ESTABLISHING A STOP CRITERION IN ARTIFICIAL NEURAL NETWORK TRAINING FOR COMPOSITE MATERIALS

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Abstract. *The main purpose of this study is to obtain a stop criterion to assess the applicability of an algorithm based on artificial neural networks (**ANNs**), in the analysis of the fatigue behavior of composite materials. Accordingly, **ANN** training was performed with the fatigue behavior data of various materials obtained from the literature. These results show the viability of the algorithm even when using a small number of **S-N** curves. This viability is confirmed by means of the correlation coefficient and the mean squared error when the **ANN** and experimental results are compared. Two composite materials for network training obtained from the literature were used for this analysis. One of these is for validating the results and the other to establish the stop criterion; in addition, these results were compared with those of previous studies.*

Keywords: *Artificial Neural Networks, Composites, Fatigue, Stop criterion.*

1. INTRODUCTION

Composite materials are composed of two or more materials, in which the compatibility between their phases is taken into consideration; that is, matrix and reinforcement. The study of these materials is extremely important for diverse application fields, which range from recreational equipment to military applications. These vast application possibilities are mainly due to the various characteristics that can be obtained from these materials, among which are their low weight, good mechanical resistance and resistance to corrosion (Freire Jr, 2005; Levy, 2006).

Metallic materials have well-defined, repeated and predictable properties, demonstrated in the classic manufacturing processes technologically consolidated over the course of several decades. On the other hand, the properties of composites are significantly influenced by a large number of parameters. If on one hand this fact makes the mathematical modeling of the mechanical behavior of composites difficult and troublesome, on the other it may enable the freedom to manufacture composite materials, endowing them with properties that meet the specific requirements of a project. Thus, composites can be effectively designed simultaneously to the structural component that is needed for a given application, providing them with unique properties to meet the specific requirements of the project (Levy, 2006).

During the design of structures and equipment submitted to cyclical loads in which composites are used as raw material, a large number of fatigue tests are needed to obtain a certain degree of material reliability. However, these tests are inconvenient because they are time consuming. High-cycle fatigue tests have more than 10^3 cycles and added to this factor is the need for labor, given that a technician is required to monitor the entire process, thus increasing costs (Freire Jr, 2005; Sutherland, 1999).

The ideal solution would be to obtain, with reasonable reliability, the fatigue response of the material with a minimum number of tests. This would enable the designer to make preliminary predictions of the likely fatigue life of the material before spending time and money on a more thorough analysis with a large number of tests (Freire Jr, 2005).

Thus, several mathematical models in the specialized literature are used to predict the life of composite materials by means of failure diagrams that analyze the overall fatigue behavior of the material. Included in these models are empirical and semi-empirical models and more recently, artificial neural networks (**ANNs**) have been introduced for this same purpose. However, in the case of **ANNs**, studies are still in the embryonic phase and much more research is required (Freire Jr 2005, Mandell et al., 1997; Bond, 1999; Beheshty et al., 1999).

2. MATERIALS OBTAINED FROM THE LITERATURE

In this paper three composite materials from previously published studies were used (Freire Jr, 2005; Freire Jr, 2009) along with those from two new investigations, one of which was used for analysis and validation of the network architecture. These results established a stop criterion to be used in two other datasets to prove the applicability of the practice.

The material used to validate the stop criterion is denominated $MAT(0)_2$ (DOE/MSU, 2003). It consists of a two-layer glass fiber plastic tested along the main fiber direction and manufactured as D155 fabric (gramature = 527 g/m^2) and the matrix is polyester-based, in which Coresina 63-AX-051 by the hand lay-up process was used, with a 48% volume of glass fiber. The material used to confirm the stop criterion is defined by the acronym **GRP** (glass fiber reinforced polyester). It is a glass-fiber plastic reinforced with polyester resin and this material was fatigue tested in the loading direction at 0º and 45º to the configuration of its fibers. The material has six unidirectional glass fiber layers where fiber density is 700 g/m^2 in the 0º layer, 450 g/m^2 in the +45º layer and 225 g/m^2 in the -45º layer, with a configuration of $[0/(\pm45)_2/0]_T$ (Philippidis, 2001; Philippidis, 1999).

3. DATA PRE-PROCESSING

To obtain the **S-N** curves of the materials studied here, generalization of the power law presented in equation 1 was applied.

$$\log(\sigma_a)=A-B\cdot[\log(N)]^P \tag{1}$$

In the above equation, **A**, **B** and **P** are constants that must be determined, **N** is the number of cycles to rupture the material and $\boldsymbol{\sigma_a}$ is the stress amplitude to which the material is subjected. The values of constant **A**, **B** and **P** for each fatigue ratio given by equation 1 are shown in tables 1, 2 and 3, where $\boldsymbol{\sigma_{ultT}}$ the value of the last tensile stress (tensile strength limit) and $\boldsymbol{\sigma_{ultC}}$ is the value of the last compression stress (compression strength limit). The values of the constants shown in tables 1 and 2 refer to the **GRP** materials in the 0º and 45º configurations and to the **MAT(0)$_2$**.

Table 1: Data obtained for the **S-N** curves and fatigue ratios of the GRP 0 º material.

Stress Ratio (R)	A	B	P	Correlation Coefficient (r)
10	2	0.0061413	1.88	0.93
-1	2.33	0.07	1	0.99
0.1	2.04	0.000401	3.5	0.93
0.5	1.79	0.000165	3.5	0.87
σ_{ultC}	-216.68 (MPa)		σ_{ultT}	244.84 (MPa)

Table 2: Data obtained for the **S-N** curves and fatigue ratios of the GRP 45º material.

Stress Ratio (R)	A	B	P	Correlation Coefficient (r)
10	2.14	0.15	0.73	0.99
-1	2.03	0.06	0.91	0.96
0.1	1.79	0.04	1.35	0.97
0.5	1.54	0.03	1.39	0.98
σ_{ultC}	-112.7 (MPa)		σ_{ultT}	139.12 (MPa)

Table 3: Data obtained for the **S-N** curves and fatigue ratios of the MAT(0)$_2$ material.

Stress Ratio (R)	A	B	P	Correlation Coefficient (r)
2	2.18	0.01	1.21	0.73
10	2.42	0.05	0.93	0.9
-1	2.77	0.06	1.06	0.98
-0.5	2.95	0.11	1	0.99
0.1	2.81	0.09	1.02	0.96
0.5	2.55	0.11	0.72	0.94
σ_{ultC}	-599.41 (MPa)		σ_{ultT}	1423.69 (MPa)

The values obtained for the correlation coefficient show that the equation used fit the experimental data well; this did not occur only for **R** = 2 of the MAT(0)$_2$ because data dispersion is very high.

4. MATEMATICAL MODEL

To create the mathematical model, the multilayer perceptron network trained by the backpropagation algorithm was used. This model contains architecture consisting of two input neurons (mean stress and number of cycles and an output neuron (stress amplitude), in order to have a function that satisfied the condition shown in equation 2.

$$\sigma_a = f(\sigma_{med}, N) \tag{2}$$

Where $\boldsymbol{\sigma_a}$ is the stress amplitude applied (maximum stress minus minimum stress divided by two), $\boldsymbol{\sigma_{med}}$ is mean stress (maximum stress plus minimum stress divided by two) and **N** is the number of cycles at which material rupture occurred (Freire Jr, 2005).

For the material used to validate the stop criterion, a hidden layer was used with 2 to 30 neurons, all with bias and sigmoid activation function in the hidden neurons and linear function in the output neuron. The retropropagation algorithm based on the rule of the moment (Freire Jr, 2005; Haykin, 2001) was used in training. Network training was carried out based on the data obtained by the **S-N** curve using equation 1 (Freire Jr, 2005).

The diagram below shows the training mode of the **ANN** (a) and the model obtained (b), where **TRE** represents the number of curves (**S-N** curves obtained from equation 1) used for training the **ANN**, **TOD** and the total number of functions used, **e** is the error between the desired response and the current **ANN** response and **w** is the matrix of synaptic weights of the **ANN** (Freire Jr, 2005).

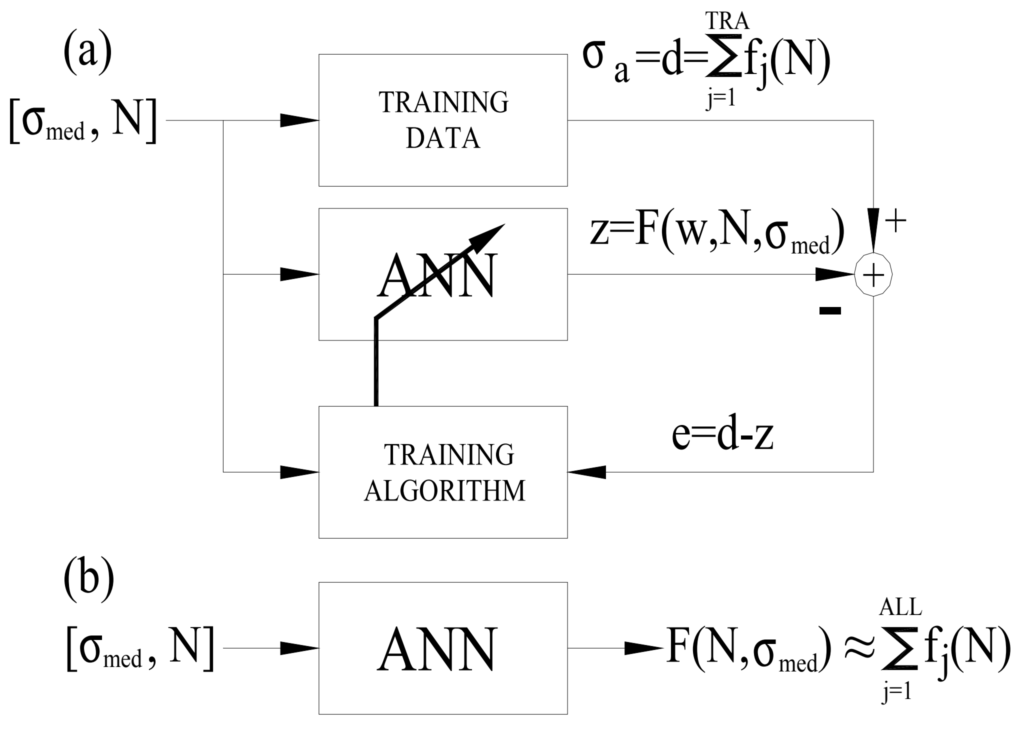


Figure 1. (a) **ANN** training method. (b) Model obtained by **ANN** training.

During training we observed **RMS** (Eq. (3)) behavior of the total data set in order to verify ANN generalization

$$\mathrm{RMS} = \frac{1}{2 \cdot Q} \cdot \sum_{1}^{Q} \sum_{i=1}^{m} (d_i - z_i)^2 \tag{3}$$

In the above equation **RMS** is the root mean square, **Q** represents the size of the data set, m the number of output neurons (for this study **m** = 1), $\mathbf{d_i}$ and $\mathbf{z_i}$ are the desired responses and the current response of the output knot, respectively.

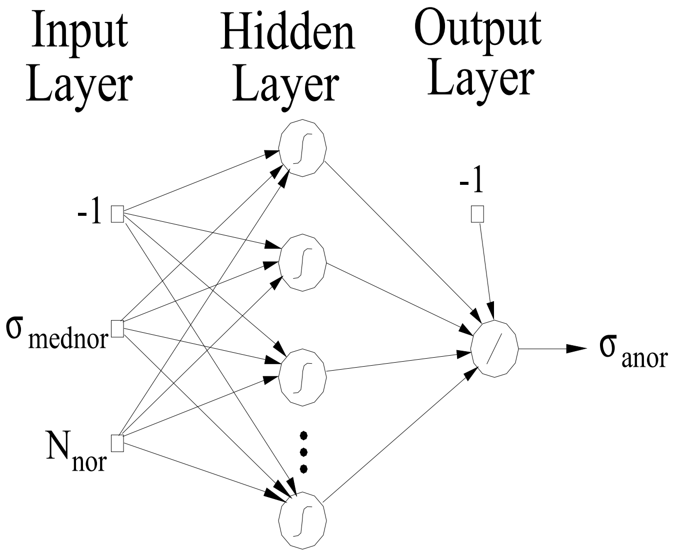
The values chosen for the constant of the moment and the learning rate were between 0.7 and 1.0. In the choice of training data, the fatigue ratios shown in table 4 were considered. The choice of this training set aimed at better data distribution within the loading regions.

Table 4: Dataset used in network training.

Dataset	MATERIAL $(0)_2$	GRP	
		0°	45°
3R	R= 10, -0.5, 0.1	R= 10, -1, 0.1	R= 10, -1, 0.1
4R	R= 10, -0.5, -1, 0.1	————	————
All	R= 10, 2, -0.5, -1, 0.1, 0.5	R= 10, -1, 0.1, 0.5	R= 10, -1, 0.1, 0.5

Data normalization was performed in both the input and output neurons. For the mean stress case, normalization was done considering its signal according to figure 2. This modification in normalization was done to achieve better data distribution, thus facilitating ANN learning (Haykin, 2001).

As previously mentioned, the behavior of a composite material submitted to fatigue was determined and the results were used to prove the stop criterion in conjunction with other materials from the literature (Freire Jr, 2005; DOE/MSU, 2003; Philipidis, 2001), where a stop criterion was established. After definition of the stop criterion, its viability in the **GRP 0º** and **GRP 45º** data set was determined, in which network architectures that varied from 10 to 11 neurons in each hidden layer were used. It is worth pointing out that all the neurons used here have bias and sigmoid activation function in the hidden neurons and linear function in the output neuron. Range analysis of the number of cycles in this study was between 10^2 and 10^7 cycles, since the experimental data analyzed are in this region. MATLAB software was used to implement all the algorithms used in this study.



$$\sigma_{mednor} = \frac{\sigma_{med}}{\sigma_{ult}} \rightarrow \begin{array}{l} \sigma_{ult} = \sigma_{ultT} \;\; if \;\; \sigma_{med} \geq 0 \\ \sigma_{ult} = \sigma_{ultC} \;\; if \;\; \sigma_{med} \leq 0 \end{array}$$

$$\sigma_{anor} = \frac{\sigma_a}{\sigma_{amax}} \rightarrow \sigma_{amax} = \sigma_a^* = \frac{\sigma_{ultT} - \sigma_{ultC}}{2}$$

Figure 2. Diagram demonstrating the **ANN** simulation model (Freire Jr, 2005).

5. RESULTS

5.1 Analysis of MAT $(0)_2$ and definition of the stop criterion

The cross-validation technique was used to analyze the results. This technique analyzed the **RMS** (mean error squared) of the training set (**RMS$_{TRE}$**) and of the total dataset (**RMS$_{TOD}$**), so that at the end of training the network synaptic weights at the lowest (**RMS$_{TOD}$**) value were chosen.

The reason for choosing the total dataset as opposed to the validation set is related to the need of obtaining an ANN that modeled fatigue behavior for all the data analyzed and not only for the training or validation set. Figure 3 shows an example of **RMS$_{TOD}$** and **RMS$_{TRE}$** behavior as a function of the number of training epochs analyzed for the MAT(0)2.

Analysis of the results obtained during cross-validation training showed that for the training set, the **RMS$_{TOD}$** and **RMS$_{TRE}$** curves exhibit the following behavior: 1) the **RMS$_{TOD}$** and **RMS$_{TRE}$** curves accompany each other with similar values or in the same order of magnitude (figure 3 (a) or 2) the **RMS$_{TOD}$** and **RMS$_{TRE}$** curves accompany each other in the same order of magnitude with separation of the curves occurring before 1500 training epochs, where the tendency of **RMS$_{TOD}$** is to stabilize at a value above that of **RMS$_{TRE}$** as can be observed in figure 3 (b). It is worth pointing out that this behavior was confirmed for the two training sets used in the MAT $(0)_2$ material.

It is interesting to note that the behavior shown in figure 3 (a) was only confirmed for this (MAT $(0)_2$) material. In previous studies this was not observed for a dataset with such a small number of composite materials (Freire Jr, 2005; Freire Jr, 2009).

Learning capacity depends on the representativity of the examples available and the complexity of the network architecture. It is known that excessive training can lead to an unsatisfactory result, since the lack of a sufficiently large dataset to validate the network is common. Very long training may lead the network to mask the results and it can even be said that the network "memorizes" these results. Other studies (Freire Jr, 2005; Freire Jr, 2007) showed that the ideal solution would be to limit the number of training epochs and the value of **RMS$_{TRE}$**.

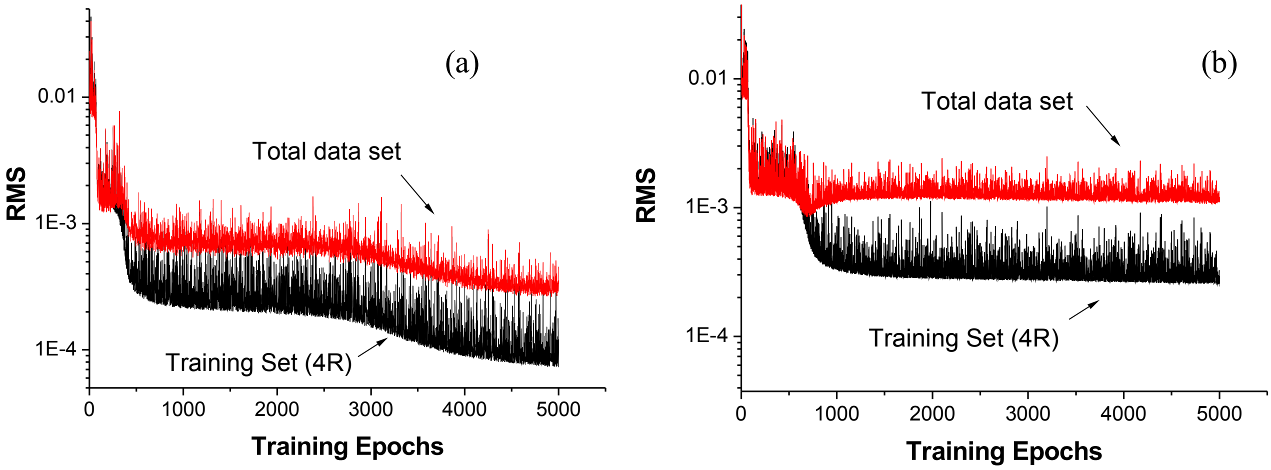


Figure 3. **RMS** curves obtained during training of an **ANN** with 15 hidden neurons (a) and 13 hidden neurons (b), with a **4R** training set for the **MAT $(0)_2$** (**R** = 10, -0.5, -1, 0.1).

The possibility of a strategy that could avoid excessive training (overfitting) was observed. From these results it can be concluded that the **RMS$_{TOD}$** must not exceed 0.0005 using only three **S-N (R)** curves and 0.0004 for four **S-N (4R)** curves, using the previously mentioned criterion.

Table 5 shows the **RMS** values for the best results obtained for each training set of the materials analyzed; the results of materials DD16, C10 and C12 were taken from other studies (Freire Jr, 2005; DOE/MSU, 2003; Freire Jr, 2007).

Table 5 shows that the use of the number of training epochs as a stop criterion would not be satisfactory, given that it varies substantially, as can be seen in the MAT $(0)_2$, which only obtains the best result for 4925 training epochs, whereas for other materials, values under 500 training epochs are obtained.

Table 5: Best results obtained for each training set (the **ANNs** used between 2 and 30 hidden neurons and up to 5000 epochs were trained).

Composite Material	Data Set	**RMS$_{TRE}$**	**RMS$_{TOD}$**	Hidden Neurons	Training Epochs
$MAT(0)_2$	3R	0.000092	0.00036	28	4069
$MAT(0)_2$	4R	0.000088	0.00026	15	4925
DD16	3R	0.00062	0.00050	8	349
DD16	4R	0.00048	0.00041	23	493
C10	3R	0.00049	0.00048	23	287
C10	4R	0.00031	0.00030	9	1721
C12	3R	0.00037	0.00040	27	289
C12	4R	0.00027	0.00029	20	3577

Figures 4 and 5 show Goodman diagrams for the $MAT(0)_2$ trained with a 3R and 4R training set, considering the best results obtained.

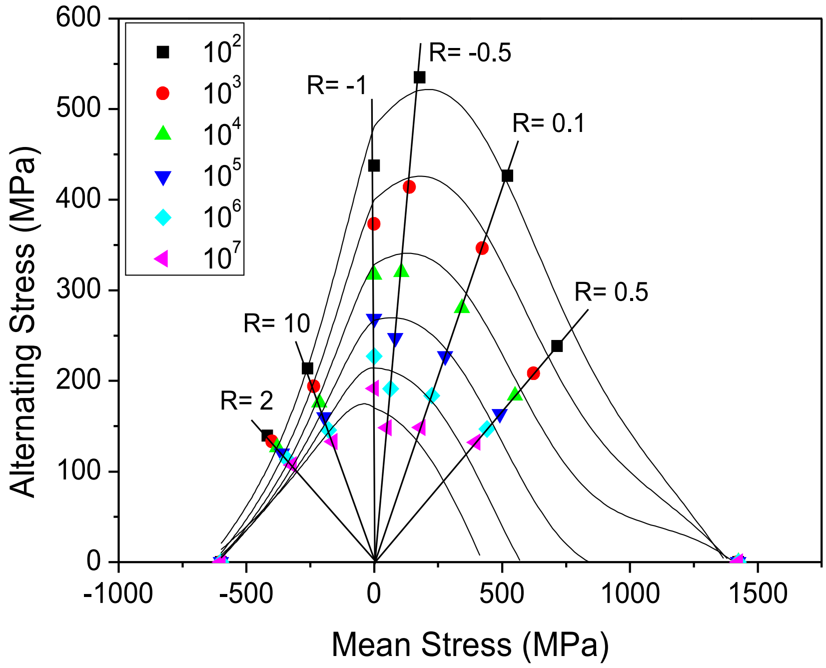


Figure 4. Goodman diagram obtained from the neural network with 28 hidden neurons trained with a MAT$(0)_2$ -3R (R = 10, -0.5, -1 and 0.1) training set with 4069 training epochs.

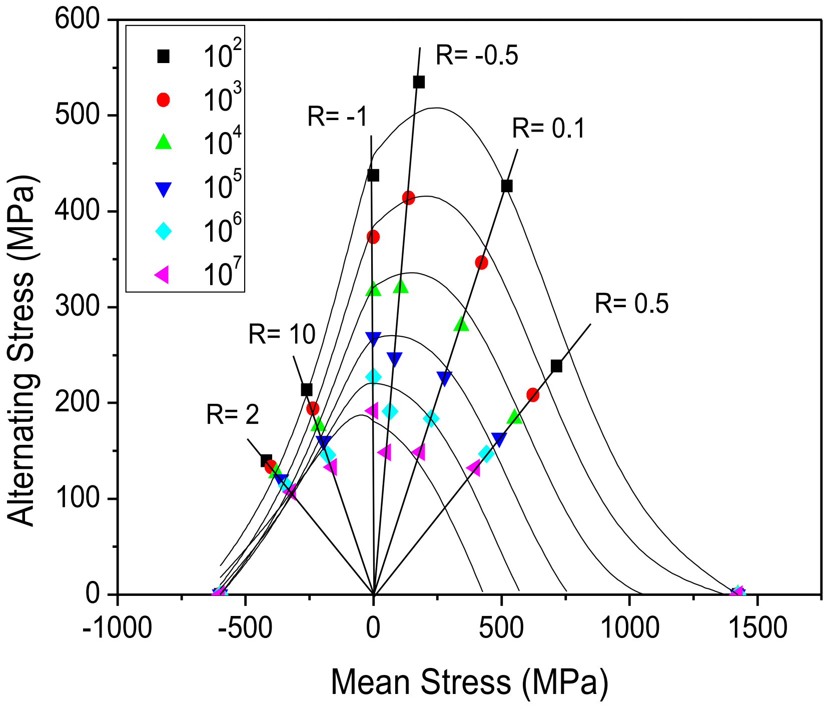


Figure 5. Goodman diagram obtained from the neural network with 15 hidden neurons trained with a MAT$(0)_2$ -3R (R = 10, -0.5, -1 and 0.1) training set with 4925 training epochs.

Analysis of figures 4 and 5 for the MAT $(0)_2$ shows that the greatest variations in results occur for R = 0.5 and R = 2; thus figures 6 and 7 were built for a qualitative assessment of these results. These figures depict the **S-N** curves obtained by the **ANN** and by equation (1) as well as the experimental data.

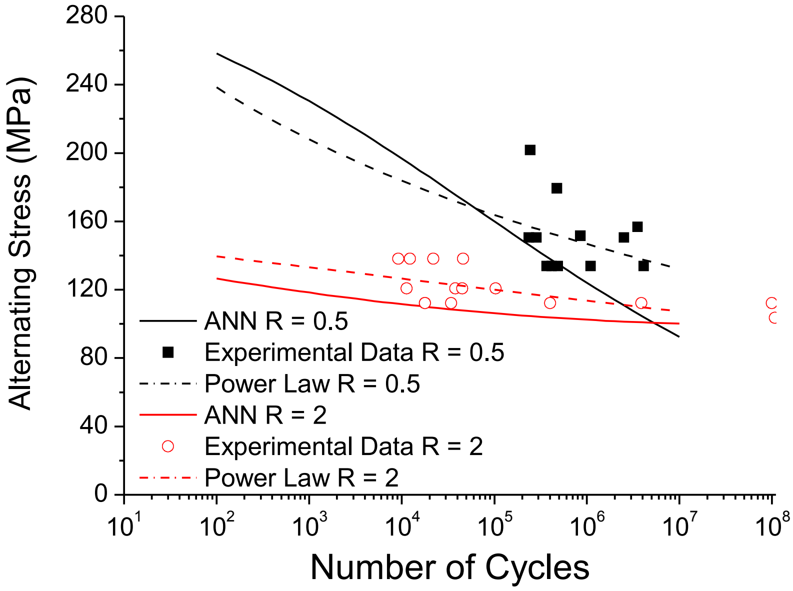


Figure 6. **S-N** curve of the MAT $(0)_2$ for R = 0.5 and R = 2, comparing the experimental data obtained from the literature, the data obtained by the **ANN** and the data obtained by the power law, trained with 3R.

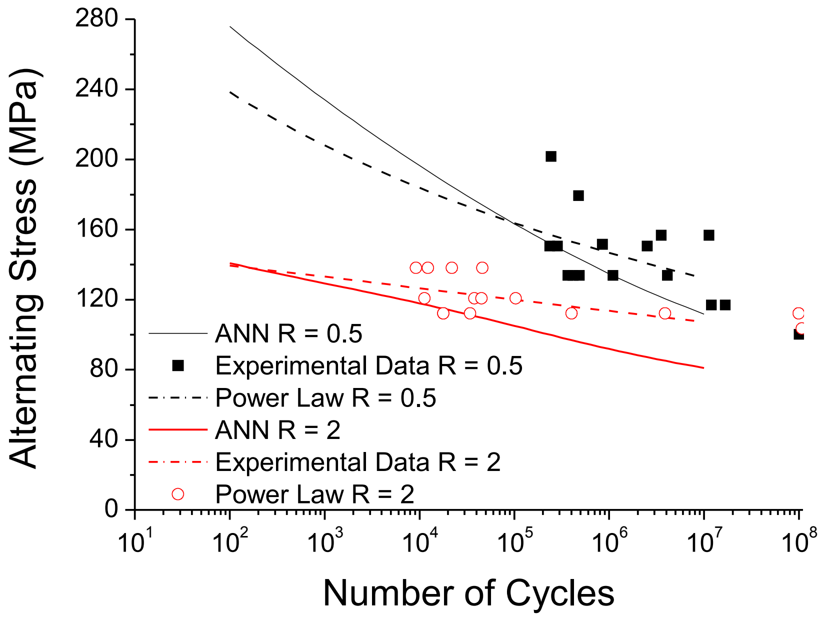


Figure 7. **S-N** curve of the MAT$(0)_2$ for R = 0.5 and R = 2, comparing the experimental data obtained from the literature, the data obtained by the **ANN** and the data obtained by the power law, trained with 4R.

5.2 Validation of the stop criterion

With the stop criterion established (0.0005 for **3R** and 0.0004 for **4R**), its applicability was determined in two other datasets (GRP 0º and GRP 45º), in which the number of hidden neurons was 10 and 11, to verify qualitatively the best architecture for the network.

Table 6 shows the results obtained for these cases. It should be pointed out that such materials cannot be used in validation owing to their small number of **S-N** curves.

Goodman diagrams for the GRP 0º and GRP 45º materials (figures 8 and 9) were built from the neural networks trained with the **3R** training set.

Analysis of figures 8 and 9 shows that the network managed to satisfactorily model material behavior. From this we can observe the benefit of using neural networks in the fatigue behavior of composite materials with a stop criterion, since good artificial neural network results are obtained using only 3 **S-N** curves.

Table 6: Results obtained for each training set of 10 and 11 hidden neurons trained up to 1500 epochs, using an $\mathbf{RMS_{TOD}}$ stop criterion of 0.0005.

Composite Material	Data Set	$\mathbf{RMS_{TRE}}$	$\mathbf{RMS_{TOD}}$	Hidden Neurons	Training Epochs
GRP 0°	3R	0.00054	0.00049	10	885
GRP 0°	3R	0.00050	0.00049	11	642
GRP 45°	3R	0.00050	0.00049	10	1182
GRP 45°	3R	0.00048	0.00050	11	1284

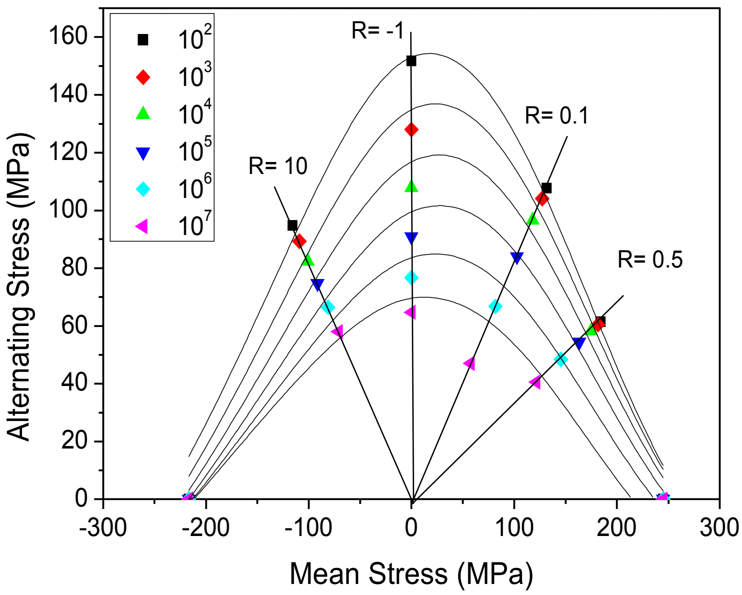


Figure 8. Goodman diagram obtained from the neural network with 10 hidden neurons trained with a GRP 0º - 3R (R = 10, -1 and 0.1) training set with 885 training epochs.

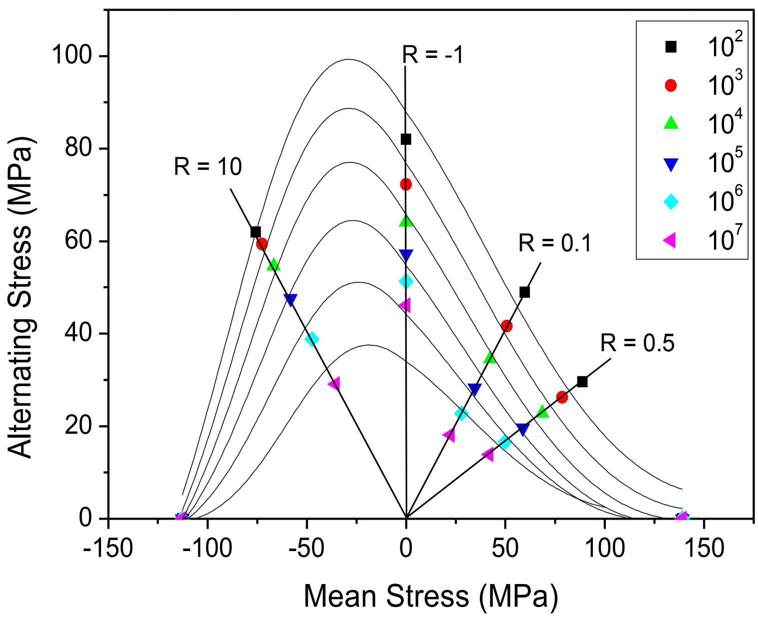


Figure 9. Goodman diagram obtained from the neural network with 10 hidden neurons trained with a GRP 45º - 3R (R = 10, -1 and 0.1) training set with 1182 training epochs.

7. CONCLUSIONS

It was found that a classic multiple-layer perceptron **ANN** was sufficient to model the fatigue behavior of the $MAT(0)_2$ using only 3 **S-N** curves.

These results and those of previous studies (Freire Jr, 2005 (b)) showed that a good stop criterion using 3 **S-N** curves for the **ANN** architecture presented here involves the use of a minimum mean squared error (**RMS**) of 0.0005 for 3 **S-N** (**3R**) curves and of 0.0004 for 4 **S-N** (**4R**) curves.

Based on this stop criterion a fatigue modeling of the **GRP** material was performed, tested at 0º and 45º, producing satisfactory results.

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9. BIBLIOGRAPHIC REFERENCES

Beheshty, M. H., Harris, B. and Adam, T., 1999, “An Empirical Fatigue-Life Model for High-Performance Fibre Composites with and without Impact Damage”, Composites – Part A: Applied Science and Manufacturing, Vol. 30, pp. 971-987.

Bond, I. P.; “Fatigue Life Prediction for GRP Subject to Variable Amplitude Loading”, Composites: Part A, Vol. 30, pp. 961-970, 1999.

DOE/MSU 2003. Composite Material Fatigue Database, Montana State University-Bozeman, p.272.

Freire Jr, R. C. S., 2005, “Building life diagrams of fatigue using artificial neural networks”, International Journal of Fatigue, Vol. 27, pp. 746-751.

Freire Jr, R. C. S., 2005, “Tese de Doutorado – Fadiga de Alto Ciclos em Compósitos de PRFV. Propriedade Mecânica e Prevenção de Falha”, Universidade Federal do Rio Grande do Norte.

Freire Jr, R. C. S., 2007, “Applying ANN Modeling in Fatigue Failure of GFRP”, 19^{th} International Congress of Mechanical Engineering.

Freire Jr, R. C. S., 2009, “Comparative study between ANN models and convencional equations in the analysis of fatigue of GRP”, International Journal of Fatigue, Vol. 27, pp. 746-751.

Haykin, S., 2001, “Redes Neurais Princípios e Prática”, 2° Ed., 893 p.

Levy, F. N. e Pardini, L. C. 2006, “ Compósitos Estruturais – Ciência e Tecnologia 1 ed.”

Mandell, J. F. and Samborsky, D. D., 1997, “DOE/MSU Composite Material Fatigue Database: test Methods, Materials and Analysis”, SAND97-3002, Sandia National Laboratories, Albuquerque, 140 p.

Philippidis, T. P. and Vassilopoulos, A. P., 2001, “Complex stress state effect on fatigue life of GRP laminates”, International Journal of Fatigue, Part I, experimental, Vol. 24, pp. 813-823.

Philippidis, T. P. and Vassilopoulos, A. P., 1999, “Fatigue of Composite Laminates under off-axis Loading”, International Journal of Fatigue, Vol. 21, pp. 253-262.

Sutherland, H. J., 1999, “On the Fatigue Analysis of Wind Turbines”, SAND99-0089, Sandia National Laboratories, Albuquerque, 133 p.