SKIN FRICTION COEFFICIENT OF IMPINGING JET FLOW: AN ESTIMATION BY LAW OF THE WALL

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Abstract. *Jet impingement flows are frequently used in industrial practice because of the high rates of heat exchange. Typical applications include many heating, cooling and drying processes such as the tempering and shaping of glass, the drying of textile and paper products, the cooling of heated components in gas turbine engines and electronic instruments. The objective of the present work is to estimate the skin friction coefficient at the impinging plate using a computational code. The inverse problem technique using the Levenberg- Marquardt method was used to implement the code in FORTRAN. An impinging turbulent jet with a semi-confined configuration was used. In this problem, fluid is injected from a round nozzle with 43,0mm inner diameter, at a nozzle-to-plate spacing of H/D=2 and Reynolds number of 35,000. Two laws of the wall were adopted in this work to determine the skin friction coefficient.*

Keywords*: Impinging Jet; Skin Friction; law of the wall, turbulent flow.*

1. INTRODUCTION

The turbulent jet impinging onto a wall has been extensively studied by the present authors in four other publications (Guerra e Silva Freire (2003, 2004a, 2004b, 2005)). However, in all of those publications, our concern was dedicated to both the velocity and the temperature fields. In fact, when the present research started four years ago, the behavior of the Nusselt number for different aspect ratios – the ratio between jet height and nozzle diameter – was our main interest. However, very soon it became clear to the present authors that an odd behavior could be observed for the law of the wall.

This fact was not a particular new information. For the simpler case of a wall jet, some authors had previously reported that some kind of law of the wall could be identified from mean velocity profiles measured with small Pitot tubes or hot-wires (see e.g., Patel (1962), Tailland and Mathieu (1967), Ozarapoglu (1973), Irwin (1973)). These same authors, unfortunately, could not agree on a same formulation for the flow description. Their reported values for the log-law parameters varied in a large interval.

At this point, we must call the reader’s attention to the relevance of a thorough study on the existence of the law of the wall for wall jets. Indeed, Wygnanski et al. (1992) showed in a very detailed research that the most reliable method for measuring the wall stress is to use the slope of the mean velocity profile near the surface. This method was tested against floating drag balances, Preston tubes and the momentum integral equation.

Having said that, it is no surprise that the previous publications of the present authors on this subject have concentrated on a description of the inner layers of the flow. The previous works specifically analyzed the existence of the so called universal law of the wall for both the velocity and the temperature fields. Here, we will deal only with the velocity field.

Özdemir and Whitelaw (1992)) had observed that, for an oblique impinging jet, a distinct logarithmic region could be identified that could be correlated through a scaling procedure based on the stream-wise evolution of the flow by the maximum jet velocity. This result was shown to be valid for both the velocity and the temperature fields. In fact, Narasimha et al. (1973) were the first to acknowledge that the traditional use of the nozzle diameter as the reference scaling for wall jet flows was not appropriate. They proposed that any reasonable scaling length should take into consideration the flow evolution.

Other authors have specifically studied the role of the scaling laws in wall jet flows. For example, Wygnanski et al. (1992) studied the relevance of the wall to the evolution of the large coherent structures in the flow.

In fact, a question that has been the object of many investigations is the behavior of the velocity field at the stagnation point. For cases where the Reynolds number is low enough so that the flow can be rendered laminar, asymptotic methods can be used to find analytical solutions in all flow regions except near the stagnation point, which presents a strong singularity. Consequently, even for this simple flow condition, calculation of flow pattern at the stagnation point is very difficult. The result is that a severe lack of information on the flow behaviour in the stagnation region exists. The reason for this is clear, due to the small scales that define this region, the placement of dedicated instrumentation is always very difficult.

The flow structure of an impinging jet produced by a nozzle can be highly complex due to the ambient fluid entrainment, flow separation, interaction of the flow with the impingement or confining walls, and generation of vortices. In this work, we will provide experimental data on turbulent semi-confined impinging jet. Results are presented for the turbulent characteristics of a round jet, with 43mm inner diameter, D, impinging on a flat plate with

3mm of thickness and 820mm of diameter. The distance of impingement is H=86mm, it means a nozzle-to-plate spacing of H/D=2. The experiments were realized with Reynolds number of 35,000.

Despite the critics of many researchers, the use of wall functions to by-pass the difficulties involved with the modeling of low Reynolds number turbulence is still an attractive means to solve problems in a simple way. Cruz and Silva Freire (1998) have proposed an alternative approach where new wall functions are used to describe the velocity and temperature fields in the wall logarithmic region. As the stagnation point is approached, these functions reduce to power-law solutions recovering Stratford's solution. The paper of Cruz and Silva Freire resorted to Kaplun limits for an asymptotic representation of the velocity and temperature fields. Results were presented for the asymptotic structure of the flow and for the skin-friction coefficient and Stanton number at the wall.

The main objective of this work is to determine wall skin friction of a turbulent jet impinging onto a flat surface using logarithmic law.

A turbulent jet impinging onto a surface is a very effective means to promote high rates of heat exchange. As an obvious implication, this geometrical arrangement has been extensively used in industrial processes that aim to achieve intense heating, cooling or drying rates. Typical applications are the tempering and shaping of glass, the annealing of plastic and metal sheets, the drying of textile and paper products, the deicing of aircrafts systems and the cooling of heated components in gas turbine engines and electronic instruments.

2. THEORECTICAL AND MATHEMATICAL APPLICATION

In the study of radial wall jet produced by a circular jet incident will be reviewed considering a geometric configuration in which the diameter D of a circular nozzle, located at a distance H normal of the incidence surface of a flat circular plate, as shown in Figure 1. The picture in question shows a schematic drawing of an unconfined jet air leaving the nozzle, impact orthogonally on the plate and spreads radially toward the edge of the plate, so thus forming a radial wall jet. The stream behavior in this configuration is different of a stream from a tangential wall jet.

In the wall jet flow is seen that at any radial position considered, r, the mean velocity u increases from zero on the wall to a maximum value U_M at $y = \delta$, then decreases to zero at some value of y. In Figure 1, r and y denote the distance in the developed flow direction on the plate and the distance in the normal direction to the surface, respectively. The speed of the jet is denoted by U_j, the external flow velocity is denoted by U_∞ in an unconfined space outside the jet and Y_M and $Y_M/2$ denote the vertical locations where the maximum radial speed U_M e $0{,}5\Delta U_M$ occur.

For the problem in study it was considered de geometric configuration of a semi-confined impinging jet as shown in Figure 2.

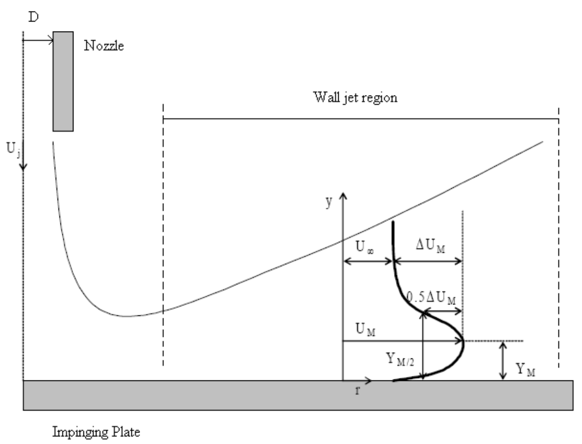

Figure 1. Theoretical wall jet velocity profile in an unconfined configuration.

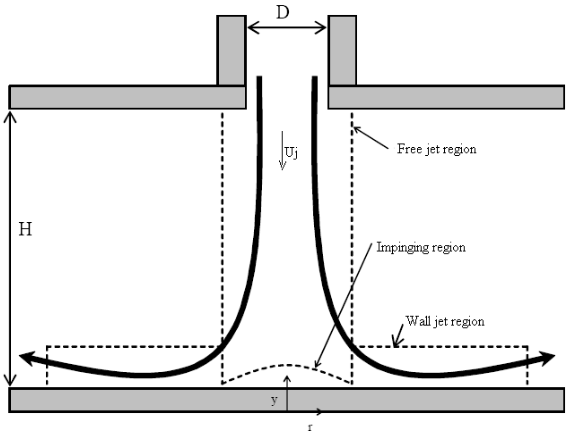

Figure 2. A semi-confined jet configuration.

As the flow field, this can be divided into two regions: one internal and other external. The region called "internal" is generally defined by the range that extends from the solid surface to the point of maximum velocity observed in each profile of radial velocity. The region called "external", extends from the point of maximum velocity to the most external edge of the profile.

The outer layer of the velocity profile of the wall jet is similar to the flow of free jet, while the internal layer has similar characteristics to those of boundary layer. Thus, the region that extends from the wall to the point of maximum velocity can be identified with the boundary layer while the area above is usually identified as a region of mixing (Rajaratnam, 1976).

The flow of a turbulent jet in the region near the wall is usually described mathematically using the maximum velocity U_M as a characteristic velocity scale, Narasimha et al. (1973) were the first to recognize that the traditional use of the diameter of the nozzle as a reference scale for the wall jet is not appropriate, they proposed with a length scale that took into account the evolution of the flow.

In this work we used the experimental data velocity profiles obtained by Guerra et al (2005), as shown in Figure 3. The set of data was obtained for a nozzle-to-plate spacing of H / D = 2 at various radial positions *r* in the range of 80 to 150mm. The positions *r*, are from the discharge of the jet toward to the plate edge. In Figure 3 there are the velocity profiles that was measured at the radial positions mentioned above, which was used to determine the skin friction coefficient in this work.

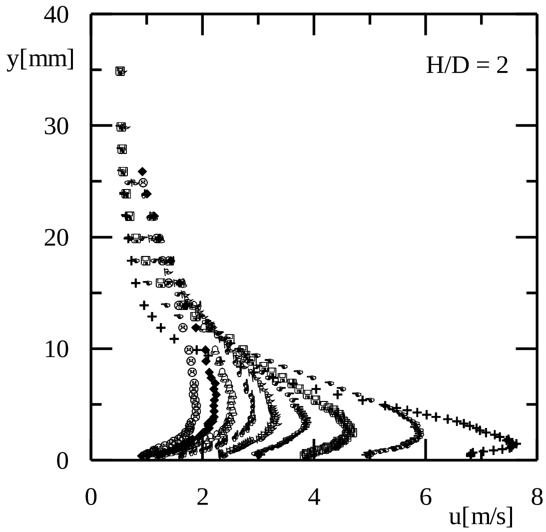

Figure 3. Mean velocity profiles at radial positions from the discharge of the jet toward to the plate edge.

2.1. Logarithmic Law

An important feature of wall turbulence is the complex, three-dimensional and instantaneous turbulent motion. The turbulent motion is subjected to viscous and inertial forces acting throughout the entire flow but with different intensities at different wall-normal positions. Due the fact that there is no y position in the flow, except the wall (the object in study) itself, it is reasonable to split wall-bounded turbulent flow into two distinct overlapping zones. This assumption is the classical view of the logarithmic law that assumes an overlap region where both viscous and inertial forces can be neglected.

The so-called ‘log law’ is widely believed to describe most turbulent wall-bounded flows, and lies at the core of the most widely used engineering computational models involving turbulent flow near surfaces. While there are several manifestations of the log law, the most important is the mean velocity profile normalized in so-called ‘inner’ variables given by:

$$\frac{u}{u_\tau} = \frac{1}{\kappa} ln\left(\frac{u_\tau}{\nu}\right) y + A \tag{1}$$

$$u^+ = \frac{u}{u_\tau} \tag{2}$$

$$y^+ = \left(\frac{u_\tau}{\nu}\right) y \tag{3}$$

The superscript + stands for normalization with wall variables, where u is the mean velocity in the streamwise direction (the r directions), y is the distance normal to the wall, κ is the von Kármán constant assumed to be 0.41, ν is the fluid kinematic viscosity, A is the universal smooth wall constant and u_τ is the friction velocity given by:

$$u_\tau = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}} \tag{4}$$

Where τ_w is the wall shear stress and ρ is the fluid density.

2.2. Skin Friction Determination

The existence of a well defined logarithmic region is particularly important for the determination of the skin-friction. Wygnanski et al. remark that in previous experiments the skin-friction was either directly assessed through floating drag balances or indirectly by wall heat transfer devices or by impact probes or Preston tubes. These same authors mentioned that, for a turbulent wall jet, the velocity profile cannot be universally represented in wall coordinates, as it can in the boundary layer. That is due to large variations in the additive constant in the law of the wall.

In fact, depending on the jet Reynolds number, logarithmic fits can be found to their data in regions defined by specific limits.

The skin-friction coefficient, c_f, is defined using the wall shear stress, τ_w, and the free stream velocity (measured relative to the surface), U_∞ as,

$$c_f = \frac{\tau_w}{[(1/2)\rho U_\infty^2]} \tag{5}$$

Is shown by equations (4) and (5) that through the wall shear stress, τ_w , we can determine the skin friction. So the determination of the wall skin friction using the logarithmic law, the so-called Clauser- Plot can be applied to determine the friction velocity, u_τ , from measured velocity distributions. This method was used here for the experimental data of velocity profiles. The second method to determine the friction velocity was the application of the classical concept of Stokes's law. A third method consists in the application of the Inverse Problem with the logarithmic law proposed by Cruz and Silva Freire (1998). For the last one, a computational program was developed in Fortran.

2.3. The Applied Methodology

Wygnansk (1992) has compared several methods for determining the friction velocity and concluded that the most reliable method to determine the wall shear stress is the use of the inclination of the mean velocity profiles. It is important to mention that the accuracy of this method depends on the quality of the positioning mechanism used to the hot-wire probe, the quality and size of the probe, and the number of points measured.

To find the values of friction velocity, the graphical method of Coles (1956) was used. By this method, the velocity profiles were plotted in the mono-logarithmic form, where the mean velocities, u, of each profile are located in the Y-axis and the logarithmic of y is located in the abscissa axis. Through the identification of the line defined directly from the expression (1) we can obtain the value of the friction velocity as the angular coefficient of the line. In fact, if θ is the angular coefficient of the law of the wall, then $\theta\kappa = u_\tau = (\tau_w/\rho)^{1/2}$. Thus, the friction velocity u_τ is introduced as a normalizing quantity and express the influence of the friction coefficient c_f.. The procedure described is illustrated by Figures 4 – 7.

In Figure 4 there are four velocity profiles due the repeatability of the experiment at radial position of 115mm, plotted in a mono-logarithmic form. The next step was the identification, in this same data, of the line defined from the expression (1), as shown in Figure 5.

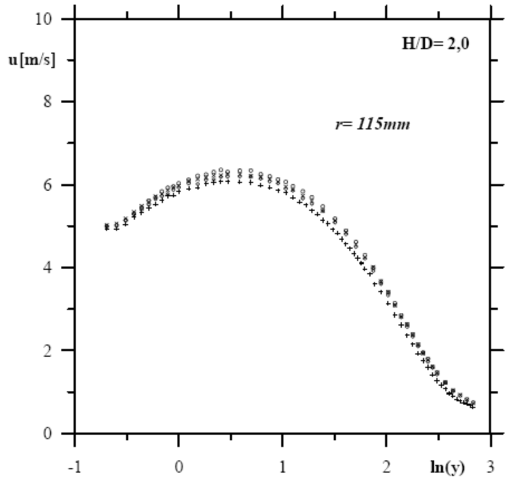

Figure 4. Mean velocity profiles at radial position r=115mm

Figure 5 is the same Figure 4 with a zoom of the velocity scale. Now it seems not all the velocity profile but only the region near the wall (impinging plate). Figures 6 and 7 follow the same procedure applied to a radial position more distant from the discharge jet.

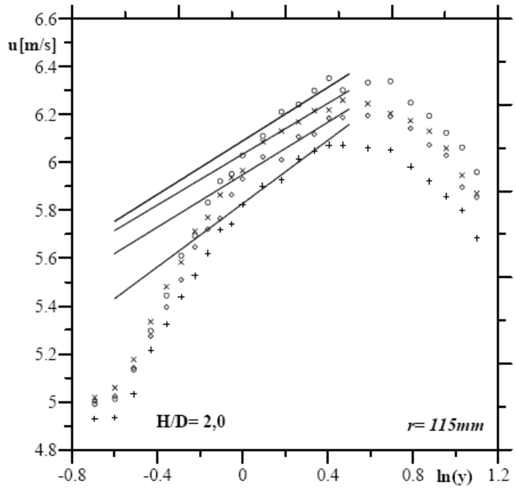

Figure 5. Graphical method for friction velocity determination at position *r*=115mm.

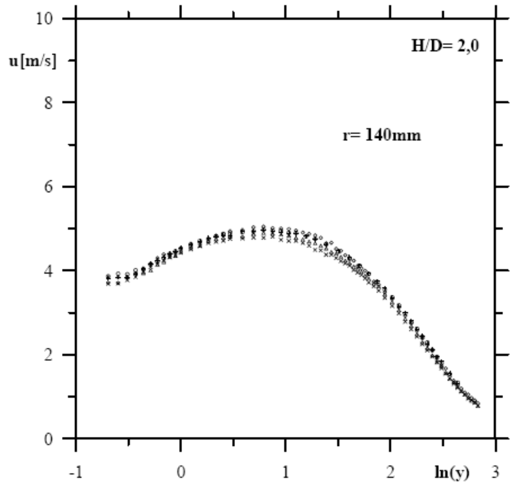

Figure 6. Mean velocity profiles at radial position *r*=140mm

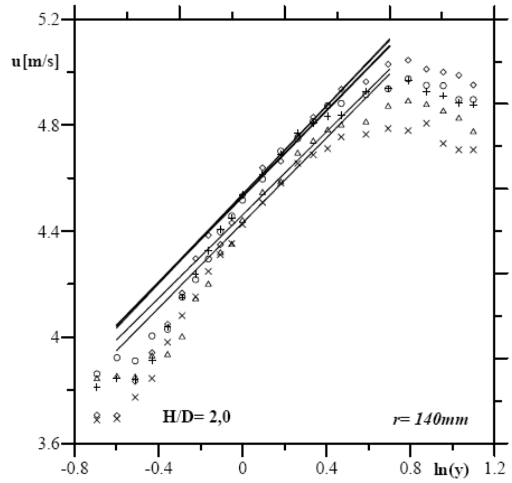

Figure 7. Graphical method for friction velocity determination at position *r*=140mm.

After applied the method above explained, attention was given to the use of the experimental data to determine the wall shear stress and consequently the skin friction coefficient. For this objective a computational program was developed in Fortran using the inverse problem to solve the logarithmic law equation.

For an evaluation of the local skin-friction coefficient the expressions developed in Cruz and Silva Freire (1998) are used on the computational program. The two unknown τ_w/ρ and $\partial P_w/\partial x$ are estimated through an Inverse Problem Solution Method. While in the direct problem the cause is given and the effect is determined, in an inverse problem an estimation of the cause is obtained through the knowledge of the effect. The solution of the inverse problem used here is obtained through the Levenberg-Marquardt Method; that is an iterative method for solving nonlinear least squares problems of parameter estimation (Özisik and Orlande, (2000)). The inverse problem is solved by a minimization of the remainder of the least-square norm which can be expressed as,

$$S(P) = \sum_{i=1}^{N} \left(Y_i - K_i(P)\right)^2 \tag{6}$$

Where $P= (P_1, P_2, \ldots P_N)$ = vector with parameters to be estimated, $K(P) = K(P,y_i)$ = quantity estimated at position y_i, $Y_i= Y(x_i)$ = measured velocity at position x_i, N = total number of parameters to be estimated, I = total number of measurements. The estimated quantities K (P) are obtained through solution of the direct problem by using a current estimation for the unknown parameters Pj, j= 1,…N.

Following the procedure of Cruz and Silva Freire (1998), the law of the wall can be written as,

$$u = \frac{\tau_w}{|\tau_w|}\frac{2}{\varkappa}\sqrt{\frac{u_\tau}{\varkappa} + \frac{1}{\rho}\frac{dP_w}{dx}y} + \frac{\tau_w}{|\tau_w|}\frac{u_1}{\varkappa}\ln\left(\frac{y}{L_c}\right) \tag{7}$$

Where

$$L_c = \frac{\sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\nu\frac{dP_w}{dx}uR - \frac{\tau_w}{\rho}}}{\frac{1}{\rho}\frac{dP_w}{dx}} \tag{8}$$

And all symbols have their classical meaning, κ is the von Kármán constant, u_τ is the friction velocity and u_R is a reference velocity. The above equation is a generalization of the classical law of the wall.

$$\tau_w = \frac{u_p {\tau_w}^{1/2}\rho\varkappa}{2\sqrt{\left|\frac{\tau_p}{\tau_{w0}}\right| + \ln\left(\frac{y}{L_c}\right)}} \tag{9}$$

3. RESULTS

The second method applied provided the data presented in Figures 8 and 9. The results achieved by the expression (7) together with the technique of inverse problem developed in Fortran code, shown a decrease in the value of the friction velocity as the flow moves away from the nozzle toward the edge of impinging surface. In Figure 8 is shown the values of τ_w/ρ varying with the measured points, NM, which constitute the measured velocity profile. The variation of NM is from the surface at y= 0,5mm to y= U_M.

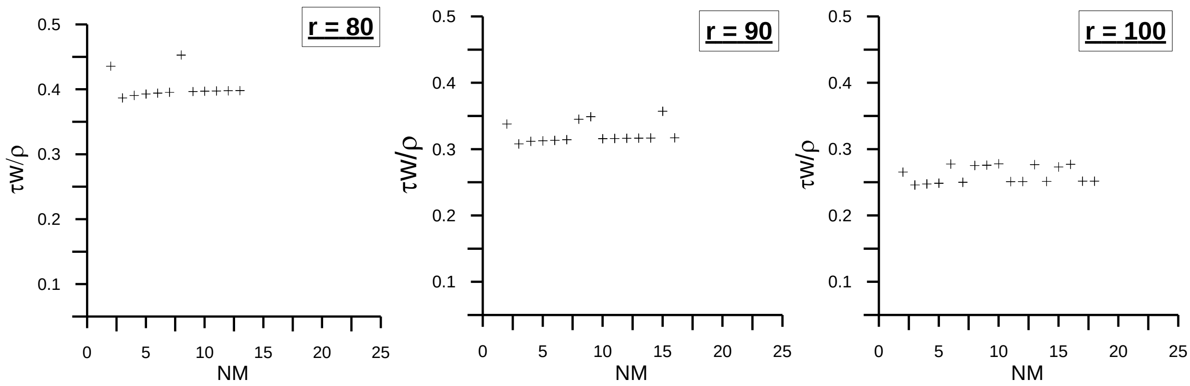

Figure 8. Results of τ_w/ρ from law of the wall expressed by equation (7).

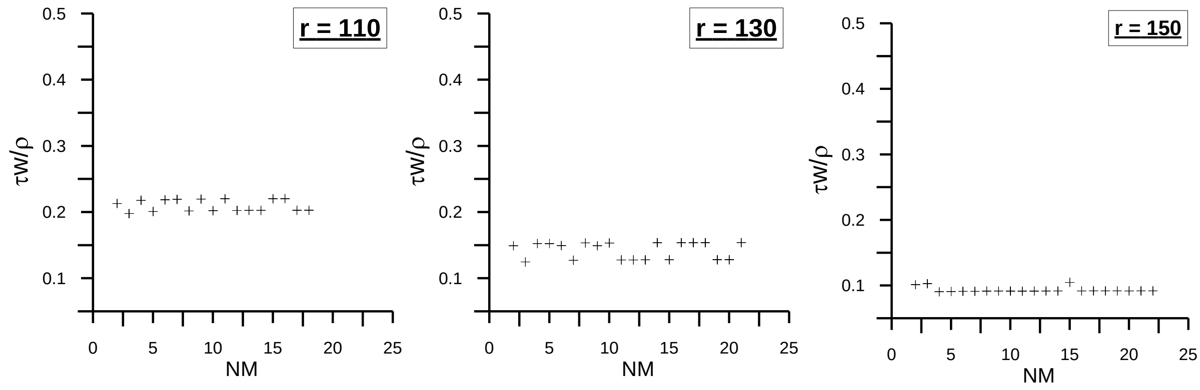

Figure 9. Results of τ_w/ρ from law of the wall expressed by equation (7).

Figure 9 follows the same behavior shown by Figure 8 for others radial positions. The results from each radial position it was evaluated the mean value and plotted in a same graphic as shown in Figure 10.

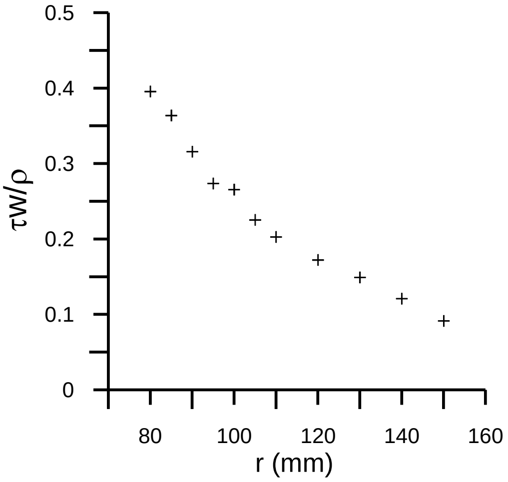

Figure 10. Mean value of τ_w/ρ obtained through equation (7) using computational code.

For the first procedure using the graphical method, the friction velocity values founded for each velocity profile, with H/D=2 at various radial positions are shown in Figure 11.

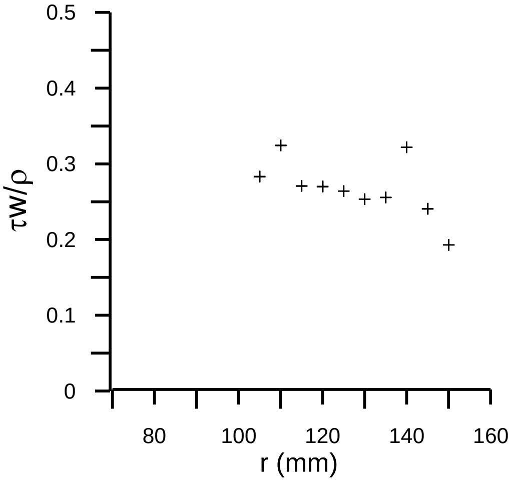

Figure 11. Value of τ_w/ρ obtained through equation (1), graphical method.

It is important to note that these last Figures represent how the applied methods can be providing the behavior of the skin friction coefficient. In this first evaluation we can observe a great difficult in apply the graphical method for the velocity profiles near the stagnation point. So that is why on Figure 11 only appear results from the radial position 110mm. With the evaluation through the computational program, we can obtain the friction velocity for all radial positions, as shown in Figure 10.

4. CONCLUSION

The present work had a very distinct goal at its beginning that is to provide a robust method for the calculation of wall jet flows promoted by impinging jets. Specially, we want to improve the calculation methods for skin-friction coefficient which were developed in the past to use the law of the wall.

Two laws of the wall were adopted in this work to determine the skin friction coefficient and the results, apparently shown that the goal has been achieved with the specification of expression (7) using a developed computational program for this objective.

The results founded in this investigation indicate that the level of the logarithmic portion of the velocity law of the wall increases with increasing maximum jet velocity.

The present research is particularly relevant due to its application for the development of methods that can be used for the determination of the local skin-friction.

This preliminary works is focus in skin-friction coefficient; these results are the first that were obtained from the experimental data of Guerra et. al (2005), so the authors are committed to continue this research.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Ashforth-Frost, S. and Jambunathan, K., 1996, “Numerical prediction of semi-confined jet impingement and comparison with experimental data”, Int. J. Numer. Methods Fluids, Vol. 23, pp. 295-306.

Baydar, E., 1999, “Confined impinging air jet at low Reynolds numbers”, Exp. Therm. Fluid Sci., Vol. 19, pp. 27-33.

Cooper, D., Jackson, D. C., Launder, B. E. and Liao, G. X., 1993, “Impinging jet studies for turbulence model assessment-I. Flow-field experiments”, Int. J. Heat Mass Transfer, Vol. 36, No. 10, pp. 2675-2684.

Craft, T. J., Graham, L. J. W. and Launder, B. E., 1993, “Impinging jet studies for turbulence model assessment-II. An examination of the performance of four turbulence models”, Int. J. Heat Mass Transfer, Vol. 36, No. 10, pp. 2685-2697.

Coles, D., 1956, “The law of the wake in a turbulent flow”, J.F.M., Vol. 1, pp. 191.

Cruz D. O. A. and Silva Freire A. P., 1998, “On single limits and the asymptotic behavior of separating turbulent boundary layers”, Inter. J. Heat and Mass Transfer, Vol. 41, pp. 2097-2111.

Dianat, M., Fairweather, M. and Jones, W. P., 1996, “Predictions of axisymmetric and two-dimensional impinging turbulent jets”, Int. J. Heat and Fluid Flow, Vol. 17, pp. 530-538.

Forthmann, E., 1934, “ Uber turbulente strahlausbreitung”, Ing. Arch., Vol. 5, pp. 42.

Guerra, D. R. S. and Silva Freire, A. P., 2003, “An experimental heat transfer study of a cold jet impinging onto a hot surface”, Congresso Brasileiro de Engenharia Mecânica, São Paulo, December.

Guerra, D. R. S. and Silva Freire, A. P., 2004, “A study of the heat transfer behaviour for a cold jet impinging upon a hot surface”, Congresso Nacional de Engenharia Mecânica, Belém, August.

Guerra, D.R.S, Su,J., Silva Freire, Atila P.,2005, “The near wall behavior of na impinging Jet”, International Journal of Heat and Mass Transfer, Vol.48, 2829-2840.

Irwin, H. P. A. H., 1973, “Measurements in a self-preserving plana wall jet in a positive pressure gradient”, J.F.M., Vol. 61, pp. 33.

Kendoush, A. A., 1998, “Theory of stagnation region heat and mass transfer to fluid jets impinging normally on solid surfaces”, Chem. Eng. Processing, Vol. 37, pp. 223-228.

Kline, S. J.; 1985, The Purpose of Uncertainty Analysis, J. Fluids Engineering, 107, 153-160.

Knowles, K., 1996, “Computational studies of impinging jets using κ–ε turbulence models”, Int. J. Numer. Methods Fluids, Vol. 22, pp. 799-810.

Knowles, K. and Myszko, M., 1998, “Turbulence measurements in radial wall-jets”, Exp. Therm. Fluid Sci, Vol. 77, pp. 71-78.

Lee, J. and Lee, S. -J., 1999, “Stagnation region heat transfer of a turbulent axisymmetric jet impingement”, Exp. Heat Transfer, Vol. 12, pp. 137-156.

Lee, J. and Lee, S. -J., 2000, "The effect of nozzle aspect ratio on stagnation region heat transfer characteristics of elliptic impinging jet", Int. J. Heat Mass Transfer, Vol. 43, pp. 555-575.

Meola, C. and Carlomagno, G. M., 1996, "Influence of shear layer dynamics on impingement heat transfer", Exp. Therm. Fluid Sci., Vol. 13, pp. 29-3.

Narasimha, R., Narayan, K. Y. and Pathasarathy, S. P., 1973, "Parametric analysis of turbulent wall jets in still air", Aeronaut. J., Vol. 77, pp. 335.

Nishino, K., Samada, M., Kasuya, K. and Torii, K., 1996, "Turbulence statistics in the stagnation region of an axisymmetric impinging jet flow", Int. J. Heat and Fluid Flow, Vol. 17, 193-201.

Ozarapoglu, V., 1973, "Measurements in incompressible turbulent flows", D.Sc Thesis, Laval University, Quebec.

Ozdemir, I. B. and Whitelaw, J. H., 1992, "Impingement of an axisymmetric jet on unheated and heated flat plates", J. Fluid Mech., Vol. 240, pp. 503-532.

Patel, R. P.,1962, "Self preserving two dimensional turbulent jets and wall jets in a moving stream", M.Sc. Thesis, McGill University, Montreal.

Sigalla, A., 1958, "Measurements of a skin-friction in a plane turbulent wall jet", J. R. Aero. Soc., Vol. 62, pp. 873.

Tailland, A. and Mathieu, J., 1967, "Jet parietal", J. Mecanique, Vol. 6, pp. 1.

Wygnanski, I., Katz, Y. and Horev, 1992, "On the applicability of various scaling laws to the turbulent wall jet, J.F.M., Vol. 234, pp. 669-690.

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