SIMULTANEOUS PIEZOELECTRIC ACTUATOR AND SENSORS PLACEMENT OPTIMIZATION AND OPTIMAL CONTROL DESIGN FOR FLEXIBLE NON-PRISMATIC BEAMS

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Abstract. *This work presents a control design for a flexible structure using piezoelectric actuators. The dynamic model of the flexible manipulator is obtained in a closed form through the finite element modal formulation. Piezoelectric actuators and sensors are added for controlling vibrations through feedback gain. An optimization problem is formulated for the location and size of the piezoelectric actuator and sensor. The naturals frequencies are calculated by the finite element method and the approximated eigenfunctions are interpolated by polynomials. Numerical experiments using Maple and Matlab were developed to verify the efficiency of the optimal control model.*

Keywords*: Piezoelectric actuators, flexible structure, vibration control, optimization.*

1. INTRODUCTION

The need of lightweight structures has attracted the researchers attention to flexible structures. These lightweight structures are essential to improve the performance in mobile applications, such as flexible robots, aircrafts, and spacecrafts. The design of these structures requires a control system, which takes into account the interaction applied forces and the elastic modes. Structure vibration suppression depends not only on control design but also on the actuator/sensor selection and placement (Li *et al.* 2001)

A flexible beam optimization and control design is composed by two parts: control gain and the placement of actuators and sensors. The proposed control must stabilize the system against the motion induced vibration. Design of a smart structure system requires more than accurate structural modeling, since both structural dynamics and control need be considered (Xu and Koko, 2004) for active vibration control. A formulation of the dynamic equations, in modal basis, can be seen in Abreu *et al.* (2003). Optimal control design for location and size and feedback gain is presented in Li *et al.* (2001), Xu and Koko (2004) and Hu and Ma (2006). This paper presents an optimal control design for actuators location, size and feedback gain, considering maximal system energy dissipation. Most papers considers just one actuator for the vibration suppression, but this paper considers two actuators and sensors with same size, suggesting a model for the optimality of their location and size. More than one actuator is suggested in Sun *et al.* (2004), but it considers linear velocity feedback and not optimal control.

Flexible structures can be built in complex geometries, which cannot be modeled by simple beam bending equations. In this work we propose a methodology for dealing complex geometry within the realm of the Euler-Bernoulli beam theory.

The use of finite elements analysis for the eigenvector determination is necessary since the analytical approach is cumbersome for complex non-prismatic beams. However, since we wish to retain the simplicity of the analytic derivation of the control, the eigenvectors are interpolated from the nodal values with polynomials (Bathe, 1976). The effectiveness of this interpolation is checked by the Rayleigh quotient (Clive and Shames, 1973).

In flexible structure, a piezoelectric actuator is applied to single-link flexible manipulators in Choi and Shin (1996) and Cho *et al.* (1999) and applied to two-link flexible manipulators in Kim *et al.* (2001). These works considered control torque of the motor, determined based on the rigid link dynamics and the oscillations caused by the torque are suppressed by applying a feedback control voltage to the piezoelectric actuator.

In this work, we propose a piezoelectric control with feedback gain. The piezoelectric actuators and sensors are fixed without considering the adhesive influence.

The lower fundamental modes are responsible for the most of the tip displacement of the beam, therefore the first three eigenfunctions are considered in the work. The theory formulated in this work can be used for generic frames, but for simplification, the simulated model has one beam. Maple and Matlab codes were created to assess the optimality and control model efficiency.

2. DYNAMIC MODEL WITH PIEZOELECTRIC ACTUATOR AND SENSOR

The deflections are obtained considering as a uniform beam with *l* length featuring a piezoceramic actuator bonded to the top face and piezofilm sensor bonded to the bottom face, as shown Fig.1.

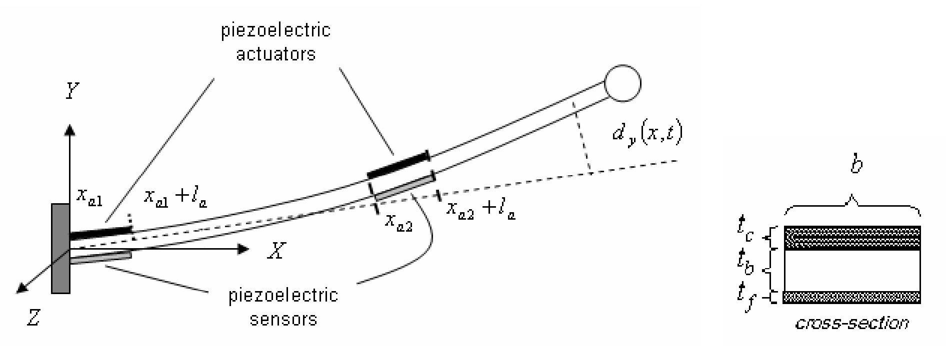


Figure1: A flexible structure with piezoelectric actuators and sensors.

This structure can be modeled as Euler-Bernoulli beam, with deflection $d_y(x,t)$ satisfying the partial differential equation

$$(E_b I_b)\frac{\partial^4 d_y(x\ ,t)}{\partial x^4}+\rho_b a_b\frac{\partial^2 d_y(x,t)}{\partial t^2}=M(x,t)\frac{\partial^2 r}{\partial^2 x}, \tag{1}$$

where ρ_b is the density of the beam, a_b is the cross-section and $(E_b I_b)$ is the flexural rigidity constant of the link (Book, 1984), $M(x,t)\frac{\partial^2 r}{\partial^2 x}$ are external forces actuating on the beam. For small displacements, the natural frequencies and modes can be considered independent of the external forces. *r* is the generalized location function expressed with the Heaviside functions, considering the first piezoelectric actuator

$$r(x)=h(x-x_{a1})-h(x-(x_{a1}+l_a)), \tag{2}$$

where l_a is the size of the actuator and x_{a1} is the actuator localization on the beam. *M(x,t)* is the bending moment acting on the beam given by (Abreu *et al.* 2003)

$$M(x,t)=\int_{t_b/2}^{t_b/2+t_c}\sigma(x,t)bxdx=\int_{t_b/2}^{t_b/2+t_c}E_c\frac{d_{31}V(t)}{t_c}bxdx, \tag{3}$$

where $\sigma(x,t)=E_c\varepsilon(x,t)$ and $\frac{d_{31}V(t)}{t_c}=\varepsilon(x,t)$, d_{31} is the piezoelectric constant E_c is the elastic modulus of the piezoceramic, t_c, t_f and t_b are the piezoceramic, piezofilm, and beam thicknesses, respectively. *V(t)* is the voltage applied to the actuator.

The evolution of the integral (3) results the following expression

$$M(x,t)=C_a V(t) \tag{4}$$

where C_a is a constant dependent on the composite system geometry computed by

$$C_a=\frac{1}{2}E_c d_{31}b(t_b+t_c)\ . \tag{5}$$

Exploring the time and space separability on Eq. (1) by the modal analysis technique, the beam deflection can be expressed as (Meirovitch , 1967)

$$d_y(x,t) = \sum_{i=1}^{m} \varphi_i(x)\delta_i(t), \tag{6}$$

where each term in the general solution of Eq. (1) is the product of a time harmonic function of the term $\delta_i(t) = e^{j\omega_i t}$ and for uniform cross section of a space eigenfunction of the form

$$\varphi_i(x_i) = C_{1i}\sin\left(\frac{\rho\omega_i^2}{(EI)}x\right) + C_{2i}\cos\left(\frac{\rho\omega_i^2}{(EI)}x\right) + C_{3i}\sinh\left(\frac{\rho\omega_i^2}{(EI)}x\right) + C_{4i}\cosh\left(\frac{\rho\omega_i^2}{(EI)}x\right), \tag{7}$$

where ω_i is the ith natural angular frequency of the eigenvalue problem for the structure and m is the finite number of eigenmodes considered in the truncated analysis. The determination of the constant coefficients C_{ki} uses clamped conditions at the beam base and mass boundary conditions representing the balance of bending moment and shearing force at the beam endpoint (De Luca *et al.* 1988).

This solution is possible when the structure geometry is prismatic or slightly non-prismatic. If the beam shape is irregular it is very difficult to obtain a closed form analytic solution. It is important to generalize the approach for other beam shapes, suggested on the next section.

By using the knowledge boundary conditions at cantilever beams and the solution (7), it can by evaluated from the Eq. (1) yields

$$\sum_{i=1}^{m} \rho_b a_b \varphi_i(x)\ddot{\delta}_i(t) + \sum_{i=1}^{m} E_b I_b \varphi_i^{IV}(x)\delta_i(t) = M(x,t)\frac{\partial^2 r}{\partial^2 x}. \tag{8}$$

Multiplying Eq. (8), on both sides, by $\varphi_i(x)$ and integrating (Abreu *et al.* 2003), it is obtained

$$\rho_b a_b \int_0^l \varphi_i^2(x)dx\,\ddot{\delta}_i(t) + E_b I_b \int_0^l \varphi_i^{IV}(x)\varphi_i(x)dx\,\delta_i(t) = M(x,t)\int_0^l \varphi_i(x)\frac{\partial^2 r}{\partial^2 x}dx. \tag{9}$$

Considering the boundary conditions of the beam and computing the integration of the right side, it yields

$$B\ddot{\delta}_i(t) + K\delta_i(t) = \frac{C_a V(t)}{\rho_b a_b}\left[\varphi'(x_{a1} + l_a) - \varphi'(x_{a1})\right], \tag{10}$$

where $B = \int_0^l \varphi_i^2(x)dx$ is the mass of the beam and $K = \frac{E_b I_b}{\rho_b a_b}\int_0^l \varphi_i^{IV}(x)\varphi_i(x)dx = \frac{E_b I_b}{\rho_b a_b}\beta_i^4$ is the stiffness. The term of modal damping D can be included in Eq. (10), as follows:

$$B\ddot{\delta}_i(t) + D\dot{\delta}_i(t) + K\delta_i(t) = \frac{C_a V(t)}{\rho_b a_b}\left[\varphi'(x_{a1} + l_a) - \varphi'(x_{a1})\right], \tag{11}$$

where $D = 2\zeta_i\omega_i$ and ζ_i is modal damping. For n actuator/sensor the Eq. (11) can be extended to:

$$B\ddot{\delta}_i(t) + D\dot{\delta}_i(t) + K\delta_i(t) = \sum_{j=1}^{n} \frac{C_{aj} V_j(t)}{\rho_b a_b} \times \left[\varphi'(x_{aj} + l_a) - \varphi'(x_{aj})\right] \tag{12}$$

The terms B, D, K, C_a, V and δ can be used in form of matrices.

3. APPROXIMATING SOLUTIONS FOR EIGENFUNCTIONS

Since structure length l is usually much larger than the cross-sectional height and depth. Thereby, considering that the control can prevent large displacements, it is possible to apply the Euler-Bernoulli theory for small displacements.

Assuming free vibration in the absence of external forces on Eq. (6), the Eq. (1), without considering external forces, is rewritten as

$$\frac{d^2}{dx^2}\left(E_b I_b \frac{d^2\varphi}{dx^2}\right) = -\rho_b a_b \omega^2 \varphi . \tag{13}$$

Approximating φ by finite element method, we have

$$\int_0^l \frac{d^2}{dx^2}\left(E_b I_b \frac{d^2\varphi}{dx^2}\right) v dx + \int_0^l \rho_b a_b \omega^2 v \varphi = 0, \qquad \forall v \tag{14}$$

where v is a arbitrary variation of φ with $v(0) = 0$.

We assume admissible solution of the form $\varphi = \sum_{i=1}^{N} Q_i \psi_i$ and $v = \sum_{j=1}^{N} V_j \psi_j$, where Q_i , V_j are scalar coefficients and N is the number of basis functions $\{\psi_1, \psi_2, \ldots, \psi_N\}$.

Recalling that $E_b I_b$ is constant on each finite element (Bathe, 1976), and using integration by parts and substituting φ and v in the results, we derive the expressions of the stiffness, mass and damping matrices, respectively

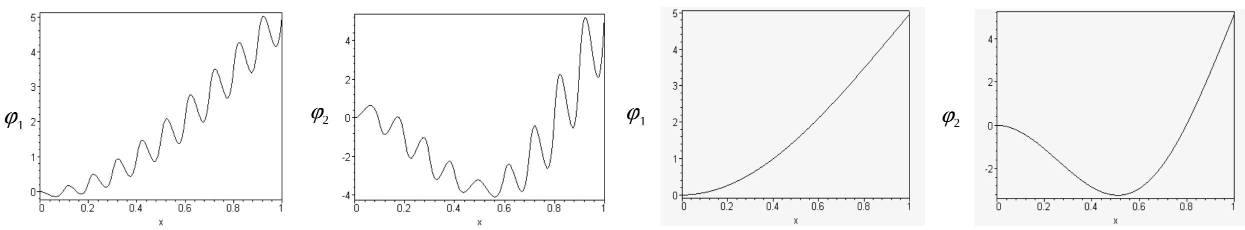
$$K_{ij} = \int_0^l E_b I_b \frac{d^2\psi_i}{dx^2}\frac{d^2\psi_j}{dx^2} dx; \qquad M_{ij} = \int_0^l \rho_b a_b \psi_i \psi_j dx; \qquad \mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{15}$$

where ψ are the elementwise interpolation functions and α, β are the Rayleigh damping constants. Usually four cubic Hermite polynomials are used as interpolation functions in each two-node finite element so the unknowns of the approximated problem are nodal displacements and its derivatives. The mass matrix can be further approximated by its lumped (diagonal) form. Then the natural modes and frequencies can be computed by the following matrix eigenvalue problem

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\varphi = \mathbf{0} , \tag{16}$$

where ω^2 are the characteristic values from the Eq. (16). The eigenvectors represent the vibration modes in nodal coordinates.

Considering that the control algorithm requires continuous twice differentiable eigenfunctions, it is necessary to create a continuous interpolation from the discrete values. The natural choice would be using the same elementwise Hermite polynomials used by the finite element approximation, but the eigenfunctions φ presents large oscillations, due to excessive sensitivity to the numerical imprecision, specially of the derivatives. Fig. 2*a*,*b* shows these oscillations for the first and second mass-normalized eigenfunction interpolation.



(a) *(b)* *(c)* *(d)*

Figure 2. Eigenfunctions which represent the vibration modes of a link fixed on *x=0* . *(a)* and *(c)* are the first mode and *(b)* and *(d)* the second mode. *(a)* and *(b)* are the eigenfunctions generate through the interpolation with Hermite polynomial in each element. *(c)* and *(d)* are the eigenfunctions generate through the interpolation with mixed Lagrange-Hermite polynomials.

It is possible to smooth these disturbances by choosing another set of interpolation functions. In this case, we chose to forgo the elementwise Hermite approximation for a global interpolation ignoring the derivatives of the inner nodes. Three alternatives are considered: interpolating all nodes with a single Hermite approximation, a mixed Hermite-

Lagrange set of polynomials which satisfies the displacements and derivatives at the outermost nodes, but only the displacements at the inner nodes, and finally a least-squares polynomial regression.

For computing the coefficients by least-squares it is necessary to considerate the pseudoinverse operator A^+ (Luenberger , 1976). This operator has the following properties: if $\mathbf{A^T A}$ is invertible, then $\mathbf{A^+} = \left(\mathbf{A^T A}\right)^{-1}\mathbf{A^T}$; if AA^T is invertible, then $\mathbf{A^+} = \mathbf{A^T}\left(\mathbf{AA^T}\right)^{-1}$. The coefficients from the polynomials are calculated from the results of the linear system $\mathbf{Ax} = \mathbf{y}$, hence $\mathbf{x} = \mathbf{A^+ y}$. The matrix **A** comes from the finite element mesh and **y** from the eigenvectors values and we choose an order of the polynomials and a number of points at the mesh.

All there three options can eliminate the oscillations on the eigenfunctions, but might be inadequate for use in the control solution, since the differentiation and the integration can generate different results. For this way, it is interesting adopt an error criterion. This work adopts the Rayleigh Quotient (Clive and Shames, 1973) as the error criterion, which can be expressed for analytic functions as $\omega^2 = \int_0^l E_b I_b \left(\frac{d^2\varphi}{dx^2}\right)^2 dx \bigg/ \int_0^l \rho_b a_b \varphi^2 dx$ and the discrete form as $\omega^2 = \frac{\varphi^T K\varphi}{\varphi^T M\varphi}$, where in this case φ are the eigenvectors.

In this work we have tested the three approximation schemes proposed. Results show that they can eliminate oscillations, but with different effect on the Rayleigh Quotient. Fig 2*c,d* shows the smoothed resulting mode shape, free from the oscillations for the mixed Lagrange-Hermite formulation.

The eigenvalue error of the smoothed eigenvectors, and the original eigenvalue from the Eq. (16) are around *1Hz* for the first and the second mode and *2Hz* for the third mode, which means around 5% for the first vibration mode and smaller for the second and third.

For geometric complex structures, the complete Hermite approximation gives smaller eigenfunctions errors, but it have to be tested for each case.

The beam was considered flexible and non-prismatic, therefore subject to motion induced vibration, which affects the trajectory of the endpoint. This beam has a linearly varying cross section.

An elementwise prismatic approach is possible for this non-prismatic beam. Specifically computation has to be considered. For the mass and for the flexural stiffness on each element, without piezoelectric material, there is considered an average on each element, the formulation of this approximation can also be found in software Ansys. For the elements, there is piezoelectric material (Crawley and De Luis, 1987), the flexural stiffness is considered as

$$EI = E_c\left[\frac{t_c^3 b}{12} + t_c b\left(t_f + \frac{t_c}{2} - t_b - d_n\right)^2\right] + E_b\left[\frac{t_b^3 b}{12} + t_b b\left(t_f + \frac{t_b}{2} - d_n\right)^2\right] + E_f\left[\frac{t_f^3 b}{12} + t_f b\left(t_f - d_n\right)^2\right], \tag{17}$$

where d_n is the distance from the bottom of the piezofilm sensor to the neutral axis.

If the elements are sufficiently small, the finite element model can represent well the real non-prismatic model of the structure.

4. PIEZOELECTRIC VIBRATION CONTROL

In this work it is propose a feedback control voltage to the piezoceramic actuator (Crawley and De Luis, 1987), expressed as

$$V(t) = -K_c \dot{V}_f(t), \tag{18}$$

and considering $G_a = \sum_{i=1}^{n} \frac{C_{ai}}{\rho_b a_b} \times \left[\varphi'(x_{ai} + l_a) - \varphi'(x_{ai})\right]$, where K_c is the feedback gain. $\dot{V}_f(t)$ is the voltage generated by the piezofilm sensor, obtained by integrating the electric charge developed at a point on the piezofilm, expressed as:

$$V_f(t) = C_s \delta = \frac{k_{31}^2 b}{C g_{31}} d_n \delta, \tag{19}$$

where k_{31}^2 is the electromechanical coupling factor, *C* is the capacitance of the film sensor and g_{31} is the piezoelectric stress constant (Banks *et al.* 1996). The resulting control law for the system Eq. (12) is expressed as

$$\mathbf{B}\ddot{\boldsymbol{\delta}}(t)+\mathbf{D}\dot{\boldsymbol{\delta}}(t)+\mathbf{K}\boldsymbol{\delta}(t)=\mathbf{G_a}\mathbf{V}(t). \tag{20}$$

The inclusion of the piezoelectric material on the flexible links is accounted by defining the beam properties with Heaviside functions to change the geometry and stiffness where the material is added. The computation of the eigenfunctions is accomplished as shown in section 3.

5. OPTIMAL CONTROL DESIGN FOR LOCATION AND SIZE OF PIEZOELECTRIC MATERIAL AND VIBRATION CONTROL

Controlling structural vibration depends not only on the control law, but also on the selection and location of the actuators and sensors (Denoyer and Kwak, 1996). In this work we propose optimal control for the actuator and sensor position and sizing optimization, based on maximizing the control energy dissipation (Li *et al.* 2001). This procedure takes into account the actuators and sensors mass and stiffness, and their effect on the mechanical behavior of the structure. This influence is combined to the control characteristics to obtain an objective function that depends on the actuators location and sizing and the control gain.

The dynamic of the flexible link with *m* piezoelectric sensors and actuators is expressed in Eq. (20), in terms of modal coordinates.

The total energy stored in the system (De Luca *et al.* 1988), can be considered a Lyapunov function as

$$W=T+U=\frac{1}{2}\dot{\boldsymbol{\delta}}^T\mathbf{B}\dot{\boldsymbol{\delta}}+\frac{1}{2}\boldsymbol{\delta}^T\mathbf{K}\boldsymbol{\delta}>0, \tag{21}$$

where **T** is the kinetic energy, **U** is the potential energy, $\boldsymbol{\delta}$ are the generalized coordinates, associated with beam deflections.

It is easily to show, that the derivate from this function (21) is negative definite, differentiating it with respect to time and simplifying with the term $\frac{1}{2}\dot{\boldsymbol{\delta}}^T\dot{\mathbf{B}}\dot{\boldsymbol{\delta}}=0$, since the matrix **B** is time independent for the cantilever beam. Isolating $\mathbf{B}\ddot{\boldsymbol{\delta}}$ on Eq. (20) and using the Eqs. (18) and (19) yields

$$\dot{W}=\dot{T}+\dot{U}=-\dot{\boldsymbol{\delta}}^T\mathbf{D}\dot{\boldsymbol{\delta}}-\dot{\boldsymbol{\delta}}^T\mathbf{G_a}\mathbf{K_c}\mathbf{C_s}\dot{\boldsymbol{\delta}}<0, \tag{22}$$

where the first and the second terms describe the removed system energy rates by the internal damping and by the control feedback, respectively.

Integrating the Eq. (22) we obtain

$$W(t_0)=W_f+W_c=\int_{t_0}^{\infty}\dot{\boldsymbol{\delta}}^T\mathbf{D}\dot{\boldsymbol{\delta}}dt+\int_{t_0}^{\infty}\dot{\boldsymbol{\delta}}^T\mathbf{G_a}\mathbf{K_c}\mathbf{C_s}\dot{\boldsymbol{\delta}}dt, \tag{23}$$

where $W(t_0)$ denote the initial energy of the system, W_f and W_c represent energy dissipated by internal damping and by the control action, respectively.

For effective vibration suppression, it is reasonable to derive a method to increase the energy dissipated by the control. We observe that W_c depends on the locations and the sizing of the actuators and feedback matrix gain K_c. Therefore, W_c can be used as an optimization criterion for control system to determine location and sizing of actuator and feedback gains.

For determining W_c, it can be write the Eq. (20) can be written in state-space form as

$$\begin{array}{l}\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{H}\mathbf{V}\\ \mathbf{y}=\mathbf{C_s}\mathbf{x}\end{array};\qquad \mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{B}^{-1}\mathbf{K} & -\mathbf{B}^{-1}\mathbf{D}\end{bmatrix};\quad \mathbf{H}=\begin{bmatrix}\mathbf{0}\\ \mathbf{B}^{-1}\mathbf{G_a}\end{bmatrix}, \tag{24}$$

where $\mathbf{C_s}$ is the output matrix.

A state feedback rather that output feedback is adopted to enhance the control performance. The quadratic cost function for the regulator problem is considered for minimizing the energy dissipation

$$W_c=\int_{t_0}^{\infty}\mathbf{x}^T\mathbf{Q}\mathbf{x}dt, \tag{25}$$

where **Q** is positive semidefinite weighting matrix, and their elements are selected connecting output of the sensor feedback with the input on the actuator

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G_a K_c C_s} \end{bmatrix}. \tag{26}$$

The feedback requires a full knowledge of states

$$\mathbf{V} = -\mathbf{K_c y}\,, \tag{27}$$

The Eq. (25) can be reduced, by standard-state transformation techniques, to the expression

$$W_c = \mathbf{x}_0^T \mathbf{P} \mathbf{x}_0\,. \tag{28}$$

In this equation $\mathbf{x}_0$ are the initial conditions.

The determination of the matrix **P** can be reduced to solving a matrix Lyapunov equation

$$\begin{aligned} &\mathbf{A_c}^T \mathbf{P} + \mathbf{P A_c} + \mathbf{Q} = \mathbf{0}\,, \\ &\mathbf{A_c} = \mathbf{A} - \mathbf{H K_c C_s}\,. \end{aligned} \tag{29}$$

It is possible observe that W_c depends on the location x_a and the size l_a of the actuators, and feedback gain $\boldsymbol{K_c}$. Therefore, W_c can be used for optimization criterion and control of the induced vibration on the beam. The optimization is for location and size of the actuator and the control with feedback gain.

6. RESULTS

The physical system considered in this work is composed by cantilevered flexible beam, but the geometry was generalized to allow non-prismatic designs. The beam was considered flexible and non-prismatic, therefore subject to motion induced vibration, which affects the trajectory of the endpoint.

This work contains significant improvements with respect to the previously published work Li *et al.* (2001), Xu and Koko (2004) and Abreu *et al.* (2003). Three vibration modes are used in the simulation and two actuators and sensors, instead of one, used in the other referenced works. In fact, it is a known fact that the best location for one actuator is on beginning of the fixed size of the beam, since there is more stress induced by the first and most significant mode. This work searches for the best location when two actuators are used.

The results were obtained using a code implemented in Maple and Matlab software. In Maple was obtained the optimum location and size for the two actuator/sensor. A finite element program was implemented for computing the eigenfunctions, as show in section 3. After computing the optimal location and size in Maple, a control procedure was simulated in Matlab, where the fourth-order Runge-Kutta method with $\Delta t = 1\,\text{ms}$ was used to integrate the equations for a half-second simulation.

We present the mechanical and geometrical properties of the piezoelectric materials (Cho *et al.* 1999, Kim *et al.* 2001) used in this work.

Table 1. Dimensional and mechanical properties of the aluminium beam and piezoelectric materials.

	Young's modulus (GPa)	Thickness (mm)	Density (kg m^{-3})	Width (mm)	Length (m)
Aluminum beam	65	at one side: 1 at other: 0.6	2890	25	1
Piezoceramic (PZT)	64	0.815	7700	25	0.4
Piezofilm	2.0	0.028	1780	25	0.4

Capacitance of the piezofilm*(C) - 380 pF cm*$^{-2}$; piezoelectric stress constant of the piezofilm(g_{31}) - *216 x* 10^{-3} *(V* m^{-1}*) (N* m^{-2} *)*$^{-1}$; electromechanical coupling factor(k_{31}) *- 0.44*; damping factors - $\zeta_1 = 0.07$, $\zeta_2 = 0.03$ and $\zeta_3 = 0.01$.

This table presents the size of the actuator and sensor placement, but this geometry was computed in the next subsection, for the best size a location in a discrete mesh.

6.1. Decision of size and location of two piezoelectric actuator and sensor

The optimization problem is formulated as

$$\begin{aligned} &\min J(x_a, l_a, K_c) = -W_c \\ &s.t. \quad 0 < x_{ai} < l \\ &\qquad\ \ 0 < x_{ai} + l_{ai} < l \end{aligned} \quad . \tag{30}$$

In the constraints of the problem (30) it can be note that the size and location of actuators/sensors can not exceed the size of the structure.

The results of the simulation in Maple are show in Fig. 3. The best location and size were chosen from a discrete set of positions [0.25,0.3,0.35,0.4,0.45,0.5,0.55,0.60,0.65]. Each value of this set represents the distance between the beginning of the first actuators/sensors to the beginning of the second and the first two was considered fixed on the beginning of the structure. Also a set of sizes was chosen as [0.01,0.025,0.05, 0.1,0.15,0.2], where each value represents the size of the actuators/sensors, considering both sizes equal. The initial conditions are $\mathbf{x_0} = [0,0,0,0.1,0.01,0.01]$. Heaviside functions were used for compute the different properties of each piece on the structures.

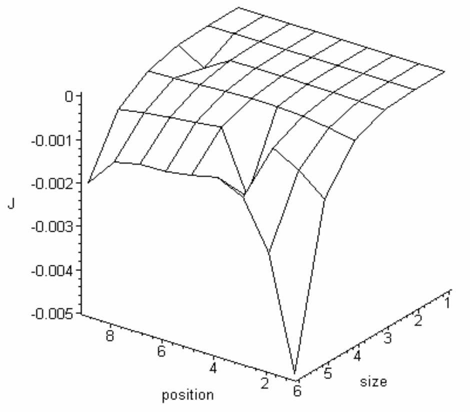


Figure 3: Cost function of dissipated energy by the system since the control action and the actuator location and size.

In the Fig. 3, the numbers from 1 to 6 and 1 to 9 corresponded to the vectors of size and position respectively. For more clarity, the number 6 from the size axis corresponds to 0.2 of the vector of size.

It can be observed from the Fig. 3 that the best position, for the control effectiveness, is merge both actuators on the beginning of the structure, or on $x = 0$. The best size is the largest. Now, these results can be used for simulating the control of the induced vibrations.

6.2 Vibrations control aspects

For the induced vibration control were used the equations of feedback gain (27) and the integration of state Eq. (24).

The tree modes of vibration, without control, are shows in Fig 4.

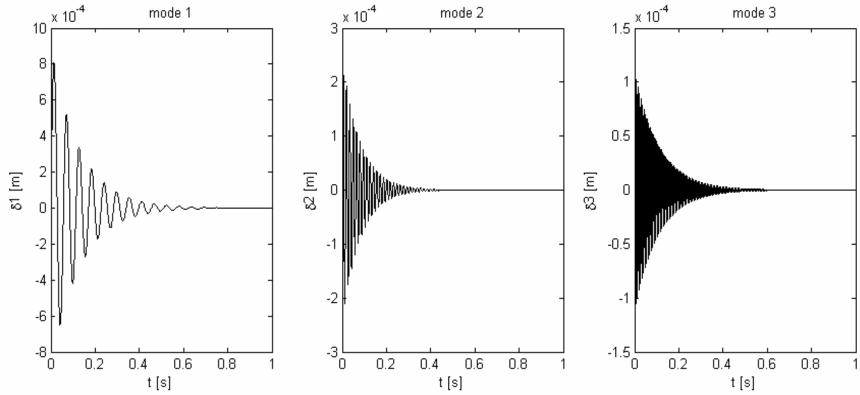


Figure 4: Response of the system without control.

The frequencies computed are *17Hz*, *66Hz*, *144Hz*, respectively. The piezoelectric material size and location change the frequencies, because this, it was chosen the best location and position from section 6.1.

The control gain can have large variations which affects significantly the control effectiveness. Two examples that show the control system and the effect of the control gain $\boldsymbol{K_c}$ are presented. In Fig. 5, the matrix $\boldsymbol{Q}$, which is dependent on $\boldsymbol{K_c}$, is computed as $\boldsymbol{Q}=diag(0,0,0,500, 500,500)$ and in Fig. 6 with $\boldsymbol{Q}=diag(0,0,0,5000, 5000,5000)$.

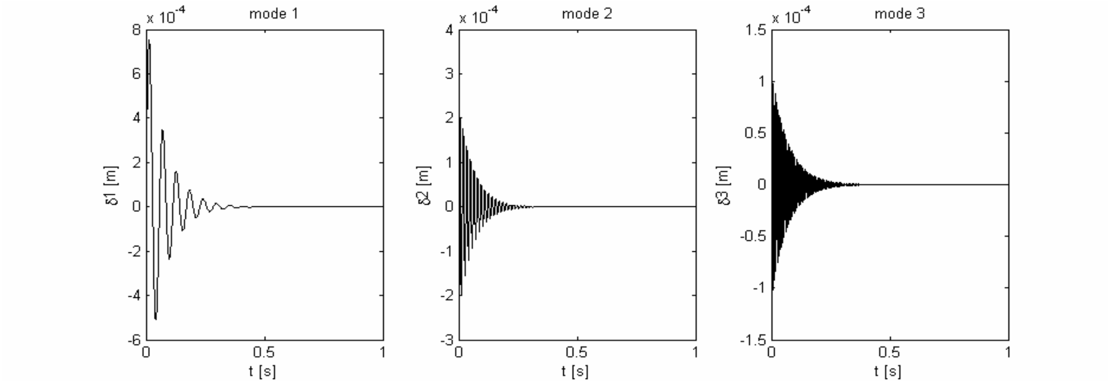


Figure 5: Response of the system with control.

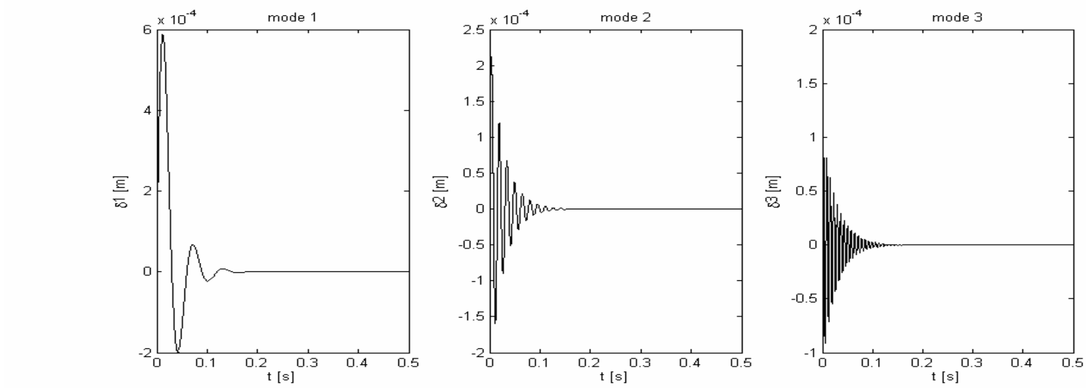


Figure 6: Response of the system with control.

With this result is possible observe that more control gain can help on the effectiveness of the vibration control. It can be see that on Fig. 6 the modal displacement stopped before than on Fig. 5.

An important aspect, that was not considered in this work, is the fact that the strain of the piezoelectric actuator is limited, but the chosen feedback gains do not extrapolate these limitations.

These simulations show competitive results with other published approaches (Xu and Koko, 2004, Sun *et al.* 2004).

The feedback gain *V(t)* from the control system show in Fig. 6, for the first vibration mode, is shown in Fig. 7.

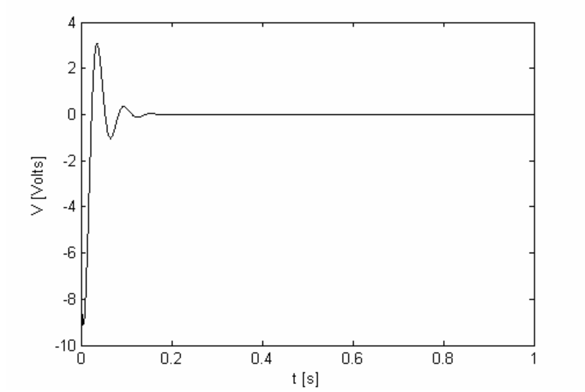


Figure 7: Closed-loop control voltage response for the first mode of the beam in vibration.

It is possible to observe in Fig. 7 that the force applied of the actuator produces a moment in contraposition of the beam deflection.

7. CONCLUSIONS AND CONSIDERATIONS

In this work we introduced a technique for optimization of location and size of piezoelectric material and vibration control of flexible beam. This technique uses the optimal control strategies for choosing the best location and size for some given discretization. Piezoelectric actuators and sensors are added to the system to control the frequency vibrations considering that the properties of the structure changes where the actuators and sensors are added. This technique can be used to build light structures with controlled vibration levels, as manipulators with flexible links, while preserving the stiffness and precision. It also reduces the energy consumption and suits the needs for aerospace systems or for tasks that demand lightness, precision and agility.

For geometric complex beams, the eigenvectors are approximated using polynomial interpolation spanning all finite elements at the beam. The Rayleigh quotient was used for the validity of the technique. Hermite polynomials interpolation proved to be the best approximation for this case.

The simulations for the control system confirmed effectiveness for this control technique. The numerical results indicate that the location and size of the actuators/sensors may have significant influence on the integrated system control performance. Also the feedback gain affects directly the control efficiency.

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10. RESPONSIBILITY NOTICE

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