THE EFFECT OF EFFECTIVE AXIAL FORCE OVER STRUCTURAL RESPONSE OF A FREE SPANNING PIPELINE LAID ON SEABED

G.F. Carvalho, gcarvalho83@gmail.com
L.C.S Nunes, luizcsn@mec.uff.br
Mechanical Engineering Program (PGMEC-TEM), Universidade Federal Fluminense – UFF, Rua Passo da Pátria, 156, Bloco E, Sala 216, Niterói, RJ CEP 24210-240, Brazil

Abstract. *The main of this work is to perform a study for natural frequency behavior in a free span pipeline located in deepwater region. In this analysis are taken into account some important conditions such as environmental conditions and the effect of surrounding water to obtain the natural frequency while pipe-soil interaction was disregarded. The concept involved is to analyze the natural frequency response behavior of the free spanning pipeline varying span length and the effective axial force.*
Keywords: *Free span pipelines, allowable free span length and structural condition*

1. INTRODUCTION

The continuous increase for petroleum field have been generated an improvement in exploration technologies of industries all around the world, having the special attention in ultra-deep water.

Associate with this development, came the equipment and methods of petroleum exploration. In a general way, petroleum exploration in deep water is performed with flexible pipeline, due several conditions, such as high temperature and high pressure. However flexible pipelines are much more expensive than rigid pipelines and thus there is a great interest in the application of rigid pipeline for deep and ultra deep water exploration. Based on this, several researches are continuously developed in order to reach an acceptable methodology to predict the response behavior of the involved phenomenon.

The principal objective of this work is to study the natural frequency behavior of a free span when subjected to several effective axial forces, which may work with tensile or compressive force in the pipeline.

2. THEORETICAL ANALYSIS OF FREE SPANNING PIPELINE

In this section, will be presented the study of a single span with span shoulders laying on flat seabed in deep water region. In this case, it intends to describe the structural response of a free span subjected to an axial force. Clough and Penzien (1975) proposed the basic theory for the dynamics of a freely vibrating beam under influence of axial force, which is given by the differential equation presented below, and illustrated in the figure (1).

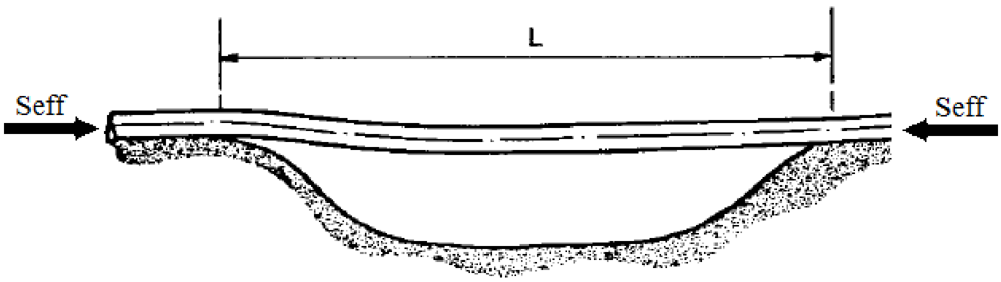

Figure (1) – Free Span Illustration

$$EI\frac{\partial^4 \omega}{\partial x^4} + F_a \frac{\partial^2 \omega}{\partial x^2} + m\frac{\partial^2 \omega}{\partial t^2} = 0 \tag{1}$$

Where $\omega=\omega(x,t)$ is the lateral deflection, x the axial co-ordinate, t is time, Fa is the effective axial force, EI the bending stiffness and m the effective mass of the beam per length (including mass of content and hydrodynamic added mass).

In order to simplify the equation (1), it is applied a variable transformation, which results in:

$$\frac{\partial^4\eta}{\partial\xi^4}+\beta\frac{\partial^2\eta}{\partial\xi^2}+\eta+\frac{\partial^2\eta}{\partial\tau^2}=0 \tag{2}$$

Where

$$\beta=\frac{F_a L}{EI}\text{ ; }\xi=\frac{x}{L}\text{ ; }\eta=\frac{\omega}{L}\text{ and }\tau=\left[\frac{EI}{m}\right]^{1/2}\frac{t}{L^2} \tag{3}$$

Through Galerkin method, presented below, it is obtained the solution of (2)

$$\eta(\xi,\tau)=\sum_{r=1}^{n}\phi_r(\xi)q_r(\tau) \tag{4}$$

$$\phi_r(\xi)=c_1 sen(\lambda_r\xi)+c_2\cos(\lambda_r\xi)+c_3 senh(\lambda_r\xi)+c_4\cosh(\lambda_r\xi) \tag{5}$$

The last equation associated with $\frac{\partial^4\eta}{\partial\xi^4}+\frac{\partial^2\eta}{\partial\tau^2}=0$, $\eta=\phi q$ give us

$$\phi''''q+\phi\ddot{q}=0\rightarrow\frac{\phi''''}{\phi}=-\frac{\ddot{q}}{q}=A_r^4 \tag{6}$$

Replacing equation (4) in equation (2) , multiplying by $\phi_s(\xi)$ and integrating for $\xi[0,1]$ domain, we got

$$\sum_{r=1}^{N}\left\{\lambda_r^4\phi_r q_r+\beta\phi_r''q_r+\phi_r\ddot{q}_r\right\}=0 \tag{7}$$

Then,

$$\sum_{r=1}^{N}\left\{\lambda_r^4\int_0^1\phi_r\phi_s dy+\beta\int_0^1\left(\phi_r''\phi_s dy\right)q_r+\int_0^1\phi_r\phi_s dy\ddot{q}_r\right\}=0 \tag{8}$$

Where,

$$\delta_{rs}=\int_0^1\phi_r\phi_s dy$$

$$C_{rs}=\int_0^1\left(\phi_r''\phi_s dy\right)$$

With this simplification, the equation (8) becomes

$$\sum_{r=1}^{N}\left\{\lambda_r^4\delta_{rs}+\beta C_{rs}+\delta_{rs}\ddot{q}_r\right\}=0 \tag{9}$$

Solving the equation, considering doing r=[1,2] and s=[1,2] we obtain the following equation:

$$\{\ddot{q}\}+[W]\{q\}=0 \tag{10}$$

Where

$$[W]=\begin{pmatrix}\lambda_1^4+\beta(C_{11}+C_{12}) & 0\\ 0 & \lambda_2^4+\beta(C_{22}+C_{21})\end{pmatrix}$$

The general solution for equation (10) is given by

$$\{q\}=\{A\}e^{i\Omega\tau}$$

$(\Omega^2[I]-[W])\{A\}=\{0\}$. Which is the non trivial solution of equation (10).

$$\det([W]-\Omega^2[I])=0 \tag{11}$$

Solving (11) the circular frequency is obtained as function of τ and t, which is given by:

$$\Omega_i^* = \Omega_i\sqrt{\frac{EI}{mL^4}}$$

$$\Omega_i^* = \sqrt{\frac{EI}{mL^4}\left[\lambda_1+\beta(C_{11}+C_{12})\right]}, \beta = \frac{F_a L}{EI}$$

Based on solution given mechanical engineers reference book 2007 for a pinned-pinned system

$\lambda_i = \pi$, considering $\dfrac{\partial^4\eta}{\partial\xi^4}+\dfrac{\partial^2\eta}{\partial\tau^2}=0$

Hence, the natural system frequency can be obtained as

$$f=\frac{\Omega_i^*}{2\pi}=\sqrt{\frac{EI}{mL^4}\left[\lambda_1+\beta(C_{11}+C_{12})\right]}$$ or

$$f=\frac{\lambda_1}{2}=\sqrt{\frac{EI}{mL^4}\left[\frac{\lambda_1{}^2}{\pi^2}-\frac{F_aL^2}{EI\pi^2}\right]} \tag{12}$$

where
f = natural frequency regarding first fundamental mode
EI = Bending stiffness of the beam
m = Effective mass of the beam considering added mass
Fa = Effective axial tension
L = Span length

Finally, the free span natural frequency can be obtained simplifying the equation (12), resulting

$$f=\frac{\pi}{2}\sqrt{\frac{EI}{mL^4}\left(1-\frac{F_aL^2}{EI\pi^2}\right)} \tag{13}$$

3. RESULTS AND CONSIDERATIONS

In order to analyze the influence of the axial force in natural frequency, let us take into consideration the results previously presented in Eq. (12). The pipeline data data used in the model are presented in Table 1. The material adopted for pipeline fabrication is the Cr-Mn API X60.

Table 1 – Pipeline Data

Parameter	**Value**	**Unit**
Outside diameter	10.75	inch
Wall thickness	0.938	inch
Corrosion allowance	3.0	mm
Fabrication Tolerance	8.0	%
Young`s Module	207000	Mpa
Poisson ratio	0.3	-
Steel mass density	7850	kg/m^3
Maximum ovalisation	2	%
Coating thickness (3LPP)	3	mm
Coating mass density	950	kg/m^3
Temperature Expansion Coefficient	1.17E-05	1/K

Regarding environmental conditions, the influence of waves and current effect are neglected in order to assess only the effect of axial force over the free span. It is important to emphasize that the effect of surrounding water is being considered as added mass in structural effective mass. Table 2 presents environmental and coating data.

Table 2 – Environmental and coating data

Parameter	Value	Unit
Concrete coating thickness	0.0	mm
Concrete coating density	2250	kg/m^3
Concrete Construction strength	0.0	MPa
Concrete deformation / slippage constant	0.0	-
Sea Water Density	1025.0	kg/m^3
Gravitacional Force	9.81	m/s^2

Figure (1) illustrates five structural modal vibrations, however in this work only the first mode is being considered for the assessment of effective axial force effect. The modal amplitude was normalized in order to provide only the mode shape and location of its maximum amplitude, considering a 40m free span.

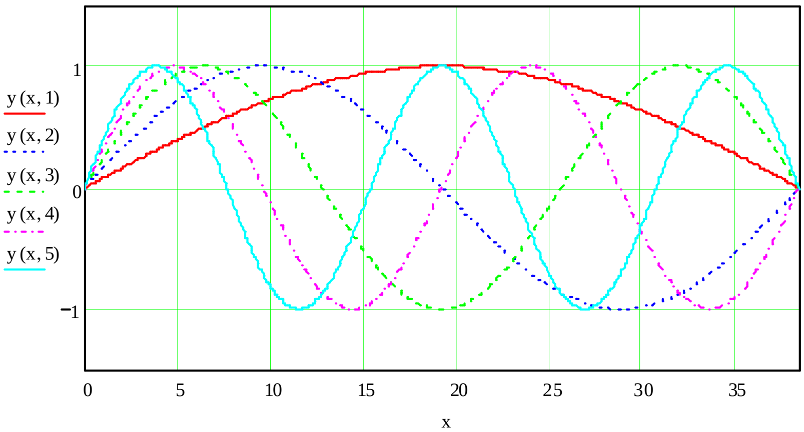

Figure (2) – Free spans modal shapes

All necessary parameters were calculated in a mathcad spreadsheet developed for this purpose and several span length value were considered in order to analyze the natural frequency for a range of effective axial forces.

The span length varies from 0m to 80m in steps of two meters and for each span length the natural frequency is evaluated for a determined effective axial force. This results can be seen in Figure(2).

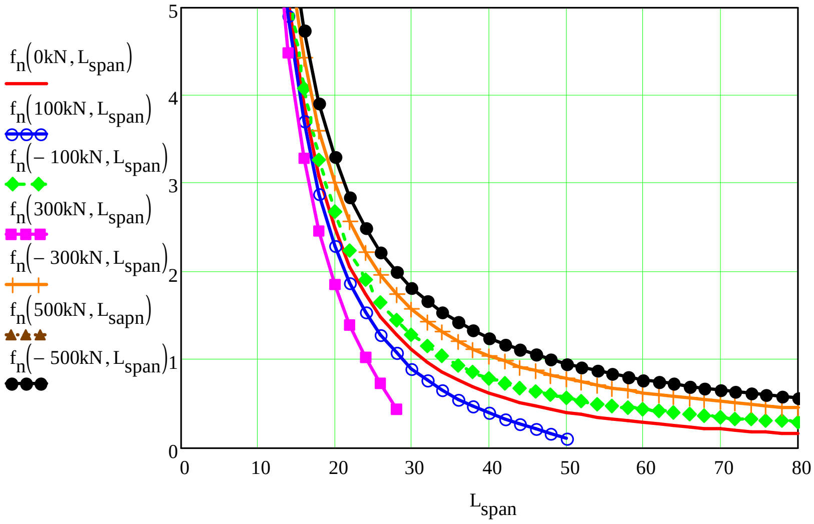

Figure (3) – Free span natural Frequencies

It can be seen in Figure (3) that the natural frequency of a fixed span increases when subjected to tensile force. In the other hand when the pipeline is subjected to a compressive force, the free span natural frequency decreases accordingly.

It is important to note that premise considered the positive signal convention for effective tension as compressive force, as it can be seen in figure (1). Hence, as figure (4) present the falling of the free span natural frequency as effective force increases, it is important to remember that effective tension represent span compression with positive signal.

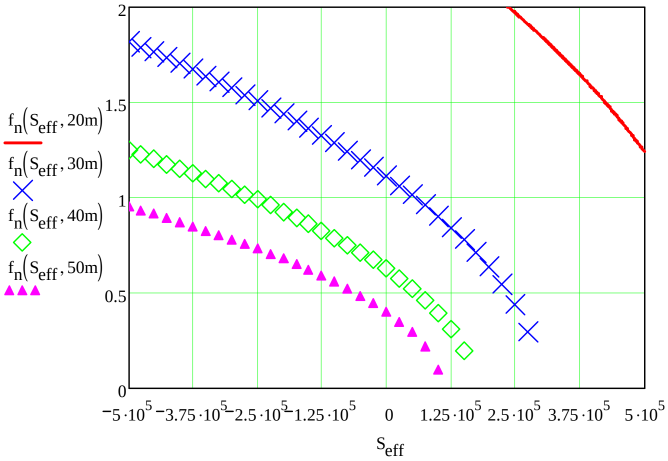

Figure (4) – Natural Frequency Variation along a single free span

4. CONCLUSION

This is a preliminary study regarding the effect of effective axial tension over natural frequency of a single free span, and thus, several conditions has been disregarded, such as effect of environmental loads and pipe-soil interaction.

In a future study this parameters shall be duly considered alongside with its respective influence over structural response.

Another possibility is to evaluate the effect of mass variation over the system, which may be caused by corrosion effect or the loss of pipeline concrete coating.

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