DYNAMICAL ANALYSIS OF AN END MILLING PROCESS

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Abstract *The cutting processes include different manufacturing procedures as milling, turning and drilling. In all of these processes, the machine tool vibration plays an important role concerning the cutting characteristics. Therefore, its correct understanding is essential in order to improve the workpiece surface quality and avoid tool breakage, chatter and squeal, undesirable phenomena related to an improper functioning. This article presents a nonsmooth two-degree of freedom model to analyze the nonlinear dynamics of end milling tools. This model establishes, at least, a caricature of the dynamic behavior of the real process establishing its vibration relative to the workpiece. The cutting force is represented as a composition of contact/non-contact of the tool and the workpiece with stick-slip behavior guided by friction force and the prescribed velocity of the tool holder. Numerical simulations are carried out showing some different profile for displacement situations related to proper and improper functioning during the cutting process machined surfaces. The general qualitative aspects of the milling process is captured by the proposed model that can be used to identify characteristics of the system responses.*

Keywords: *End milling, force prediction, nonlinear dynamics, nonsmooth systems, chatter, machining.*

1. INTRODUCTION

Machining process is associated with a complex dynamics that involves the coupling of different phenomena. During cutting processes, for example, it is common to have temperature variations that induce dramatic changes in expected behaviors. Although all these complexity, some simple models may be used to obtain useful information concerning tool and workpiece behaviors as dynamic tool prediction. The machine tool vibration during the cutting process plays an important role concerning the workpiece surface quality and also the tool durability. Chatter and squeal are some undesirable phenomena related to an improper functioning. During certain cutting conditions, the motions of the workpiece-tool system are characterized by large amplitudes, which are not desirable for obtaining a good surface finish. The undesirable motions, which are often referred to as chatter, can result in wavy surfaces on the workpiece, inaccurate dimensions, and excessive tool wear.

The analysis of machining using a dynamical approach has been done by different research efforts. The first works in this way is due to Tobias (1965) , Merritt (1965) and Altintas (2000) that investigated the chatter on the process. Their research provided instability diagrams to inform the safe parameters to avoid chatter. Afterwards, a new approach has been developed using nonlinear dynamic tools as bifurcation analysis to provide better comprehension of the process (Moon, 1978) .

Another references treated machining operations by dynamical approach. For turning operations, Chandiramani and Pothala (2006) analyzed the dynamics of cutting considering a two-degree of freedom system (2-dof) and orthogonal cutting modeling to predict chatter. Pratt and Nayfeh (1999) studied the boring bars for turning, which commonly presents chatter problems, using experimentally determined modal properties of the tool. Also dedicated to turning, Kalmar-Nagy *et al* (2001) showed the existence of a subcritical Hopf bifurcation in the delay-differential equation used to describe the machining equations of motion. The stability of the milling process was investigated by Zhao and Balachandran (2001), Insperger *et al* (2003) and Gradisek *et al* (2005) by considering single and two degree of freedom systems for different experimental conditions. The mathematical model is represented by delay-differential equations.

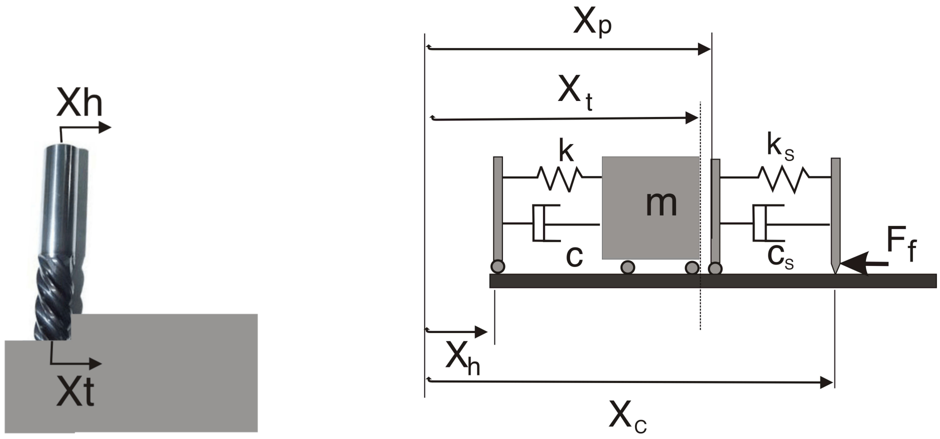
Milling process has the peculiarity to have a contact/non-contact behavior due to its geometry and due to tool vibration. This kind of behavior is related to nonsmooth systems that are usually associated with either the friction phenomenon or the discontinuous characteristics as intermittent contacts (Savi *et al*, 2007) . This article investigates the milling process representing the end milling tool by a mass-spring-dashpot system and the contact with the workpiece by a nonsmooth contact/non-contact system. The process of cutting is related to a stick-slip behavior that defines whether the chip is being removed from the workpiece. The tool holder displacement is prescribed and the local force that allows the slip motion is related to the workpiece shear stress. Under these assumptions, the equation of motion is represented by a differential equation that is solved employing the Runge-Kutta method. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.

2. DYNAMICAL MODEL

The milling process is modeled by assuming a full immersed milling in the x direction. Figure 1(a) shows the general view of the tool and the workpiece, where it is identified the prescribed displacement, X_h, which is defined by the feed velocity and eventually by the spindle speed when there is an eccentricity, and the tool tip displacement, X_t. The model is a nonsmooth system composed by a primary system that represents the tool and a secondary system, representing the workpiece. The primary system consists of a linear spring-dashpot-mass oscillator with parameters m, k, c and displacement X_t.

The workpiece system has a weightless slider connected to a linear spring-dashpot system with parameters k_s, c_s. The movement of the secondary system is a progressive motion activated when the tool force exceeds the dry friction force, F_f. An internal displacement X_p is related to the gap that separates the mass to the secondary system: the workpiece. This internal displacement cannot be measured but can be tracked with the other parameters.

Similarly to the stick-slip phenomena reported by Marian (2001) , the progressive motion, represented by the displacement X_c, occurs when the force acting on the slider exceeds the threshold of the dry friction force, considering that there is no cutting fluid.



(a) Tool (b) System model

Figure 1. Tool and system modeling

As the system may operate in stick-slip phases and contact-non contact situations, its dynamic behavior is nonlinear and its equations should consider each case. The system dynamics may be understood as a three stage motion as represented in Figure 2: non-contact, contact in stick and contact in slip. The non-contact stage may be understood as a two independent systems representing the situation where the tool is not in contact with the workpiece. Figure 2(a) shows that the displacements X_t and X_h are not connected with X_c and X_p by the mass.

An auxiliary displacement g is used to locate the work-piece position related to the tool position, this variable remains equal to X_p (as in Figure 2) in non-contact case and in slip phase, as it is shown on Figure 2c. The displacement g changes only when the slider slips simulating the chip removal. Note that this is an auxiliary variable used to represent the equilibrium point of the secondary system.

As long as g is greater than X_t, the system remains separate. When there is no more gap between X_t and X_p, both systems are in contact and the dynamic changes, as shown in Figure 2(b), for stick stage, and Figure 2(c) for slip stage. In stick stage, the slider displacement does not change and the system is restricted to a fixed boundary. In slip stage this boundary is free and X_c displacement changes. The equations of motion are formulated by considering each kind of response separately.

The primary system is described by the dynamic equation of the single degree of freedom, as follows:

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = F \tag{1}$$

2.1 Non-contact Stage

In non-contact stage, when $X_t < g$, the secondary system has no movement and its displacements does not change $\dot{X}_p = \dot{X}_c$. The primary system is free and the contact force F is zero.

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = 0 \tag{2}$$

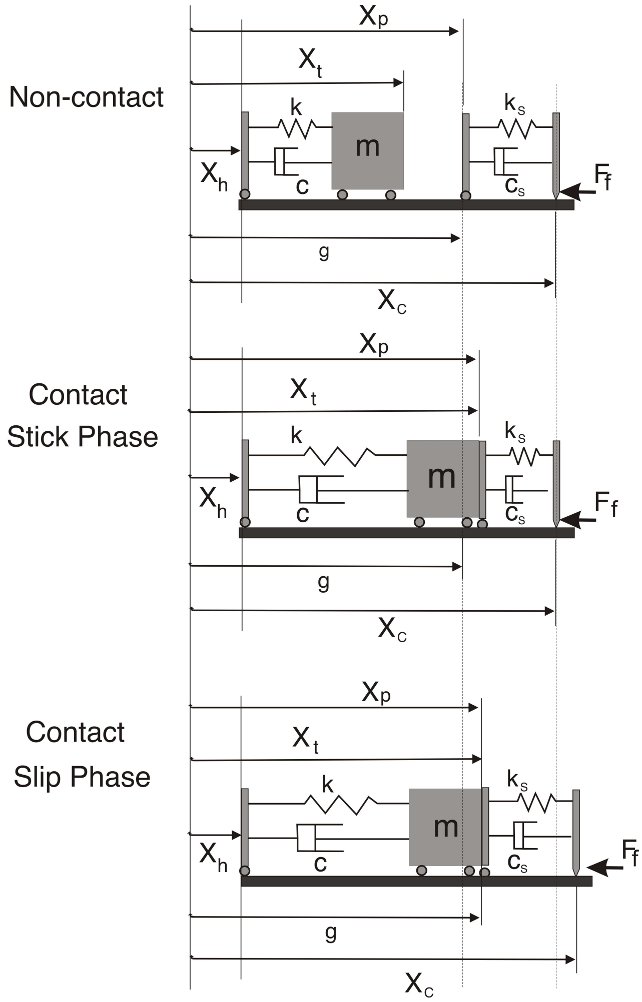


Figure 2. Contact / Non-contact situations

2.2 Contact - Stick Stage

The chip, represented by the slider, is not removed in stick stage. This case occurs when the friction force F_f is less than a maximum friction force supported by the workpiece material F_{fmax}:

$$F_f < F_{fmax} = A_s \tau_s \tag{3}$$

which can be considered as a function of the shear area and the maximum shear stress. The friction force is calculated, in stick phase, by:

$$F_f = \Big(k_s(X_t - X_c) + H[\dot{X}_t] c_s \dot{X}_t) \Big) \tag{4}$$

Note that the damping coefficient when the tool retracts from the workpiece ($\dot{X}_t < 0$) is not considered, so the Heaviside function is used: $H(*) = 0$ when $(*) < 0$ and 1 else if. For this case the contact force F is equals to the friction force F_f:

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = F_f \tag{5}$$

2.3 Contact - Slip Stage

During slip stage, when the chip is being removed, the maximum friction force $F_{fmax}$is achieved. The chip position X_c follows the workpiece displacement, $X_c = X_p +$ *constant*, and g assumes the work-piece position $g = X_p$. The contact force is, in slip phase, the value of maximum friction force F_{fmax}.

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = F_{fmax} \tag{6}$$

2.4 Equations of motion

Under these assumptions, for either contact and non-contact situations, the system dynamics may be represented by a nonsmooth system as follows:

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = F \tag{7}$$

where F is:

$$F = \begin{cases} 0 & \text{if } X_t < g \\ F_f & \text{if } F_f < F_{fmax} \\ F_{fmax} & \text{otherwise} \end{cases} \tag{8}$$

3. NUMERICAL SIMULATIONS

Numerical simulations are carried out in order to evaluate the model capability to describe milling process. The governing equations are solved by considering the forth order Runge-Kutta method with time steps less than 5.10^{-4}s. The end milling tool parameters as diameter, run-out, damping (Mann, 2005)) and system parameters are presented in Table 1. The depth of cut was 10 mm and the distance between the tool holder and the workpiece in $t = 0$ is 1 mm. Moreover, it is assumed that the workpiece material is under plastic behavior, then the damping coefficient for the workpiece is defined as 10 times the damping tool coefficient ($c_s = 10c$). Note that for $\dot{X}_t < 0$, when the tool mover back from the workpiece, c_s vanishes.

The machine tool is represented by the tool holder and it has a prescribed displacement X_h defined by the feed velocity v_f, the spindle speed w and the run-out distance ρ between the the tool axis and the spindle speed axis.

$$X_h(t) = v_f t + \rho \sin(wt) \tag{9}$$

The milling tool is considered as a flexible body and linear elastic behavior is assumed for the end milling tool material and, therefore, the tool parameters m, k and c are calculated by considering solid mechanics principles from the tool geometry and material properties.

The stiffness of the primary system is defined by considering the tool as a circular beam in bending. Under this assumption, there is a relation given by:

$$k = \frac{3E_t.\pi.d^4}{64.L^3} \tag{10}$$

where tool material Young modulus E_t is considered as 200 GPa, d is tool diameter and L is the free tool length.

The workpiece stiffness is evaluated by assuming that the workpiece is a compression beam, therefore:

$$k_s = \frac{E_c.d.h_c}{10.L} \tag{11}$$

where workpiece material Young modulus E_c is considered as 200 GPa and h_c is the depth of cut.

The feed per tooth for a two flute milling tool is written as a function of z, the number of teeth:

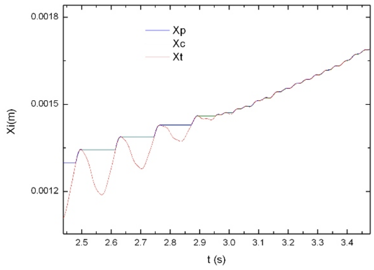
$$f_t = \frac{v_f}{wz} = \frac{v_f}{2w} \tag{12}$$

For the sake of simplicity, the cutting force is assumed to be represented by the dry friction. Actually, cutting force in end milling is a function of the chip thickness and several cutting pressures (or specific pressures) depending on the type of modeling (Araujo, 1997). The dry fiction is evaluated from the shear strength calculated by the shear area as a function of the depth of cut h_c and approximated by:

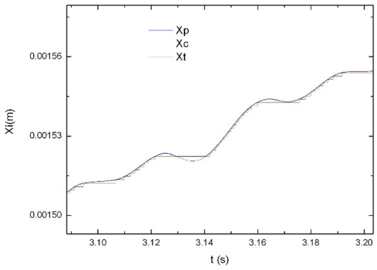
$$A_s = \int_0^\pi h_c f_t sin\phi d\phi \tag{13}$$

Table 1. Tool, workpiece and machine tool properties.

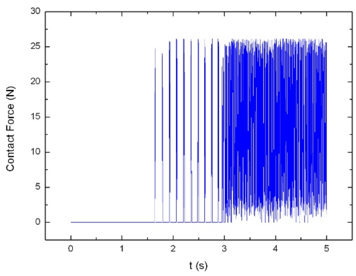
Diameter (d)	Height (L)	Damping (c)	Elast.Mod. (E_t)	Run-out (ρ)	Spindle (w)	Depth of cut (h_c)
10 mm	100 mm	20 N s/m	200 GPa	0.05 mm	1800 rpm	10 mm



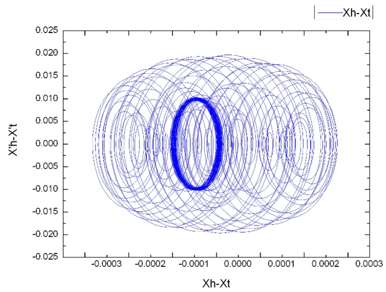
(a) Displacement



(b) Displacement - focused window 0.1 s

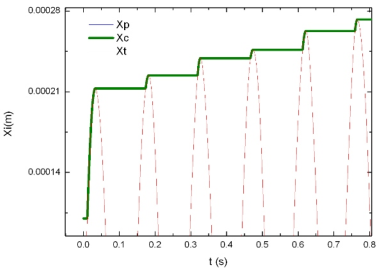


(c) Contact force - F

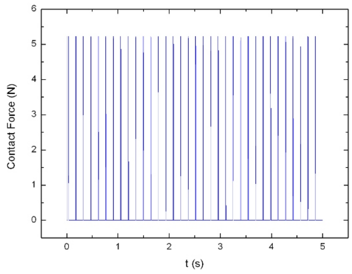


(d) Phase space

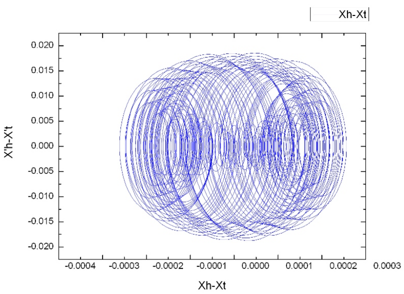
Figure 3. $v_f = 0.5$mm/s, $\tau_s = 100$MPa and $w = 1800$rpm



(a) Displacement



(b) Contact force - F

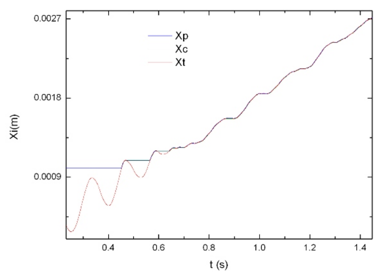


(c) Phase space

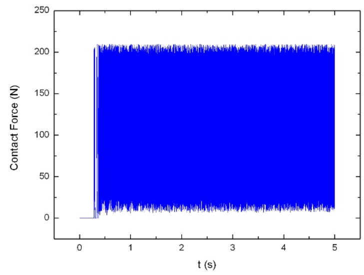
Figure 4. $v_f = 0.1$ mm/s, $\tau_s = 100$MPa and $w = 1800$rpm

This model represents a qualitative description of the milling process. The tool tip displacement, X_t, can be experimentally measured with a capacitor sensor as in Mann *et al* (2005). The internal displacement X_p cannot be directly measured but can be tracked with the other parameters. The contact force is a simple representation of the cutting force and its description can be improved as described in Araujo (1997) and Ehmann (1997). The tool force can be measured by a dynamometer when the tool is in contact to the workpiece and it is responsible to the chip removal. The system dynamics is investigated by considering different operational conditions of the milling process. The idea is to represent different aspects related to proper and improper functioning. In order to describe these conditions, different machine tool velocities and workpiece material are used.

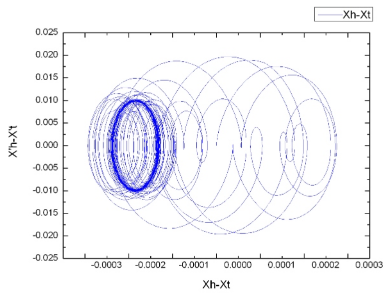
Initially, let us consider a condition that represents an ideal functioning condition during the milling process. The feed velocity and the shear stress limit is $v_f = 0.5$mm/s and $\tau_s = 100$MPa. Figure 3(a) presents the displacement time history of different parts of the system, namely the tool, the work-piece and the chip. Figure 3 also shows the contact force (3b) and the phase space of the response (3c). It can be noted the moment that the tool holder makes the tool to touch the workpiece between 2 and 3s. Afterwards, the contact force varies from zero to 25N, the force that the tool cuts the workpiece.



(a) Displacement



(b) Contact force - F



(c) Phase space

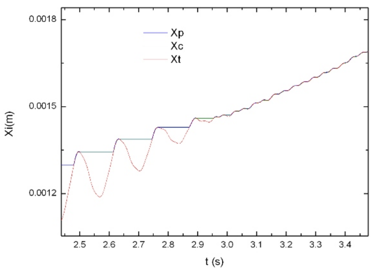
Figure 5. $v_f = 2$mm/s, $\tau_s = 100$MPa and $w = 1800$rpm

The chip thickness may be estimated from the displacement steps that the work-piece material let the tool cut. In order to improve the surface quality, the chip steps must be as smaller as possible. In the same way, the tool cannot touch and leave the work-piece with large displacements since this behavior is related to impacts on the tool.

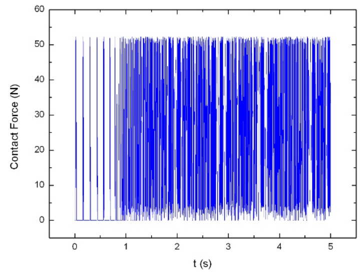
The increase of system velocities can change the dynamical response of the system and this can be associated with surface quality and operational conditions. In order to show this kind of behavior, different spindle speeds and feed velocities are now considered. Under these conditions, the response is changed. Another important parameter that may cause different dynamical response is related to the material work-piece property, represented by the shear stress limit and its impact on the stick-slip behavior. Therefore, system velocity and material properties define the main dynamical aspects of the system and for a softer material, the velocities can be higher without an impact on chip thickness.

Figures 4a and 5a present the time history of the displacements on the system with $0.1mm/s$ and $2mm/s$ respectively. It can be noted that for higher feed velocities the chip formation is smoother and the movement described by the tool holder is reflected on the chip movement. Figures 4 and 5 also show the contact force (4b and 5b) and the phase space of the response (4c and 5c).

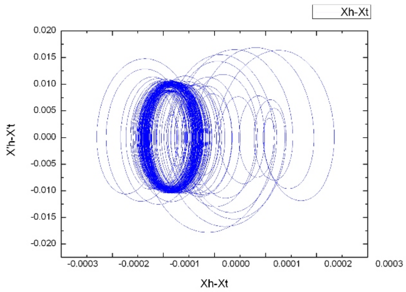
At this point, material property is changed in order to evalute its influence in system dynamics. Figures 6a and 7a present the displacement time history of the system with $200MPa$ and $500MPa$, respectively. For higher stress limits the system presents large steps that represents a improper surface quality. The contact forces, shown in Figures 6b and 7b, are higher for harder materials. Phase spaces of the response are presented in Figures 6c and 7c showing the difference of the steady state response that tends to be more regular for softer materials.



(a) Displacement

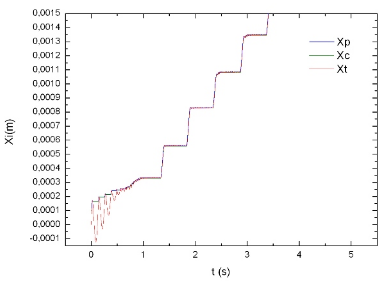


(b) Contact force - F

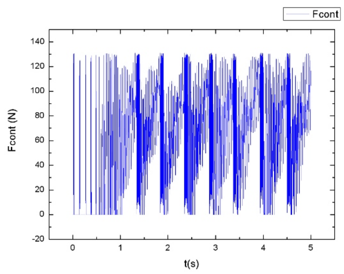


(c) Phase space

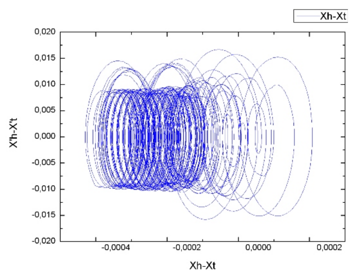
Figure 6. $v_f = 0.5$mm/s and $\tau_s = 200$MPa and $w = 1800$rpm



(a) Displacement

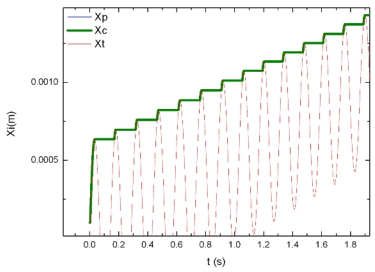


(b) Contact force - F

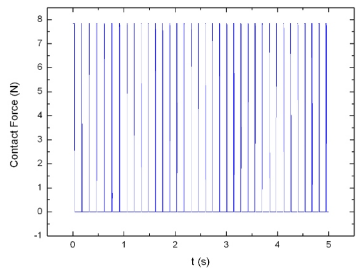


(c) Phase space

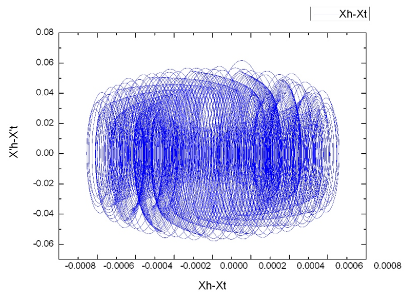
Figure 7. $v_f = 0.5$mm/s, $\tau_s = 500$MPa and $w = 1800$rpm



(a) Displacement

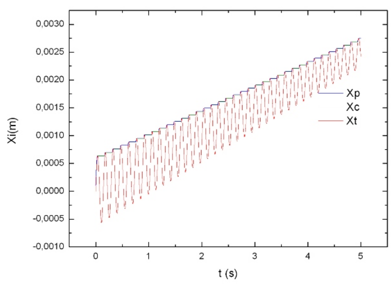


(b) Contact force - F

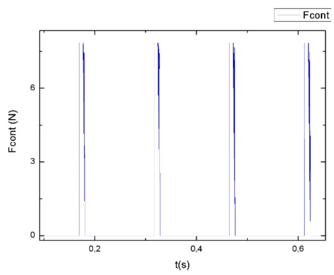


(c) Phase space

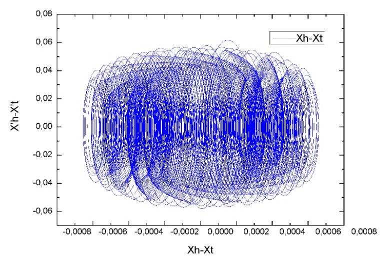
Figure 8. $v_f = 0.5$mm/s, $\tau_s = 100$MPa and $w = 3000$rpm



(a) Displacement



(b) Contact force - F



(c) Phase space

Figure 9. $v_f = 0.5$mm/s, $\tau_s = 100$MPa and $w = 6000$rpm

Figure 8ca and 9a presents the time history of the displacements on the system with $3000 rpm$ and $6000 rpm$ respectively. Both spindle speeds conducted to chip formation that had no smooth displacements. Figures 8 and 9 also show the contact force (8b and 9b) and the phase space of the response (8c and 9c).

4. CONCLUSIONS

Machining process involves the coupling of different phenomena and can be associated with complex dynamics behavior. Nevertheless, useful information concerning machining process can be achieved from a dynamical approach analyzing simple dynamical systems. This article deals with the nonlinear dynamical analysis of the milling process using a nonsmooth contact/non-contact system. The process of cutting is related to a stick-slip behavior that defines whether the chip is being removed from the workpiece. The tool holder displacement is prescribed and the local force that allows the slip motion is related to the workpiece shear stress. Numerical simulations show that the proposed model captures the main characteristics of the dynamic behavior of the end milling. Some situations related to proper and improper functioning during the cutting process are discussed. The general qualitative behavior represented by the proposed model may be confirmed by experimental tests that measure forces and displacements related to the system dynamics.

5. ACKNOWLEDGEMENTS

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