)!}$$

are binomial coefficients. Equation (1) can be expressed in its parametric form by splitting the abscissa and ordinates:

$$\begin{aligned} x(u)&=\sum_{i=0}^{n} x_i B_{i,n}(u) \\ y(u)&=\sum_{i=0}^{n} y_i B_{i,n}(u) \end{aligned} \tag{3}$$

where x_i and y_i are the coordinates of the control points. These points are responsible to control and adjust the shape of the curve, with the parameter u varying from 0 to 1. Further details can be found in Harrington (1983).

The main issue in applying Bézier (or any other smoothing method, for that matters) functions to the boundary of a domain under optimization is to determine which portions of it were modified by the process, and therefore need smoothing. Some segments of the boundary must not be changed because they still belong to the original design, and consequently were not distorted by material removal, or contain important data regarding the boundary conditions. Furthermore, one cannot treat the whole boundary as a single Bézier curve, since this would produce unwanted rounded corners. In order to overcome this problem a routine was developed in the present work to identify which curves should be smoothened during the iterative optimization process. This routine takes action inside the optimization loop, after each material removal step. Figure 4 depicts schematically the algorithm of identification and smoothing used.

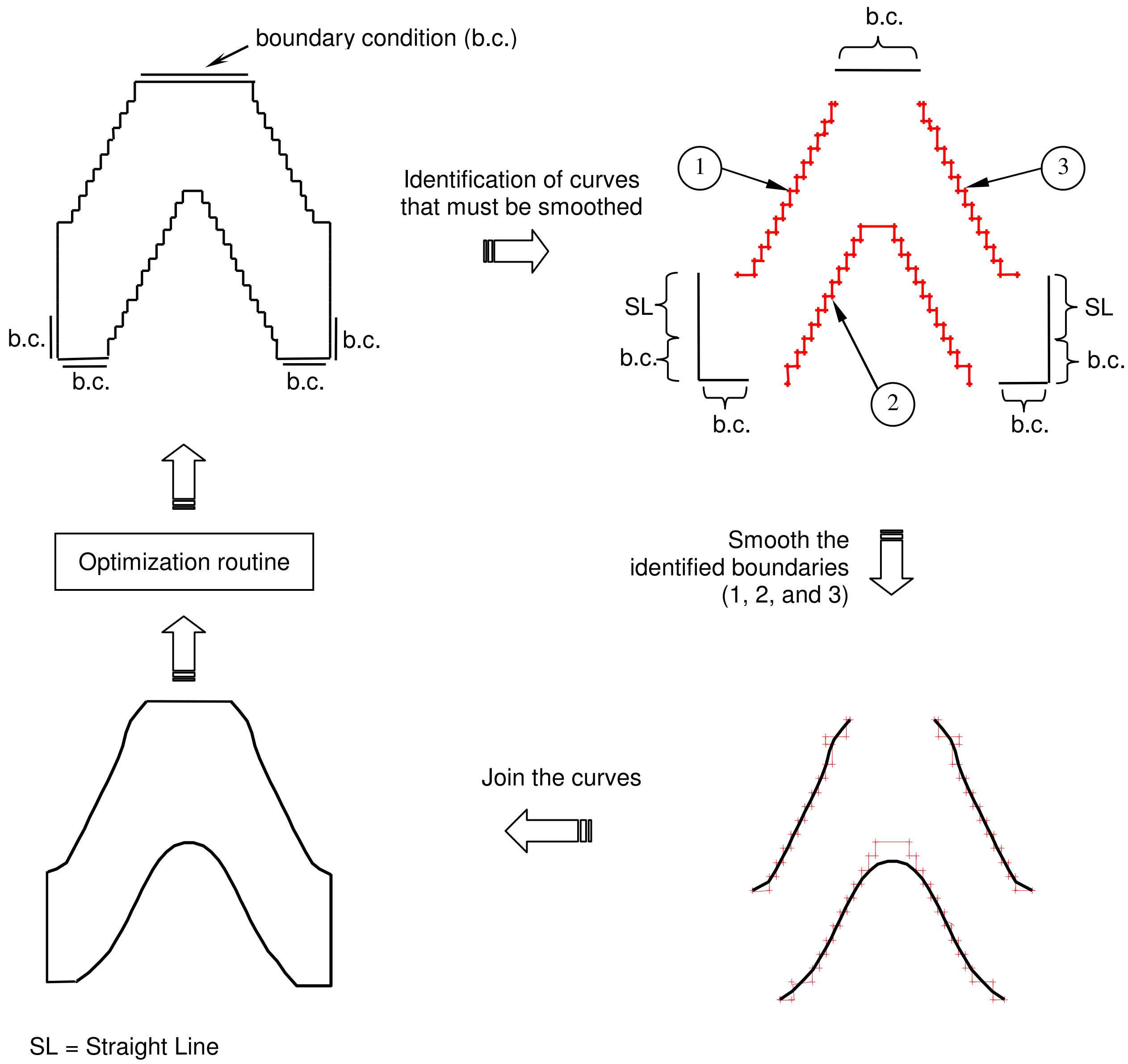


Figure 4 – Scheme for curve identification and smoothing.

4. NUMERICAL EXAMPLE

A case analyzed previously (Anflor and Marczak, 2006) is revisited here in order to assess the developed algorithm. This case refers to linear steady state heat transfer problem, where the material is removed to optimize the potential energy. The iterative process is halted when a target material volume fraction (A_f / A_0, where A_f and A_0 are the final and initial volume, respectively) is achieved. This provided a simplified criterion to compare the topologies generated using Bézier curves with the raw results obtained without any boundary regularization.. Linear discontinuous boundary elements integrated with 4 Gauss points are used in all cases.

4.1 Heat conductor with Neumann boundary conditions on the cavities

A rectangular 20×30 domain subjected to a prescribed temperature (T_1 = 373K) on its left edge and convection boundary conditions (T_0 = 298K and h_0 = 5.677 W/m^{2}K) on the remaining ones is to be optimized (Fig.5). The isotropic material used is Aluminum (k = 236 W/mK). For this case, Neumann boundary conditions were prescribed on the cavities open during the optimization process.

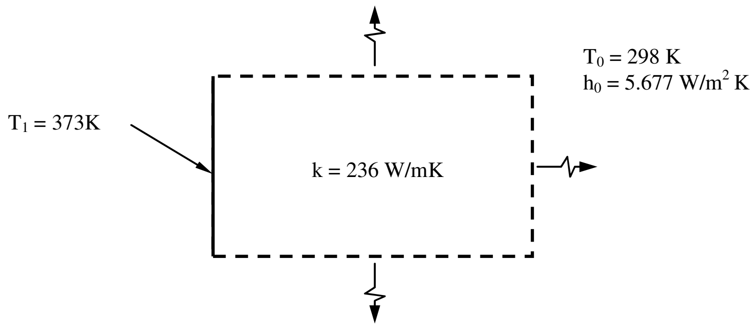
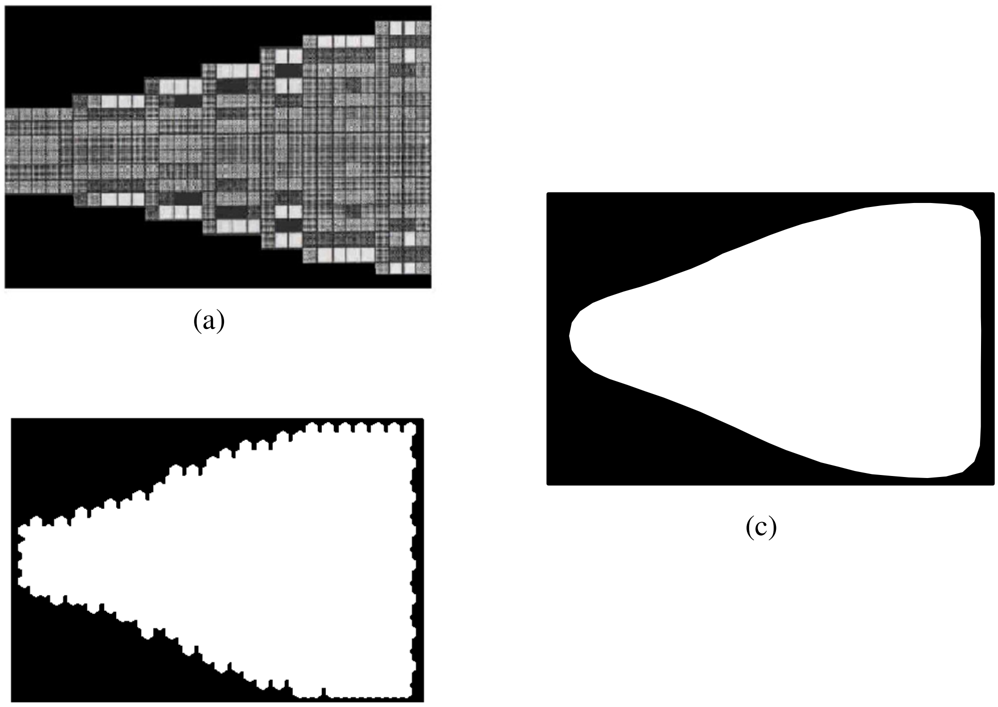


Figure 5 - Design domain and boundary conditions for the heat conductor.

Park (1995) also solved this problem by using ESO techniques and the FEM. Figure 6a compares the results obtained by Park (1995), with the raw geometry obtained in the present work without any regularization (Fig.6b) and using the Bézier smoothing (Fig.6c). All cases were optimized until a final volume of 30% of the original value is reached. Besides the good agreement between all three solutions, it is evident that the jagged aspect of the results in Fig.6b disappeared after the boundary regularization.



(a)

(b)

(c)

Figure 6 - Final topologies: (a) FEM (Park, 1995); (b) Present work (BEM) without regularization; (c) Present work (BEM) after Bézier regularization.

Figure 7 shows a few intermediary steps until the final result of fig.6c is obtained. The final and intermediary topology resulted much more regular than the original results of Anflor and Marczak (2006).

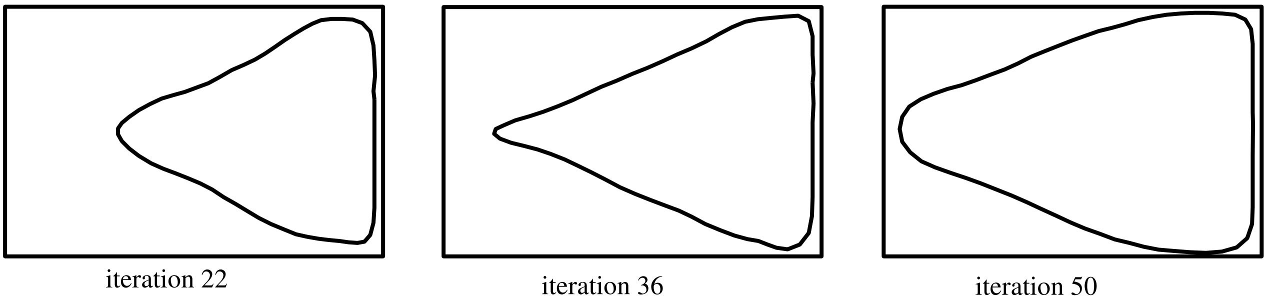


Figure 7 – Intermediary solutions.

The mean value of the normal heat flux on the left side edge of the plate was chosen to further compare the original and the smoothing algorithms. These values are plotted in Fig.8 for each iteration in the optimization process. It is worth to note that the Bézier smoothing did not modified significantly the overall behavior of the process, and therefore it is not destroying or modifying any requisite for a successful optimization.

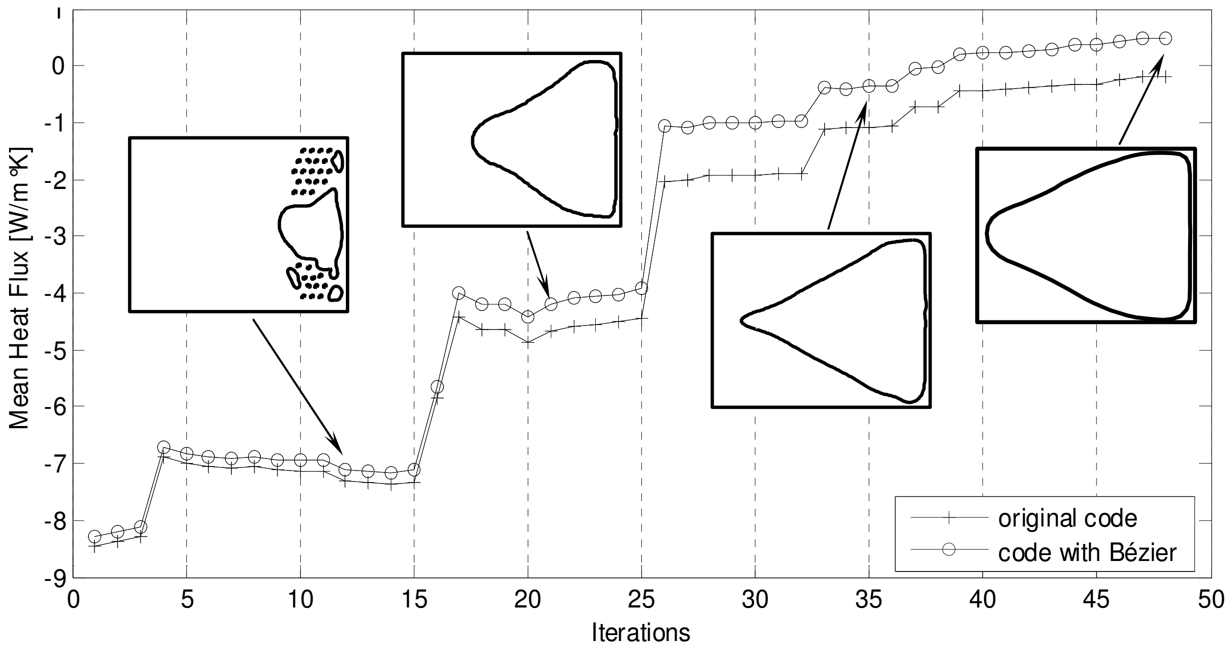


Figure 8 - Evolution history of heat flux along the left side edge.

5. ACKNOWLEDGEMENTS

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6. CONCLUSIONS

The presented work uses the DT (Topological Derivative) to calculate the sensitivity of the domain when a small hole is introduced. Those portions with lower sensitivity have material removed by an optimization code. This procedure results in an irregular shape due to the way of the material is punched out. The objective of this paper was attained with the development of an algorithm of smoothness. The case studied here had its intermediary geometries smoothed as the iterative optimization process had evolved. The smoothness algorithm allowed avoiding a post-processing procedure in order to eliminate those sawtooth shape boundaries. The particularities of the joint of DT and

BEM were preserved, such as, no mesh dependence and low computational cost. This new methodology presents a suitable procedure without lost accuracy turning the present optimization code (BEM + DT + SMOOTH) attractive.

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