A MULTI-OBJECTIVE OPTIMIZATION DESIGN FOR PARALLEL STRUCTURES

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Abstract. *Manipulators with parallel architecture have inherent advantages in some applications with respect to serial manipulators, like high stiffness, accurate positioning and can move at high velocities. Therefore, they address great interest in some industrial applications and medical fields. In this paper, a multi-objective optimization process is proposed in order to enhance the design of parallel structures. Characteristics of parallel structures like workspace, singularities and compliant displacements are considered in order to propose design criteria obtaing a computationally efficient objective functions. The proposed procedure has been applied to a 5R Symmetric Parallel Manipulator.*

Keywords*: Parallel Manipulators, Singularities, Compliant displacements, Multi-objective optimization.*

1. INTRODUCTION

A parallel manipulator typically consists of a moving platform that is connected to a base by several serial chains and/or closed mechanisms, called limbs. Features of such system can present better stiffness and payload capacity with respect to the serial architectures, and high velocity and acceleration during the operation. Furthermore, errors in the joints are not cumulative, which contributes for its overall accuracy. Due to their characteristics they have been studied extensively both from theoretical and practical viewpoints. Prototypes have been conceived and built together with the development of theoretical investigations on kinematics and dynamics. The attention are focused to a number of possible industrial applications such as manipulation (Gonçalves and Carvalho, 2008a; Macho, et al., 2008), packing and assembly/disassembly machines (Figielski et al., 2007), motion simulation (Stewart, 1965), milling machines (Hess-Coelho et al., 2001), toys and sensors. However, they have some disadvantages such as small and complex workspace with internal singularities and the complexity of their forward kinematics (Gosselin and Angeles, 1990; Macho et al., 2008; Gonçalves and Carvalho, 2008b).

In many industrial applications the project and the performance of a robotic structure can be improved through a suitable optimization procedure. In fact optimization methodologies have long been applied to mechanism synthesis in order to obtain high performances and suitable mechanism dimensions which several performance criteria can be taken into account for design purposes.

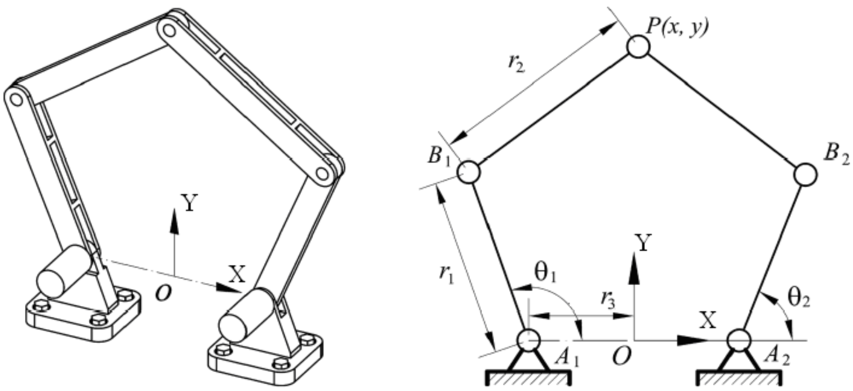
Modern design of manipulators can to consider simultaneously several aspects on its procedures using optimization methods whose can be solved by using well-established mathematical techniques in commercial software packages (Ceccarelli et al., 2005).

Performance indices can be used to characterize a manipulator which is associated to an applied optimization criteria. Then, the development of manipulators to perform a wide range of tasks can be achieved by different optimization criteria depending on the resources and general nature of tasks to be performed. The designer problem consists in choosing the performance criteria and justifies the optimality of different designs since each performance criteria and optimization method, in general, gives different results.

In this paper is presented a formulation for optimum design of parallel structures that considers the workspace, singularities and stiffness. The analysis of stiffness and singularity consider the methodology proposed by Gonçalves and Carvalho (2008a, 2009) to formulate objective functions. In order to show the procedure the methodology is applied to a 5R symmetric parallel manipulator to obtain its design parameters. As the workspace of the 5R symmetric parallel manipulator has a complex shape, it has been represented through an equivalent area.

2. THE 5R SYMMETRIC PARALLEL MANIPULATOR

A five-bar manipulator is a typical parallel manipulator with the minimal degrees of freedom, which can be used for positioning a point on a region of a plane. A 5R parallel manipulator consists of five bars that are connected end to end by five revolute joints, two of which are connected to the base and actuated, as sketched in Fig. 1a. Such a manipulator with a symmetric structure has attracted many researchers, who have investigated its position analysis (Liu et al, 2006; Alici and Shirinzadeh, 2004), workspace (Macho, et al., 2008), assembly modes (Cervantes-Sánchez et al., 2001, singularity (Macho et al., 2008; Mbarek et al., 2007; Figielski et al, 2007; Gonçalves and Carvalho, 2009a), performance atlases (Liu et al, 2006) and kinematic design (Cervantes-Sánchez et al., 2001; Alici and Shirinzadeh, 2004).



(a) (b)

Figure 1. The 5R Parallel Manipulator (Liu et al., 2006). a) The typical 5R linkage; b) Kinematics parameters.

Its kinematics can be analyzed using the parameters shown in Fig. 1b, where each actuated link A_iB_i has an active joint A_i and a passive one B_i (I = 1,2), the non-actuated links B_iP are coupled in a common passive joint P. To describe its kinematic behavior an inertial frame OXY has been assumed fixed to base A_1A_2 with X-axis as coincident with the line joining O to A_2, Y-axis orthogonal to A_1A_2 and upward, and the origin O coinciding with the center point of link A_1A_2. The kinematic variables are the input angles θ_1 and θ_2 as sketched in the Fig. 1b. The 5R symmetric linkage the length of the actuated links are equals and the length of non-actuated links too, i. e., $A_1B_1 = A_2B_2 = r_1$ and $B_1P = B_2P = r_2$. The length of the base link $A_1A_2 = 2\ r_3$.

The kinematic model relates the position of point P, given by its coordinates x and y, to the input angles θ_1 and θ_2, as follows.

2.1. Inverse Kinematics

The input angles θ_1 and θ_2 can be obtained, from the inverse kinematics, when the position of point P is known using vector relations.

The kinematic analysis can be done from Fig. 1b, considering both equations:

$$\overrightarrow{OP} = \overrightarrow{OA_2} + \overrightarrow{A_2B_2} + \overrightarrow{B_2P} \tag{1}$$
$$\overrightarrow{OP} = \overrightarrow{OA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1P} \tag{2}$$

or in a scalar form

$$(x - r_1\cos\theta_1 + r_3)^2 + (y - r_1\sin\theta_1)^2 = r_2^2 \tag{3}$$
$$(x - r_1\cos\theta_2 - r_3)^2 + (y - r_1\sin\theta_2)^2 = r_2^2 \tag{4}$$

If the coordinates of the point P are known, the inputs angles, θ_1 and θ_2, for reaching this position can be obtained from Eqs. (3) and (4) as

$$\theta_i = 2\tan^{-1}(z_i)\,,\quad \text{i = 1,2} \tag{5}$$

where

$$z_i = \frac{-b_i \pm \sqrt{b_i^2 - 4a_ic_i}}{2a_i}\,,\ \text{i = 1,2} \tag{6}$$

and

$$\begin{aligned}
a_1 &= r_1^2 + y^2 + (x+r_3)^2 - r_2^2 + 2(x+r_3)r_1 \\
b_1 &= -4yr_1 \\
c_1 &= r_1^2 + y^2 + (x+r_3)^2 - r_2^2 - 2(x+r_3)r_1 \\
a_2 &= r_1^2 + y^2 + (x-r_3)^2 - r_2^2 + 2(x-r_3)r_1 \\
b_2 &= b_1 = -4yr_1 \\
c_2 &= r_1^2 + y^2 + (x-r_3)^2 - r_2^2 - 2(x-r_3)r_1
\end{aligned} \tag{7}$$

Equation (5) give four solutions for inverse kinematics problem of the 5R manipulator.

2.2. Direct Kinematics

The direct kinematics problem consists in obtaining the coordinates of point P it the inputs angles θ_1 and θ_2 are known from Eqs. (3) and (4) one can write

$$x^2 + y^2 - 2(r_1\cos\theta_1 - r_3)x - 2r_1\sin\theta_1 y - 2r_1r_3\cos\theta_1 + r_3^2 + r_1^2 - r_2^2 = 0 \tag{8}$$

$$x^2 + y^2 - 2(r_1\cos\theta_2 + r_3)x - 2r_1\sin\theta_2 y + 2r_1r_3\cos\theta_2 + r_3^2 + r_1^2 - r_2^2 = 0 \tag{9}$$

The Equations (8) and (9) yield to

$$x = ey + f \tag{10}$$

where

$$e = \frac{r_1(\sin\theta_1 - \sin\theta_2)}{2r_3 + r_1\cos\theta_2 - r_1\cos\theta_1} \tag{11}$$

$$f = \frac{r_1r_3(\cos\theta_2 + \cos\theta_1)}{2r_3 + r_1\cos\theta_2 - r_1\cos\theta_1} \tag{12}$$

The y coordinate can be obtained substituting Eq. (10) into Eq. (9) giving

$$dy^2 + gy + h = 0 \tag{13}$$

so

$$y = \frac{-g \pm \sqrt{g^2 - 4dh}}{2d} \tag{14}$$

where

$$\begin{aligned}
d &= 1 + e^2 \\
g &= 2(ef - er_1\cos\theta_1 + er_3 - r_1\sin\theta_1) \\
h &= f^2 - 2f(r_1\cos\theta_1 - r_3) - 2r_1r_3\cos\theta_1 + r_3^2 + r_1^2 - r_2^2
\end{aligned} \tag{15}$$

Equations (10) and (14) provide two solutions for the forward kinematic problem of the 5R manipulator.

3. A MULTI-OBJECTIVE OPTIMIZATION DESIGN FOR PARALLEL STRUCTURES

Once the numerical technique is chosen or is advised for solving a proposed multi-objective optimization problem, the main efforts can be addressed to the formulation of common algorithms for numerical evaluation of optimality criteria and design procedure constraints. In the following, main aspects are overviewed by emphasizing the common numerical evaluations for parallel manipulators in terms of workspace, singularity and stiffness.

3.1 General concepts

A multi-objective optimization solves numerically problems subject to constraints as: obtaining the set of design variables x which will

$$minimize\left[f_1(x), f_2(x), ..., f_k(x)\right] \tag{16}$$

for k objective functions $f_i : \Re^n \to \Re$ subject to equality and inequality constraints. For the vector of decision variables, $x = \left[x_1, x_2, ..., x_n\right]^T$, the task is to determine the set F of all vectors which satisfy the constraints and the particular set of optimal values $x^* = \left[x_1^*, x_2^*, ..., x_n^*\right]^T$.

As soon as there are several objectives functions to be optimized simultaneously, usually there is no longer a single optimal solution but rather a whole set of solutions. When several objectives are optimized at the same time the search space becomes partially ordered. To obtain the optimal solution there will be a set of optimal trade-offs between the conflicting objectives.

In this context, best solution means a solution not worst in any of the objectives and at least better in one objective than the other. An optimal solution is the solution that is not dominated by any other solution in the search space, which is called a Pareto-optimal and the entire set of such optimal trade-offs solutions is called a Pareto- optimal set.

Even though there are several ways to approach a multi-objective optimization problem, most works is concentrated on the approximation of the Pareto set.

Given a set of alternatives, the problem of choosing the best alternative depends on the way the data is classified. One of the most popular evaluation methods is to associate to each alternative a real value, and the best alternative is chosen as the one with the largest or the smallest value.

In a higher dimension the notion of the smallest and the largest values is not available. In this case, the concept of partial order in a multidimensional space can be applied.

The Pareto cone: Let $\Re_+^n$ be the positive octant of the n-dimensional Euclidean space. Then, for two vectors $x = (x_1, ..., x_2)$, $y = (y_1, ..., y_n)$ in $\Re^n$, one has $x \le y$ if and only if $x_i \le y_i$, $i = 1,...,n$. The cone $\Re_+^n$ is called the Pareto cone because the original Pareto optimality is defined by the order generated by this cone. When $n=1$, the usual order of real numbers is exactly this order. The order is total in the sense that any two numbers x and y are comparable: either $x \ge y$ or $y \ge x$. On the other hand, when $n > 1$ this order is not total.

By using the concept of partial order it is possible to define the concept of optimal solution. However, in a real world situation, a decision making (trade-off) process is also useful to evaluate optimal solutions. Therefore, in this paper a procedure to determine the Pareto frontier is presented. An up-to-date discussion about this subject is presented by Pardalos and Du (2008).

3.2. Optimum workspace for planar parallel manipulators

In general the kinematic model of parallel structure is highly non linear and the end-effector position and orientation are coupled. Furthermore its workspace has a complex shape. Then, in order to draw its workspace discretization algorithms are usually applied because they are general and can be applied to any kind of kinematic architecture. The method consists in discretizing the three-dimensional space, solving the inverse kinematics at each point of the space, verifying the constraints that limit the workspace (Ceccarelli et al., 2005).

The optimization process can to consider the workspace as an objective to be optimized. In this paper the workspace is computed through geometric approach, i. e., the formulation uses geometric entities like parallelepiped, cylinder, sphere, rectangle and so on, that can consider the workspace in positioning and the orientation simultaneously. In such cases a fixed area, for planar geometric entities, or volume for solids, is the goal to be achieved (Gonçalves et al., 2007).

As the 5R linkage is a planar structure, in this work the methodology is applied to maximize the area of the workspace. For the analysis purposes the workspace area can be approximated by the smallest rectangle area A_p, containing the workspace. Thus, the problem consist in finding the size of design parameters such the workspace area A_p, which is a numerical approximation of the real volume, is as close as possible to a prescribed area A.

Therefore, if x and y are the sides of a rectangle, the objective function of maximum area can be achieved through the expression, Fig. 2,

$$f_1 = x.y \tag{17}$$

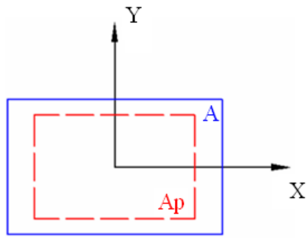


Figure 2. The rectangle geometry.

3.3. Optimum stiffness and singularity for planar parallel manipulators

One of the important limitations of parallel mechanisms is that they may lead to singular configurations in which the stiffness of the mechanism is compromised. The physical meaning of a singularity in kinematics refers to those configurations in which the number of degree of freedom (dof) of the mechanism changes instaneously. The concept of singularity has been extensively studied and several classification methods have been defined. Gosselin and Angeles (1990) suggested a classification of singularities for parallel manipulators into three main groups. The first type of singularity occurs when the manipulator reaches internal or external boundaries of its workspace and the output links loses one or more dof. Second type of singularity is related to those configurations in which the output link is locally movable even if all the actuated joints are locked. Third type is related to linkage parameters and occurs when both first and second type of singularities is involved. Tsai (1990) classify the tree type of singularity by: inverse singularity; direct singularity and combined singularity respectively. Their method is based in finding the roots of the determinant of the manipulator's Jacobian matrices. Another alternative approach to obtain singular configurations for parallel architectures is based in the analysis of stiffness matrix (Gonçalves and Carvalho, 2009).

Another problem of parallel manipulator are the compliant displacements that are changes on geometry due to the applied forces (Rivin, 1999). These compliant displacements in a parallel robotic system produces negative effects on static and fatigue strength, wear resistance, efficiency (friction losses), accuracy, and dynamic stability (vibration). The growing importance of high accuracy and dynamic performance for parallel robotic systems has increased the use of high strength materials and lightweight designs improving significant reduction of cross-sections and weight. Nevertheless, these solutions also increase structural deformations and may result in intense resonance and self-excited vibrations at high speed. Therefore, the study of the stiffness becomes of primary importance to design multibody robotic systems in order to properly choose materials, component geometry, shape and size, and interaction of each component with others. Some examples of design procedures based on stiffness analysis can be found in (Yoon et al., 2004; Deblaise et al., 2006)

Thus, the overall stiffness of a manipulator depends on several factors including the size and material used for links, the mechanical transmission mechanisms, actuators and the controller (Tsai, 1999). In general, to realize a high stiffness mechanism, many parts should be large and heavy. However, to achieve high speed motion, these should be small and light. Moreover, one should point out that the stiffness is greatly affected by both the position and the values of the mechanical parameters of the structure parts (Yoon et al., 2004).

There are three main methods have been used to derive the stiffness model of parallel manipulators (Deblaise et al., 2006). These methods are based on the calculation of the *Jacobian matrix* (Company et al., 2005); the *Finite Element Analysis* (FEA) (Bouzgarrou et al., 2004) and the *Matrix Structural Analysis* (MSA) (Deblaise et al., 2006; Przemieniecki, 1985; Dong et al., 2005; Gonçalves and Carvalho, 2008a).

The methods based on calculation of the Jacobian matrix are simple and they supply one initial estimation of the stiffness matrix. The uses of Finite Element Analysis models are reliable, but these models have to be remeshed over again, involving very tedious and time-consuming routines. However these models are well adapted to validate analytical models, or some experimental results. Methods based on matrix structural analysis are simple and easy for computational implementation.

In this paper, the stiffness matrix is obtained from the method *Matrix Structural Analysis* (*MSA*), also known as the *displacement method* or *direct stiffness method* (*DSM*). The methods of structural analysis is based on the idea of breaking up a complicated system into component parts, discrete structural elements, with simple elastic and dynamic properties that can be readily expressed in a matrix form. The discrete structure is composed by elements which are joined by connecting nodes. When the structure is loaded each node suffers translations and/or rotations, which depend on the configuration of the structure and the boundary conditions. For example, in a fixed linkage no displacement occurs. The nodal displacement can be found from a complete analysis of the structure. The matrices representing the beam and the joint are considered as building blocks which, when fitted together in accordance with a set of rules derived from the theory of elasticity, provide the static and dynamic properties of the whole structure (Przemieniecki, 1985).

The stiffness matrix k_j of a *j-th* three-dimensional straight bar with uniform cross-sectional area is

$$k_j = \begin{bmatrix} k_{bj} & -k_{bj} \\ -k_{bj} & k_{bj} \end{bmatrix} \tag{18}$$

where k_{bj} is given by:

$$k_{bj} = \begin{bmatrix} \frac{A_j E_j}{L_j} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12E_j I_{zj}}{L_j^3} & 0 & 0 & 0 & \frac{6E_j I_{zj}}{L_j^2} \\ 0 & 0 & \frac{12E_j I_{yj}}{L_j^3} & 0 & -\frac{6E_j I_{yj}}{L_j^2} & 0 \\ 0 & 0 & 0 & \frac{G_j J_j}{L_j} & 0 & 0 \\ 0 & 0 & -\frac{6E_j I_{yj}}{L_j^2} & 0 & \frac{4E_j I_{yj}}{L_j} & 0 \\ 0 & \frac{6E_j I_{zj}}{L_j^2} & 0 & 0 & 0 & \frac{4E_j I_{zj}}{L_j} \end{bmatrix} \tag{19}$$

On Equation (19) E_j and G_j are, respectively, the modulus of elasticity and the shear modulus of element *j; I_{yj}, I_{yz}* are the moment of areas about the *Y* and *Z* axes, respectively. J is the Saint-Venant torsion constant and A_j is the cross-sectional area.

The stiffness of a joint is given by (Gonçalves and Carvalho, 2008a):

$$k_{joint} = \begin{bmatrix} k_c & -k_c \\ -k_c & k_c \end{bmatrix} \tag{20}$$

Where $k_c = diag(k_{tx}, k_{ty}, k_{tz}, k_{rx}, k_{ry}, k_{rz})$; k_{tx}, k_{ty}, k_{tz} are the translation stiffness and k_{rx}, k_{ry}, k_{rz} the rotational stiffness along the axes.

Application of *MSA* needs to write the stiffness matrices of all elements in the same reference frame. This transformation, element by element, must be held before the assembly of the stiffness matrix of the structure. This transformation matrix, T_j, can be obtained from algebra linear.

Thus, the stiffness matrix of the elements in a common reference frame (elementary stiffness matrix), for segments, k_j^e, and for joints, k_{joint}^e, are:

$$[k_j^e] = [T_j][k_j][T_j]^T \tag{21}$$

$$[k_{joint}^e] = [T_j][k_{joint}][T_j]^T \tag{22}$$

After obtaining the stiffness matrix of each beam and joint in a common reference frame, the stiffness matrix of whole structure can be obtained using the *MSA*. Based on how the structure elements are connected, from their nodes, it is possible to define a connectivity matrix. As each segment and joint stiffness are known, the global stiffness matrix is obtained by a superposition procedure. This global stiffness matrix is singular because the system is free. After application of the boundary conditions, for example, where the displacements are known, the new matrix is invertible and the compliant displacements can be done by:

$$\{U\} = K^{-1} \{W\} \tag{23}$$

Where *U* are the compliant displacements and *W* are the external applied wrenches. This procedure is described in detail in (Gonçalves, 2009).

In a singular position the stiffness is compromised, and the inverse stiffness matrix of the whole structure, Eq. (23), in this configuration is badly scaled, identified by using a condition number. A large condition number indicate a nearly or singular position.

The condition number, *cond*, of a square matrix is the product of the norm of the matrix and the norm of its inverse (Meyer, 2000), Eq. (24).

$$cond(K) = \|K\| * \|K^{-1}\| \tag{24}$$

There are different ways to evaluate the matrix norm (‖ . ‖). In this paper the norm is calculated by:

$$\|K\|_1 = \underset{j}{Max} \sum_{i=i}^{n} \|k_{ij}\| \quad \text{(Absolute maximum sum of columns of K)} \tag{25}$$

As a result, a general computational routine for mapping workspace of a parallel robotic structure is given, since the stiffness matrix is dependent of the configuration of the structure. Simultaneously with the mapping of workspace, the method *MSA* is applied to obtain the stiffness matrix of structure and the computation of the conditional number.

Figure 3 represents the discrete structural elements of the 5R linkage where each structural element is defined by two nodes. Then, from Figs. 1b and 3 one has: link A_1B_1 is represented by nodes 1-2, link B_1P by nodes 3-4, link PB_2 by nodes 5-6, link B_2A_2 by nodes 7-8, the link base A_1A_2 by nodes 1-8, the passive joint B_1 by nodes 2-3, the passive joint P by nodes 4-5, the passive joint B_2 by nodes 6-7. In nodes 1 and 8 are the active joints that, for the *MSA* methodology are considered as blocked in order to obtain the compliant displacements.

The segments are built with steel ($E = 2 \times 10^{11}\ N/m^2$ and $G = 0.8 \times 10^{11}\ N/m^2$); the cross-sectional area is circular with $R = 0.005m$ diameter and $r_1 = 0.1m;\ r_2 = 0.1m$ and $r_3 = 0.1m$. The boundary conditions are given by actuators considered as blocked in nodes 1 and 8. In order to obtain compliant displacements an external force and torque are applied on node 5, which is considered center of the end-effector. The others joints are passive and modeled with $k_{tx} = k_{ty} = k_{tz} = 2 \times 10^{11}\ N/m;\ k_{rx} = k_{ry} = 2 \times 10^{11}\ N/rad$ and $k_{rz} = 0\ N/rad$ like proposed by Gonçalves (2009).

Applying the methodology *MSA* for the 5R manipulator is possible to map the stiffness, Eq. (22) simultaneously with calculation of the singularities positions, given by the conditional number, Eq. (23).

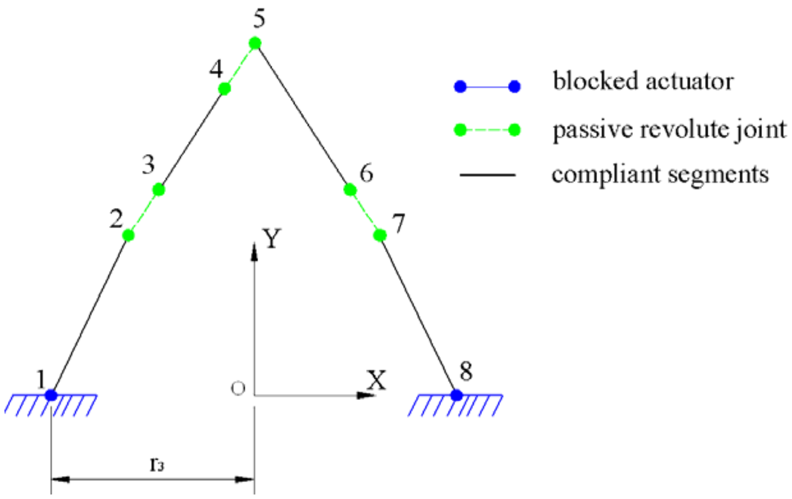


Figure 3. Nodes of the 5R mechanism to apply the *MSA* method.

The objective function for the analysis of the singularity can be given by

$$f_2 = \frac{cond(K)}{[cond(K)]_0} \tag{26}$$

where $[cond(K)]_0$ is the initial value.

The objective function that evaluates the stiffness, obtained by Eq. (23), considers the compliant displacements of point P, node 5, corresponding the linear compliant displacements x and y, and the rotational compliant displacement about axis z. The procedure for obtained the compliant displacements is described in details in Gonçalves (2009) and Gonçalves and Carvalho (2008a). The corresponding objective functions can be given as:

$$f_{3,1} = \frac{(f_{3,x})_d}{(f_{3,x})_g}; \qquad f_{3,2} = \frac{(f_{3,y})_d}{(f_{3,y})_g}; \qquad f_{3,\phi} = \frac{(f_{3,\phi z})_d}{(f_{3,\phi z})_g} \tag{27}$$

where $(f_{3,x})_d$, $(f_{3,y})_d$ and $(f_{3,\phi z})_d$ are the compliant displacements obtained by Eq. (23) and, the $(f_{3,x})_g$, $(f_{3,y})_g$ and $(f_{3,\phi z})_g$ are the initial values whose may be different for each problem.

3.4 Weighting Objective Formulation

To formulate the performance criterion that takes into account all the objective functions in such a way that an overall multi-criterion objective function can be written, the Weighting Objective Method is used. The minimization process leads to a Pareto optimal solution or, alternatively, to a set of optimal solutions. The scalar objective function that represents the performance criteria altogether is written as:

$$f(x) = \sum_{i=1}^{k} \alpha_i f_i(x) \tag{28}$$

where $\alpha_i \geq 0$ are weighting coefficients that represent the relative importance of each separate criterion. From the numerical point of view the minimization process depends also on the numerical values that express the objective functions. Due to scaling problems, the numerical values that express the objective functions should be adjusted. Otherwise, α_i will not represent the relative importance of the objective functions (Deb, 2001). Consequently, Eq. (28) should be rewritten as follows:

$$f(x) = \sum_{i=1}^{k} c_i f_i(x) \tag{29}$$

where c_i are scaling factors. Usually, satisfactory results are obtained if $c_i = \frac{\alpha_i}{f_i^0}$, where f_i^0 represents the minimum of the objective function f_i calculated separately (Eschenauer et al., 1990). Equation (29) was used in the optimization process shown in this paper.

4. NUMERICAL RESULTS

Numerical simulations were performed to evaluate the proposed objectives in a unified approach where the maximum area f_1, the singularity avoidance f_2 and the maximization of the stiffness f_3 are considered in the multi-objective formulation.

To obtain an optimal design of the structure the goal is to maximize the workspace, given by the objective function f_1, minimize the singularities, given by the objective function f_2, and maximize the stiffness, given by the objective function f_3. Then, without loss of generality, a minimization objective function can be given by

$$F = -\left(\frac{f_1}{f_1^0}\right)^2 + \left(\frac{f_2}{f_2^0}\right)^2 - \left(\sum_{i=1}^{3} \frac{f_{3,i}}{f_{3,i}^0}\right)^2 \tag{30}$$

The proposed formulation, Eq. (30), can be solved as a maximization problem by multiplying this objective function by (-1).

The design variables are the link diameter R, and the links length r_1, r_2 and r_3 of the 5R linkage which are comprised in the following intervals: $0.001 < R < 0.05\ m$, $0.01 < r_1 < 1\ m$, $0.01 < r_2 < 1\ m$ and $0.01 < r_3 < 1\ m$, respectively.

Since an initial design (R_0, $r_{1,0}$, $r_{2,0}$, $r_{3,0}$) are given, weighting factors need to be determined that to obtain an appropriate value, the objective function is evaluated without such constants, that is, $(f_i^0 = 1)$. The values of the objective functions when using such design are set as weighting factors f_i^0.

Deterministic and heuristic optimization methods were used to find the optimal solution of the problem.

For a deterministic evaluation, a Sequential Quadratic Programming (SQP) was adopted, since it belongs to the state of the art in nonlinear programming methods. At the major iterations, a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated using the BFGS method (Powell, 1978).

Results obtained by this methodology are presented in Table 1.

Table 1. Optimal design provided by a deterministic procedure.

Initial design (R, r_1, r_2, r_3)	f_1^0	f_2^0	$f_{3,1}^0$	$f_{3,2}^0$	$f_{3,3}^0$	Optimal design (R, r_1, r_2, r_3)
(0.005, 0.1, 0.1, 0.1)	2.7e-12	3.3e+18	9.4e+3	0.3	2.0e+5	(0.05, 0.01, 0.01, 1.00)
(0.01, 0.3, 0.3, 0.3)	2.4e-11	8.0e+17	4.0e+4	2.4	1.4e+5	(0.037, 0.010, 0.031, 0.010)
(0.03, 0.01, 0.01, 0.01)	2.7e-14	5.4e+20	7.9343	3.3e-004	4.1e+5	(0.05, 0.01, 0.13, 1.00)

The problem under consideration is highly nonlinear then, it follows that small deviation in the design values may lead to big deviations in the objectives. In this context the deterministic optimization is well suited to perform a fine tuning of the design parameters aiming to improve the overall performance of the system. The interpretation of this behavior is: if the small decrease of any performance will lead to a bigger increase of other performance, this new configuration will be preferred.

A second analysis was carried out by means of a heuristic optimization methodology, the so called Differential Evolution Methodology. This strategy is based on genetic algorithm and has been proven to be suitable to deal with a number of problems.

A feature of such methodology is that initial design is not required. Furthermore, local minima are not a problem, since the search direction does not requires information about the gradient of the objective function.

Without loss of generality, weighting parameters were chosen the same used in the first experiment. It should be pointed out that different values may influence the optimal design. This behavior can be used to provide a higher priority to some objective against others.

The optimal results are presented in Table 2. Different evolution criteria can be set in the procedure. The current implementation provides nine strategies of evolution (lines 1 to 9, respectively). Each strategy was evaluated three times (columns 1st , 2nd and 3rd respectively).

Table 2. Optimal design provided by a heuristic procedure.

	Optimal design											
Strategy	R			r1			r2			r3		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
1	0.0410	0.0305	0.0453	0.8700	0.7141	0.6138	0.0936	0.2295	0.6215	0.4058	0.1262	0.8608
2	0.0018	0.0044	0.0345	0.9842	0.3264	0.5511	0.1755	0.5356	0.4315	0.1152	0.6579	0.6480
3	0.0318	0.0345	0.0010	0.7743	0.7070	0.8668	0.9335	0.4479	0.6164	0.9730	0.0294	0.9901
4	0.0071	0.0454	0.0392	0.4955	0.5384	0.3442	0.8545	0.1181	0.6118	0.8752	0.8276	0.7438
5	0.0063	0.0217	0.0476	0.6354	0.3660	0.4495	0.1352	0.5627	0.0694	0.1430	0.7451	0.8681
6	0.0120	0.0105	0.0148	0.3798	0.8969	0.7337	0.0966	0.1081	0.1464	0.6437	0.0537	0.8384
7	0.0159	0.0193	0.0206	0.0557	0.5973	0.8336	0.2035	0.8738	0.1430	0.7230	0.9342	0.0699
8	0.0311	0.0080	0.0011	0.8205	0.5638	0.1973	0.8874	0.0145	0.1511	0.9318	0.7690	0.2754
9	0.0015	0.0205	0.0455	0.5370	0.3810	0.5567	0.2866	0.1398	0.0426	0.9468	0.4407	0.0633

In this case the method is not attracted by a local minimum. Differences on the optimal design are justified by a highly nonlinear nature of the problem and a high sensitivity of the objectives regarding small changes on the design variables.

5. CONCLUSION

In this paper a methodology to obtain design parameter of a parallel robotic structure was presented.

First, inverse and direct kinematic equations of a 5R symmetric parallel manipulator were presented. It was followed by general concepts of multi-objective optimization concepts, optimum workspace formulation, stiffness and singularity analysis.

A key point to evaluate multi-objective problems is the computation of weighting factors to correctly express objective priorities.

The current study considered objectives with the same priority. It was achieved by using results of an initial design as weighting factors. Other applications can to use different weighting for the objective functions.

Two strategies were used for the optimization process. The first strategy was a Quadratic Sequential Programming that could to improve the initial design by means of a local optimization. This method is recommended when a fine tuning of design variables is required. The improvement of the overall performance index is sometimes achieved by means of the penalization of individual objectives. The second strategy consists in a Differential Evolution Methodology which is a heuristic method to search for a global optimum. Different evolution parameters were considered in multiple runs.

The results show that there is no unique solution for this problem. Then, it follows that multiple designs lead to similar objective values. The proposed formulation is able to deal with the complexity of the parameters evaluated because the problem is highly nonlinear and coupled.

Future research includes the analysis of the Pareto frontier when a qualitative analysis is considered and the use of stochastic optimization methods to consider uncertainties in the parameters.

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