CONTROLLED BRAKE SYSTEM DESIGN FOR FOUR-WHEELED VEHICLES USING A SIMPLIFIED DYNAMIC MODEL

Leonardo Bartalini Baruffaldi, leobart@fem.unicamp.br
Faculdade de Engenharia Mecânica, Universidade Estadual de Campinas
Rua Mendeleyev, 200 – Cidade Universitária Zeferino Vaz - Campinas – SP – Brazil

Marcelo Augusto Leal Alves, malalves@usp.br
Departamento de Engenharia Mecânica, Escola Politécnica da Universidade de São Paulo
Av. Prof. Mello Moraes, 2231 – Cidade Universitária – São Paulo – SP - Brazil

Abstract. *Nowadays, it's usual to employ complex multi-body systems to analyze road vehicle behavior, and to design control systems to conform its response. However, as models gain in complexity and reality, the computational costs grow as well. This paper shows a simplified four-wheel vehicle mathematical model, which is used to the design a lateral/longitudinal attitude brake based control system. The control strategy, developed in MATLAB, is then used to act over a more sophisticated ADAMS vehicle model and the results confirm that, even a simplistic model can yield satisfactory results at small processing times.*

Keywords*: vehicle dynamics, control, multi-body dynamics*

1. INTRODUCTION

The development of vehicular stabilization devices, e.g. ABS, ESP and EBD, past the last twenty years has been focused on brake system actuators. The control actions on the tires' rotation benefits from the high amount of power delivered by hydraulic brakes and the recent development of low weight and faster valves has considerably improved the response time and the steady-state error of these equipments. New stability control systems, however, are facing two basic troubles regarding basic hydraulic circuits: response delay – which may cause system instability – and fluid leakage. With the income of hybrid and electric vehicles some researches are being made towards the development of drive-by-wire and brake-by-wire, systems that do not rely on mechanical links between the commands done by the driver, e.g. accelerating and turning the steering wheel, and the actuation components, e.g. the throttle plate and the steering mechanism.

According to Gombert (2006), new vehicle control technologies shall not only analyze surrounding traffic and environment conditions but also deliver active assistance to the driver. From the point of view of a handful of automakers, the by-wire technologies are the answer to this thread for they provide an effective way of integrating all vehicle's fundamental systems, like brakes, steering and suspension. The final goal is to get a car that can be fully adjustable to the driver's taste and yet safe in dangerous situations. For the past decade, the automotive industry has been introducing new electronic technologies, hoping to get the consumers used to the by-wire concept (Bretz, 2001). Nowadays, throttle-by-wire and steering-by-wire are at the production line of several carmakers. Indeed, by-wire technologies match with the auto-industry tendency to produce more and more hybrid and electric vehicles to replace internal combustion mechanically linked cars.

However, the substitution of hydraulic brake lines for wired control dramatically changes the dynamical behavior of the discrete-time control made by the micro controllers used nowadays into ABS or ESP modules. The natural lag of fluidic systems allows for slower computation times but when electronic actuators make the control loop, a new perspective opens up, allowing designers to really command every move. Due to the non-linearities that exist in vehicle dynamics modeling and to the wide range if operational conditions commercial vehicles are subjected to, the car's control systems must adapt itself to the environment, i.e., the control gains must change accordingly to yield the maximum performance. This calls up to a computational efficient model, which can be solved several times in a short period.

For the last ten years many authors have addressed the problem of modeling and controlling an electro-mechanic brake caliper (Hartmann et al., 2002, Krishnamurthy et al., 2005, Klode et al., 2006, Manzie et al., 2008) and some others developed new control techniques of the whole vehicle's lateral dynamics (Antonov, 2008, Park and Ahn, 1999) or of tire-only behavior (Baslamisli et al., 2007). All of these, however, rely on classic lateral dynamics models like the ones presented in Genta (1997), Rill, (2007) or Gillespie (1992) that disregard the differences between left and right wheels' slip angles and do not provide a easy way of considering the load transfer due to curving maneuvers. These models, usually called "bicycle models," are interesting as a first approach, but they are not proper for extracting the maximum performance of tires' resistive forces. This paper presents a more complete yet simplified model of a vehicle negotiating an arbitrary curve, aiming precision and ease when solving the differential equations. A sample pole placement control strategy is then developed based upon this model and the system's ability to deal with non-linear situations is tested via multi-body dynamics analysis.

2. TIRE/ROAD INTERACTION

When it comes to the analysis of a ground vehicle dynamics, the utilization of an adequate tire mathematical model is very important. Tires represent the link between the vehicle and the ground, collecting and responding to geometric information the affects the whole behavior of the car. Traditionally, tire models can be classified into two main groups:

1. *Semi-empiric tire models*: are adjusted curve representations of experimental results. These curves generally are graphically given by a function of the *slip rate*, the ratio from the tire's peripheral speed to its center of mass velocity. The main advantage of this kind of model is that it accelerates numerical simulations, for its mathematical form is just a polynomial. The most widely known semi-empirical tire model is Pacejka's Magic Formula (Pacejka, 2006);
2. *Numerical or structural tire models*: are results of structural modeling of tire/road interface, usually using finite element methods. Have much better transient and frequency responses at high computational costs. Recent growth in processing capacities and new advances in theoretical structural modeling, however, are encouraging their use in multi-body vehicle dynamics simulations.

Figure 1 shows forces and moments acting on the tire/road contact area and defines the coordinate system used to described tire's movements: z-axis points up, normal to the contact plane; x-axis points towards the vehicle front edge and y-axis is the cross-product of x- and z-axis unity vectors.

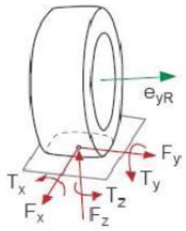

Figure 1. Efforts at tire/road contact patch (adapted from Rill, 2007).

Beside the forces acting on the three directions, torque components T_z and T_y are specially important to the vehicle dynamic and are called, respectively, *auto-aligning torque* and *rolling resistance torque*. Both result from the displacement of the action line of the contact forces. As it is presented on Genta (1997), a standstill tire or that rolls without slipping presents the normal force F_z aligned with its geometric center (right tire in Figure 2). When this same tire is subjected to a torsional moment that imposes the motion, the elastic characteristics of the tire contact patch act to alter the position of F_z application point (left situation in Figure 2). It is generated, then, a moment associated with F_z that acts parallel to the y-axis and opposes the driving torque and this is the rolling resistance torque. The auto-aligning torque is born in a similar way, when the tire is subjected to a moment that imposes it to spin around the z-axis (and that is the case of a cornering vehicle, when the steering bars force the wheels to turn).

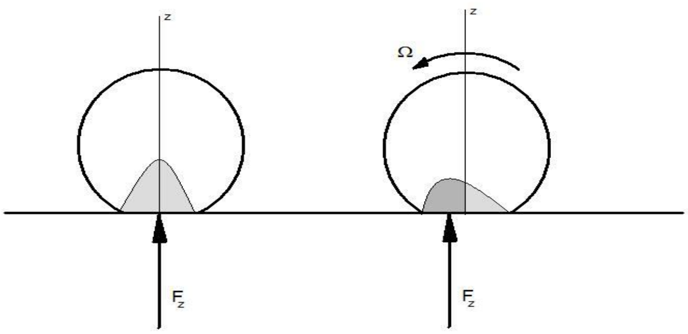

Figure 2. Displacement of the vertical force line of action. The shaded areas represent the contact pressure distribution.

Empiric data shows that the forces generated by the tires in longitudinal direction can be modeled as a function of the longitudinal slip or, mathematically

$$F_x = s_x F_z = \frac{u - R\Omega}{u} F_z \tag{1}$$

where s_x is the longitudinal slip, u is the wheel's geometric center longitudinal velocity seem from the car's coordinate frame (non-inertial), R is the effective rolling radius, and Ω is the wheel's angular velocity. Comparing carmaker's nominal data with simulation results for longitudinal dynamics, given in Baruffaldi et al. (2008), one can observe that adopting a simpler force expression like $F_x = \mu F_z$, with constant friction coefficient μ, do not imply in degraded results if the solution is for non-extreme situations.

Lateral forces have a similar formulation, as a function of tire's slip angle α, which is also a measure of tire slip velocity. For small values of α, the force is given by

$$F_x = C_\alpha \alpha F_z \approx C_\alpha \frac{v}{u} F_z \qquad (2)$$

where v is tire's lateral velocity (also from the car's coordinate frame) and C_α is an approximately constant value known as lateral (or cornering) stiffness. For numerical implementation, the formulae in Eq. (1) and (2) are singular at $u = 0$. This anomaly can be overcome by using a very small virtual speed ε added to u at the denominators as suggested in Rill (2007).

3. VEHICLE DYNAMIC MODEL

Considering a vehicle with a rigid body negotiating an arbitrary plane curve without turning its wheels, Figure 3, wheels' velocities can be written as functions of the center of mass lateral, longitudinal and yaw (rotation in y-axis) speeds. Let r be the vehicle's body yaw angular velocity on its own coordinate frame and b_i and a_i the geometric parameters defined in Figure 3. Then the slip angles at each wheel take the form:

$$\alpha_1 = \frac{v + r \cdot a_1}{u - r \cdot b_1} \qquad (3)$$

$$\alpha_2 = \frac{v + r \cdot a_1}{u + r \cdot b_2} \qquad (4)$$

$$\alpha_3 = \frac{v - r \cdot a_2}{u - r \cdot b_1} \qquad (5)$$

$$\alpha_1 = \frac{v - r \cdot a_2}{u + r \cdot b_2} \qquad (6)$$

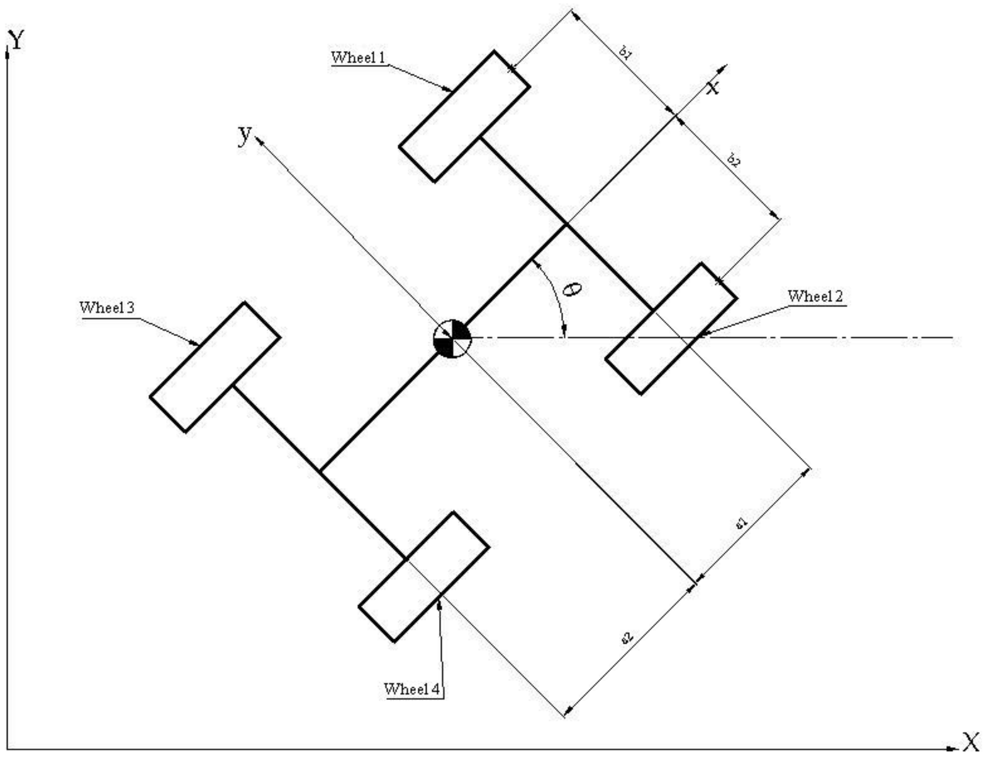

Figure 3. Vehicle describing a curve.

The equations of motion are, assuming that aerodynamic forces are applied at the center of mass, given by:

$$\begin{cases} m\dot{u} + mrv &= \sum F_{x,i} - \sum F_{r,i} - \frac{1}{2}\rho A_x C_x u^2 \\ m\dot{v} - mru &= \sum F_{y,i} - \frac{1}{2}\rho A_x C_y v^2 \\ I\dot{r} &= a_1\left(F_{y,1} + F_{y,2}\right) - a_2\left(F_{y,3} + F_{y,4}\right) - b_1\left(F_{x,1} - F_{r,1} + F_{x,3} - F_{r,3}\right) + b_2\left(F_{x,2} - F_{r,2} + F_{x,4} - F_{\cdot}\right. \end{cases} \quad (7)$$

where I is the yaw moment of inertia of the vehicle, A_x is the frontal area, m is the vehicle's total mass, C_x and C_y are the aerodynamic coefficients with respect to the y-z and x-z planes, respectively and forces $F_{r,I}$ represent the rolling resistance that appear on the contact areas resulting from the already discussed phenomenon of line of action displacement.

Equations (7) can be easily linearized to represent the system in space state form. The approach is to reject second order terms of a Taylor series expansion:

$$\dot{x}_j = f_j\left(x_{j_0}\right) + \left.\frac{\partial f_j}{\partial x_j}\right|_{x_0} \Delta x_j + \left.\frac{\partial f_j}{\partial F_{x,i}}\right|_{x_0} \Delta F_{x,i} \quad (8)$$

with j = 1,2,3, representing the state variables u, v and r and i = 1,…,4 representing the four wheels, as pointed out in Figure 3, and x_0 are the states at the linearization point. Functions f_i are, then, represented by:

$$f_1(u,v,r) = \frac{F_{x,i} - F_{r,i}}{m} - rv - \frac{R_x}{m}u^2 \quad (9)$$

$$f_2(u,v,r) = \frac{k}{m}F_{z,i}\cdot\alpha_i + ru - \frac{R_y}{m}v^2 \quad (10)$$

$$\begin{aligned} f_3(u,v,r) = & -\frac{b_1}{I}\left(F_{x,1} + F_{x,3} - F_{r,1} - F_{r,3}\right) + \frac{b_2}{I}\left(F_{x,2} + F_{x,4} - F_{r,2} - F_{r,4}\right) + \\ & + \frac{a_1 k}{I}\left(F_{z,1}\cdot\alpha_1 + F_{z,2}\cdot\alpha_2\right) - \frac{a_2 k}{I}\left(F_{z,3}\cdot\alpha_3 + F_{z,4}\cdot\alpha_4\right) \end{aligned} \quad (11)$$

where R_x and R_y represent, respectively, the x- and y-directions aerodynamic forces.

Substitution of Eq. (9), (10) and (11) in Eq. (8) yields the linear model. To write the equations in state space form, one must define which the control inputs to the plant are. The actuators (brake calipers) force wheel angular velocity to diminish, changing, thus, the contact force F_x so the brake torque and the longitudinal force are related in a non-linear manner. In this work, it is supposed that the control system can act directly on the generation of F_x (respecting tire's saturation that naturally occurs), skipping the mathematical treatment of the function that relates the brake torque to the contact force. Future works must also include this for it is the connection between the outer loop of the control system, which focus on the whole car dynamics, with the inner control loop, which determines the caliper and wheel dynamics.

The state space is usually represented as $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, with **A** being the dynamic matrix, **B** the actuator matrix, and **u** the input vector. The matrixes are shown bellow:

$$\mathbf{A} = \begin{bmatrix} -\frac{2}{m} R_x \cdot x_{1_0} & -x_{3_0} & -x_{2_0} \\ \frac{k}{m}\left(\sum_{i=1}^{4} F_{z_i} \frac{\partial \alpha_i}{\partial x_1} \right) & \frac{k}{m}\left(\sum_{i=1}^{4} F_{z_i} \frac{\partial \alpha_i}{\partial x_2} - 2 \cdot R_y \cdot x_{2_0} \right) & \frac{k}{m}\left(\sum_{i=1}^{4} F_{z_i} \frac{\partial \alpha_i}{\partial x_3} \right) + x_{1_0} \\ \frac{k}{I}\begin{bmatrix} a_1\left(F_{z_1} \frac{\partial \alpha_1}{\partial x_1} + F_{z_2} \frac{\partial \alpha_2}{\partial x_1} \right) + \\ -a_2\left(F_{z_3} \frac{\partial \alpha_3}{\partial x_1} + F_{z_4} \frac{\partial \alpha_4}{\partial x_1} \right) \end{bmatrix} & \frac{k}{I}\begin{bmatrix} a_1\left(F_{z_1} \frac{\partial \alpha_1}{\partial x_2} + F_{z_2} \frac{\partial \alpha_2}{\partial x_2} \right) + \\ -a_2\left(F_{z_3} \frac{\partial \alpha_3}{\partial x_2} + F_{z_4} \frac{\partial \alpha_4}{\partial x_2} \right) \end{bmatrix} & \frac{k}{I}\begin{bmatrix} a_1\left(F_{z_1} \frac{\partial \alpha_1}{\partial x_1} + F_{z_2} \frac{\partial \alpha_2}{\partial x_1} \right) + \\ -a_2\left(F_{z_3} \frac{\partial \alpha_3}{\partial x_2} + F_{z_4} \frac{\partial \alpha_4}{\partial x_2} \right) \end{bmatrix} \end{bmatrix} \quad (12)$$

$$\mathbf{B} = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{b_1}{I} & \frac{b_2}{I} & -\frac{b_1}{I} & \frac{b_2}{I} \end{bmatrix} \quad (13)$$

Matrix **B** shows that the second state, i.e. the lateral velocity, does not suffer direct impact of longitudinal force control.

4. CONTROL SYSTEM DESIGN

Space-state control system of the linearized dynamics can be achieved through several ways, like the ones presented in Ogata (2001). A simple method for controlling this system is using a proportional gain controller designed to place the closed-loop poles in specified locations, determining thus the behavior of the vehicle. Due to the highly variable conditions to which the car may be exposed, the controller must be adaptative, changing the gains accordingly to keep the system stable and safe. This paper do not focus on designing this type of controller, but instead concentrates on one set of selected gains and test the closed-loop system's performance to a safety-critical situation using ADAMS/Car, a multi-body simulation environment specialized on vehicle dynamics.

Analyzing the poles of the system to a conventional set of linearization conditions, e.g. straight line, low friction drive, one can notice the vehicle is asymptotically stable. However, when subjected to perturbations, e.g. subtle fall on road adherence, the driver can easily loose car's control, leading to a potentially dangerous situation. In other words, even if the vehicle is stable, it has small stability margins. The pole placement then enlarges these margins, preventing loss of control.

The tuning simulations were made with the vehicle driving forward and, at t = 1.00 s, losing adherence at wheel four. After a set of simulations, it was established that placing the poles at –2.5, -3.2±j0.5 lead to satisfactory results as shown in Figure 4. With control activated the vehicle could get back to its original trajectory in considerably less time, as it can be noticed from the lateral and yaw velocities time traces. Furthermore, this effect is obtained with a smaller decrease in longitudinal speed. The yaw speed overshoot was deliberately left at a high value to give the driver a feeling of when the vehicle may be getting out of control.

Though the specified gains turned out to be adequate for most straight line situations, a better approach to the problem would be iteratively determined a set of optimal controller parameters based on information send from the wheel sensors. The tire force characteristics can be estimated from wheel's speed, as showed by Ono *et al.*(2003).

The values of the lateral speed were estimated using a minimal order state observer, which consists of reproduction of the system that is co-simulated within the loop and gives a best approach to the unknown value. It can be easily substituted by a sensor value, which is more reliable and fast. The observer brings the inconvenient of adding an extra pole to the system dynamic, which must be faster than the others to guarantee that the yaw estimation will converge before the system can change considerably.

Figure 5 exhibits system's full block diagram with the observer, linear transformation that reconstruct the full state vector and the actuators subsystem. Inside each wheel another control is demanded to control caliper actuation or, in the case of traditional hydraulic brakes, the caliper control is replaced by a valves control.

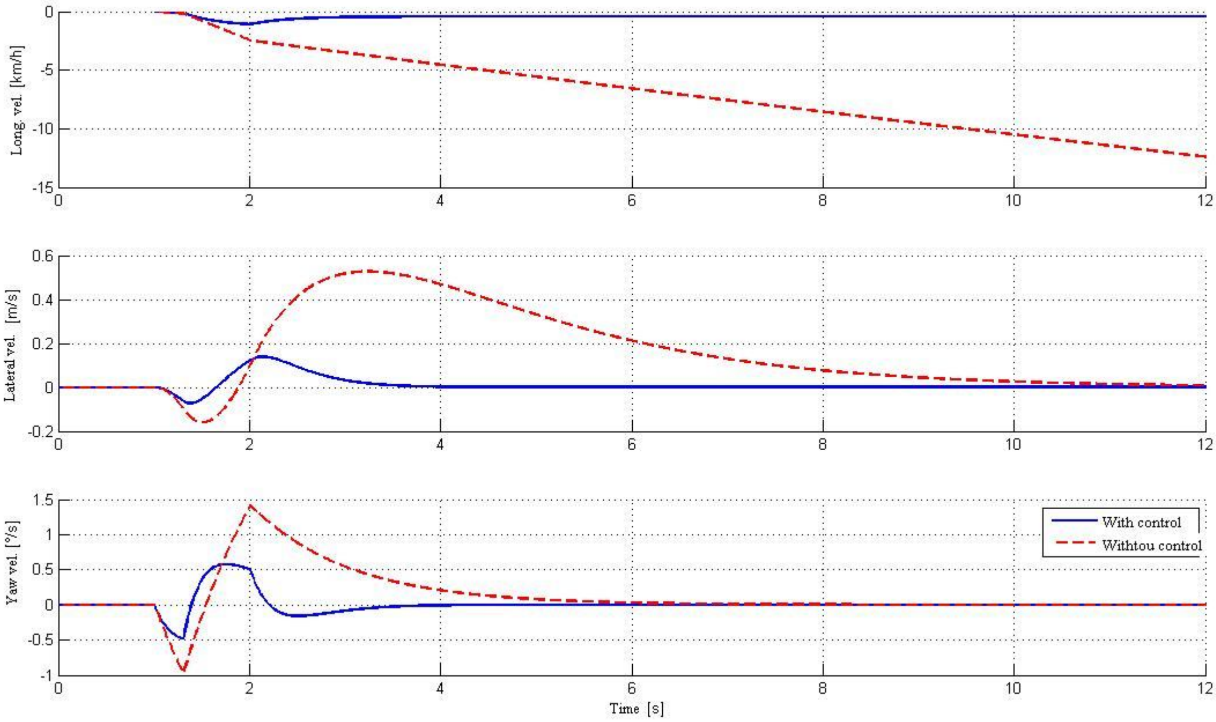

Figure 4. System response to subtle loss of friction at wheel four.

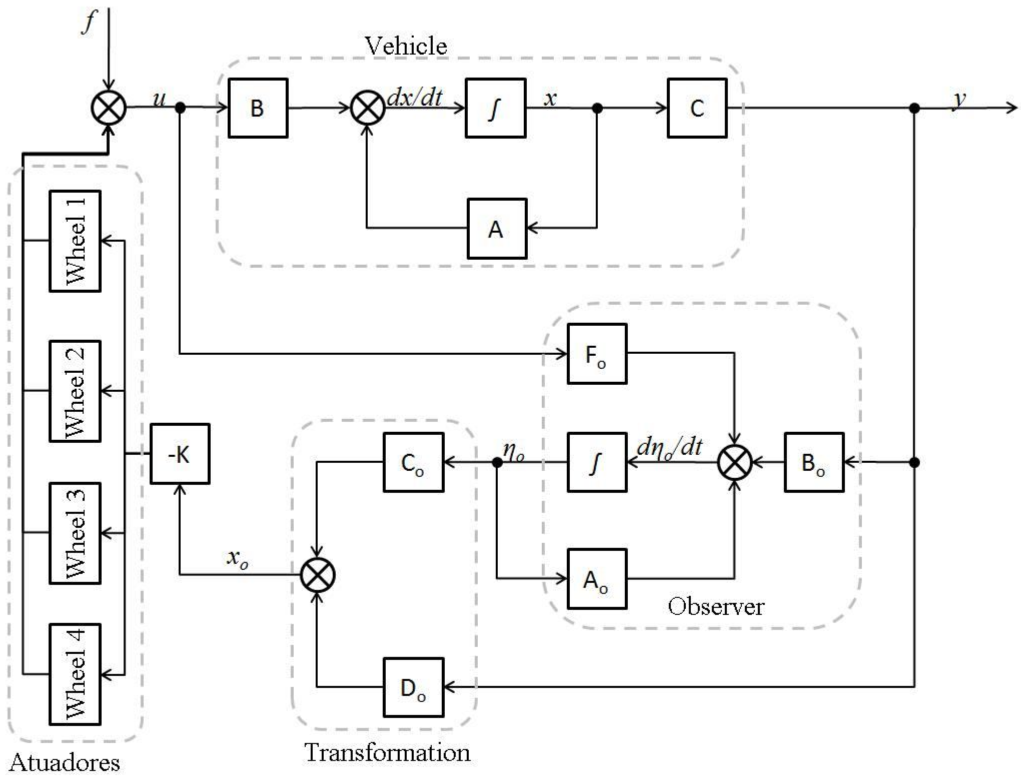

Figure 5. System's block diagram.

5. NON-LINEAR VERIFICATION

Though the projected controller showed an adequate behavior in a linear environment, it possible that in real usage, where the conditions are unpredictable and hence non-linear, the system may not respond properly. To verify this, the projected controller was tested on a virtual prototype assembled on the multi-body simulation software MSC.ADAMS. The interface between ADAMS and MATLAB was broadly used to provide the communication between the vehicle dynamics and the proposed controller. Figure 6 shows the basic topology of the vehicle model, whose inertial characteristics are similar to those of a small Brazilian city car with total mass of 1050 kg. The vehicle is conducted by a virtual driver already available in ADAMS.

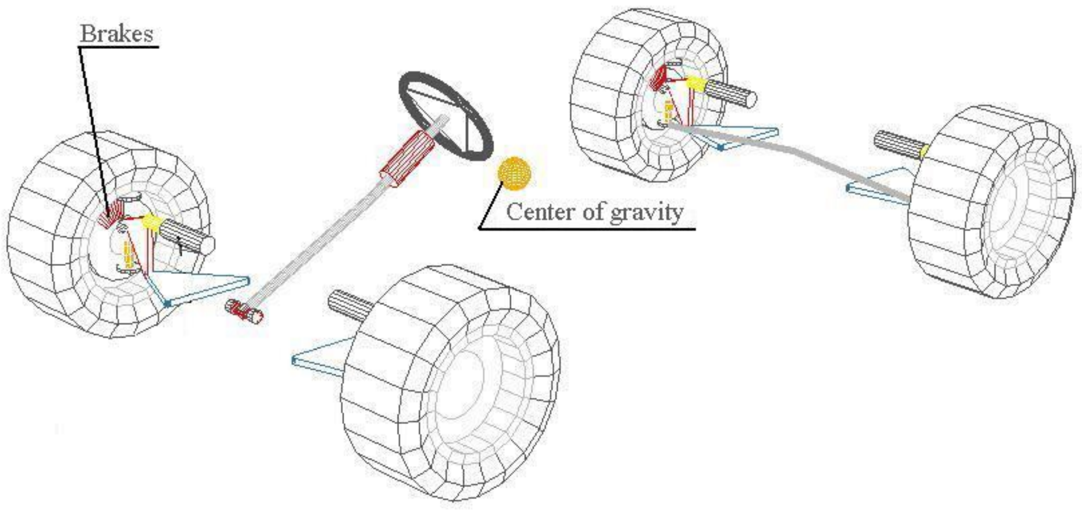

Figure 6. ADAMS virtual prototype

Figure 7 shows the block diagram developed in Simulink to run the simulations. The controller was subject to some modifications to adequate it to more real conditions. Though control gains are continuously calculated, the actuators are only if sideslip angle θ exceeds 8° for this value indicates lateral movements are beginning to take over. Furthermore, all positive control actions were discarded for the calipers cannot accelerate the wheels; it is likely that a future development of automotive vehicle's controls and driving systems may be based on in-wheel electric motors, which permit both acceleration and braking commands. By rejecting controller's positive signals, the control system looses efficiency because part of its energy spent calculating control gains is in vain. A more suitable controller should prior negative commands and will be subject of future investigations.

ISO standard 3888 was used to simulate lane change maneuvers always with 100 km/h initial longitudinal speed and road peak coefficient of friction varying from 1.0 (regular pavement) to 0.1 (very slip pavement). The controller behaved well, showing small influence on good road conditions and acting to increase vehicle safety when slip risk is great.

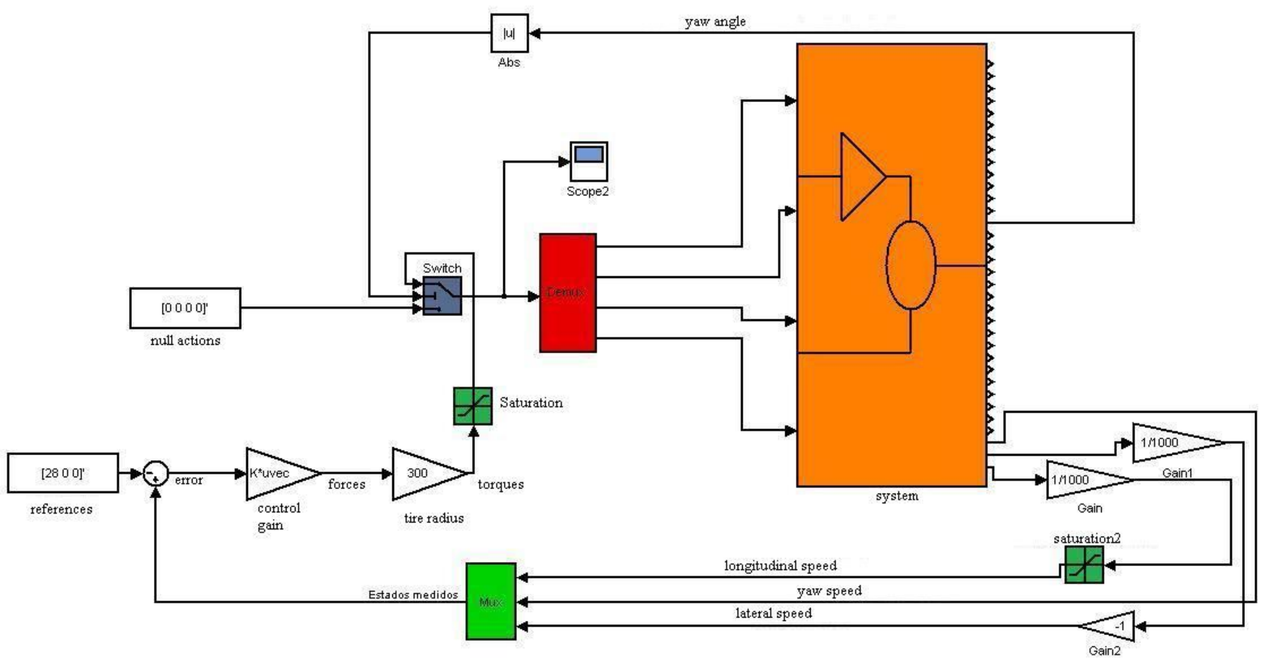

Figure 7. Simulink block diagram. The system block represents the interface with ADAMS via S-function.

Figure 8 shows a comparison between two analyses with very slipy road (case E) and one with full adherent road (case A). It is perceptible that without control the lane change begins almost 25 m later and the vehicle regain the original trajectory only in 300 m and with still high lateral velocity, suggest the vehicle still presents high slipping tendency. When the control is active, the driver regains trajectory in less space with greater stability. Third curve shown is the result of an uncontrolled simulation for the best conditioned road. Comparing this last curve with the controlled trajectory for case E, one can see the positive effects of controlled braking in vehicle's lateral stability.

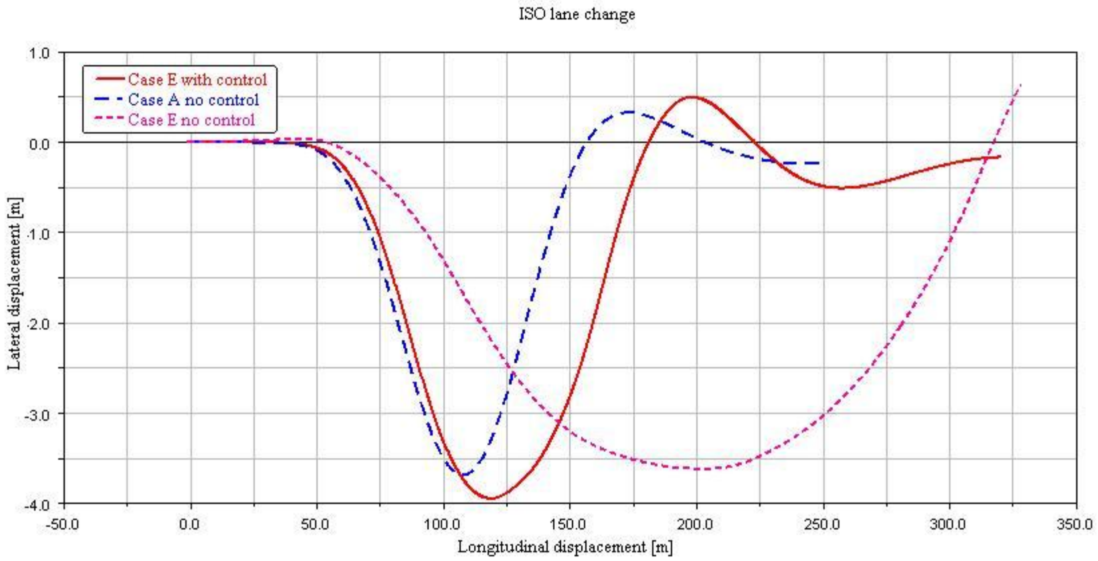

Figure 8. ISO lane change comparative analysis. In case A adherence is at maximum and in E, at minimum.

6. CONCLUSION

By analyzing vehicle behavior when negotiating a curve and choosing an appropriate simplification for the tire's behavior, it was possible to develop a simple model to describe the system's dynamics. A pole placement controller was then design to increase the stability margins to improve system response to critical conditions.

The simulation of controller performance in non-linear conditions was proven using software in the loop configuration involving ADAMS and MATLAB. Results show that even in non-linear environment the control system behaves as wished.

Future works may address the control system design and connection with caliper actuators.

7. REFERENCES

Antonov, S., et alii, 2008, "A New Flatness-Based Control of Lateral Vehicle Dynamics," Vehicle System Dynamics, v. 49, n. 9, pp. 789-801.

Baruffaldi, L. B. et alii, 1999, "Desempenho em Aceleração e Frenagem," Technical Report, Escola Politécnica, Universidade de São Paulo, São Paulo, Brasil.

Bretz, E. A., 2001, "By-Wire Cars Turn the Corner," IEEE Spectrum, April 2001.

Baslamisli, S. C., et alii, 2007, "Robust Control of Anti-Lock Brake System," Vehicle System Dynamics, v. 45, n. 3, p. 217-232.

Genta, G., 1997, "Motor Vehicle Dynamics," World Scientific, Cingapura.

Gillespie, T. D., 1992, "Fundamentals of Vehicle Dynamics," Society of Automotive Engineers, Warrendale, USA.

Gombert, B., 2006, "Wedge Brake Design Boosts by-Wire Stopping Performance," Siemens VDO press release, Germany.

Hartmann, H., et alii, 2002, "eBrake® – The Mechatronic Wedge Brake," SAE Paper 2002-01-2582.

Klode, H., et alii, 2006, "The Potential of Switched Reluctance Motor Technology for Electro-Mechanical Brake Applications," SAE Paper 2006-01-0296.

Krishnamurthy, P. et alii, 2005, "A Robust Force Controller for an SRM Based Electromechanical Brake System," Proc. of the 44th IEEE Conference on Decision and Control, and the European Control Conference, Seville, Spain, pp. 2006-2011.

Manzie, C., et alii, 2008, "Electromechanical Brake Modeling and Control: from PI to MPC," IEEE Transactions on Control Systems Technology, v. 16, n. 3, pp. 446-457.

Ogata, K., 2001, "Modern Control Engineering," 4th ed., Prentice Hall, USA.

Ono, E., et alii, 2003, "Estimation of Automotive Tire Force Characteristics Using Wheel Velocity," Control Engineering Practice, v. 11, pp. 1361-1370.

Pacejka, H. B., 2006, "Tyre and Vehicle Dynamics," 2nd ed., Elsevier, England.

Park, J. H., Ahn, W. S., 1999, "H_∞ Yaw-Moment Control with Brakes for Improving Driving Performance and Stability," Proc. of the 1999 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Atlanta, USA, pp. 747-752.

Rill, G., 2007, "Short Course on Vehicle Dynamics," Lecture notes, Universidade Estadual de Campinas, Brazil.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.