TRUSS OPTIMIZATION ON SHAPE AND SIZE WITH DYNAMIC CONSTRAINTS USING A PARTICLE SWARM OPTIMIZATION ALGORITHM

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Abstract. *In this paper, truss optimization for size and shape is performed, taking into account frequency constraints. It is well-known that structural optimizations on shape and size are highly non-linear dynamic optimization problems since this mass reduction conflicts with the frequency constraints especially when they are lower bounded. Besides, vibration modes may switch easily due to shape modifications. This paper intends to investigate the use of a Particle Swarm Optimization (PSO) algorithm as an optimization engine in this type of problems. This choice is based on reported well-behavior of such algorithm as global optimizer in other areas of knowledge. Another feature of the algorithm is taken into account for this choice, as the fact that it is not gradient based, but just based on simple objective functions evaluations. This is of major importance when highly non-linear dynamic optimization problems with several constraints are treated, avoiding bad numerical behavior due to gradient evaluations. The algorithm is briefly revised highlighting its most important features. It is presented four examples regarding the optimization of trusses on shape and sizing with frequency constraints. The examples are widely reported and used in the related literature as benchmark examples. The results show that the algorithms performed similar to other methods and better in other cases.*

Keywords*: particle swarm optimization, truss optimization, frequency constraints, shape and sizing optimization.*

1. INTRODUCTION

The optimization on shape and sizing of truss structures with frequency constraints is a non-linear optimization problem that is not completely addressed. This type of optimization is very useful when the dynamic performance is intended to be enhanced with regard to narrow band or even wide band frequency excitations. As indicated by Grandhi *et al.*(1993), in most of low-frequency vibration problems the response of the structure is a primarily function of its fundamental frequency and mode shape, so the ability to manipulate these fundamental frequency can significantly improve the performance of the structure and avoid resonance phenomenon. As indicated, when optimizing for mass, vibration modes can switch, for example, from a bending mode to a torsional or axial mode, and this can lead to frequencies changes, causing convergence difficulties. Methods based on sensitivity (gradients) of the frequency to the design variables had difficulties when dealing with multiple repeated eigenvalues (structural symmetry), since the gradient is not formally defined. Furthermore, by its gradient based nature, it may converge to local optima.

2. BRIEF BIBLIOGRAPHICAL REVIEW

One of the first papers that addressed the problem of minimum mass-truss optimization with constraints in frequency was after Bellagamba and Yang (1981) which presented a constrained parameter optimization technique. The procedure employed an exterior penalty function to transform the constrained function into an unconstrained one and uses the Gauss method to solve simultaneous linear equations with the variation of the parameters as the unknowns to perform the unconstrained minimization. It was later introduced local buckling constraints. In the end, stable thermal loads were introduced and equality constraint was imposed on the fundamental natural frequency (which uses a four degree-of-freedom axial force bar element) of the analyzed structures. Another paper that reviews methods and applications on structural optimization with frequency constraints was due to Grandhi (1993). In this paper a series of revised papers on truss optimization with frequency constraints are listed and a large number of papers covering a wide variety of related problems is revised, as well. One of the findings in the review is that the common problem in frequency optimization is the switching of vibration modes due to structural size and shape modifications and this is reported as a cause of convergence difficulties for the optimization algorithms. Another problem found in the revised papers was the fact that some structures exhibit repeated eigenvalues even though the initial design did not have any. A series of papers that deal with the problem of sensitivity analysis for structures with repeated eigenvalues are listed..

Yang *et al.* (1999) presented an evolutionary method for structural topology optimization subjected to frequency constraints. The method is based on the idea of gradually removing inefficient material and allowing new material to be added. In this paper, three kinds of optimization objectives were investigated: maximizing a single frequency, maximizing multiple frequencies and designing structures with prescribed frequencies. It was analyzed four cases and the results showed that the new methodology seems to be computationally accurate in most of the cases. Roux *at al.*(1998) presented the idea for a response surface methodology to bypass the expensive structural analysis. In the paper it is investigated aspects related to best regression equation, location and size of the region of approximation. As optimization engine, it is used a variation of the Sequential Linear Programming (SLP). As result, it is reported that the use of the two investigated method does not significantly improved the results due to a lack of accuracy in the approximated cost function. Simple examples of 2, 3 and 10-bar trusses are used to validate the results.

The problem of multiple fundamental eigenvalue in the optimum design was addressed by Ohsaki *et al.* (1999). It was presented a Semi-Definite Programming Algorithm (SDPA) with utilizes extensively the sparseness of trusses matrices in the optimization problem for specified frequencies. As a gradient based method, it is demonstrated that the SDPA has advantages over existing methods in view of computational efficiency and accuracy of the solutions. As example, a five fold fundamental eigenvalue optimal topology truss is analyzed.

Tong *et al.* (2000) in their paper presented a basic theory for determining the solution existence of frequency optimization problems for truss structures. A practical method is presented based on the fact that frequencies remain unchanged when the truss is modified uniformly. So, using a first order derivative of certain eingenvalues with respect to design variables it was possible to conclude when a specific natural frequency is achievable. The presented method requires separated checking for each frequency constraints which may be costly for problems with large number of frequency constraints; however it is indicated that approximation methods would be preferable in practical checking process to avoid such eigenvalue analysis. The procedure allows obtaining an extreme value of the corresponding natural frequency or a small confined range of design variables that contains the extreme value. Examples ranging from a two bar optimization problem to a 72-bar space truss were analyzed.

Tong and Liu (2001) presented a procedure for minimum weight optimization with discrete design variables for truss structures subjected to constraints on stresses, natural frequencies and frequency responses. The procedure consisted of two basic steps: firstly it was determined a feasible start point using a difference quotient method and secondly it was evaluated the discrete values of the design variables converting the structural dynamic optimization problem into a linear zero-one programming by means of a binary number combinatorial algorithm. Three examples were presented, two of them analyzing a 10-bar plane truss and the last regarding a 25-bar space truss, demonstrating the feasibility of the optimization procedure. Sedeghati *et al.* (2001) proposed an integrated finite element analysis with an integrated force method for frequency analysis jointly with a mathematical programming technique. Three structures, composed of truss and frame type were studied and the results were compared with literature benchmarks. It was shown that the multiple frequency constraints significantly affected the final optimum design although the force method resulted in a lighter computational cost design. Xu *et al.* (2003) presented a practical methodology based on a topology group concept for finding optimal topologies of trusses. It was considered constraints like, natural frequencies, stress, displacement, Euler buckling, and multiple loading conditions. In this paper special attention was given to meaningless topologies which are excluded. The algorithm used for the optimizations was a Binary Number Combinatorial Algorithm joined to an unconstrained direct search algorithm. So, there were a certain number of fixed nodes and it was allowed to change the total number of nodes, taking into account mechanisms formation (rigid body motion) or meaningless topologies. To avoid problems related to the elimination of bars/nodes and maintain the computational dimension, a so called Imaginary Bar Method was implemented. The efficiency of the proposed algorithm was demonstrated by two typical examples of trusses. The optimally criterion based on differentiation of the Lagrangian function was used by Wang *et al.* (2004). It is used to solve the three-dimensional truss structures optimization with multiple constraints on its natural frequencies. Nodal coordinates and cross-sectional areas were treated simultaneously in the weight minimization. In this methodology the optimal solution was achieved gradually from an infeasible starting point with minimum weight increment and the structural weight was indirectly minimized. Four typical truss examples were analyzed and solved with the proposed methodology and it was shown to be quite effective and reliable.

More recently, Lingyun *et al.* (2005) presented a paper where an enhanced genetic algorithm with float point codification was proposed to solve de non-linear optimization problem of mass minimization of trusses with frequencies constraints. To account for constraints it was used the well-known and effective constraint handling method, the Penalty Method. In this work, a hybrid algorithm, formed by simplex search and genetic algorithms was developed following a nature based scheme of Niche. The algorithm, called NGHA (Niche Genetic Hybrid Algorithm), chooses groups of individuals from population to form a niche. Each niche is searched for optimum values using a simplex algorithm and then another simplex search is used to search among the best niches. Several parameters, like those used to define the niche (some distance measure in terms of cost function) and the probability of use of the simplex search have to be set a priori. This was intended to give the algorithm a balance between exploration and exploitation. Three truss structures were analyzed and compared with literature results, in most of the cases the new methodology presented best results than those reported in the references.

Gholizadeh *et al.* (2008) and Salajegheh *et al.* (2007) presented a study where a Genetic Algorithm (GA) and a Neural Netowork (NN) were used together to find the optimal weight of structures subjected to multiple natural frequency constraints. The evolutionary algorithm used was the Virtual Subpopulation (VSP) method which was responsible for the optimization. To reduce the computational time in the optimization process, the structural analysis was replaced by a properly trained neural network with radial basis function (RBF) and a wavelet radial basis function (WRBF) neural network. The paper demonstrates that the best results were found using the VSP method with a WRBF network. It was analyzed two examples, the first one a simple planar truss with 10 bars and a second one, a more complex problem, a double layer grid with 200 bars. In a similar work, Torzakdeh *et al.* (2008) presented four different methods for the optimum design of structures subjected to multiple frequency constraints based on gradient evaluations. It was presented a new third order approximation function for the structural responses quantities, as function of cross sectional properties and the four different methods for the optimum design were defined based on this approximate function. The key idea was to create the approximate eingenvalues with respect to the cross-sectional properties, so using this higher order approximation could enhance the frequencies evaluation and thus enhance the quality of the optimization. In the paper 3D steel framed structures with one and eight stories was optimized with constraints in the first 3 natural frequencies. The results were compared in terms of accuracy and computational time.

3. SWARM OPTIMIZATION ALGORITHM

The particle Swarm optimization (PSO) has been inspired by the social behavior of animals such as fish schooling, insects swarming and birds flocking. This method is used to search for the global optimum of wide variety of arbitrary problems. It was first introduced by Kennedy and Everhart (1995). The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredictable choreography of a bird flock, the aim of discovering patterns that govern the ability of such bird flock to fly synchronously and suddenly change direction with regrouping in an optimal formation. Rigorously speaking, it is a stochastic, population based evolutionary computer algorithm. The basis for the method relies on the social influence and social learning which enable persons to maintain cognitive consistency. So, the exchange of ideas and interactions between individuals may lead them to solve problems. The particle swarm simulates this social plot. As stated by Li *et al.*(2007), the method involves a number of particles, which have a defined position and velocity and they are initialized randomly in a multidimensional search space of an objective function. Each particle represents a potential solution of the problem and the measure of this potentiality is its objective function. The set of particles are generally referred as “swarm”. These particles fly through the multidimensional space and have two essential reasoning capabilities: their memory of their own best position and knowledge of the global or their neighborhood's best. In a minimization optimization problem, "best" simply means the position of the particle ($\mathbf{x}_i$) with the smallest objective value $\min f(\mathbf{x}_i)$. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on this information of good positions. So, related to each particle there are a set of design variables ($\mathbf{x}_i$) and the respective velocities ($\mathbf{v}_i$) that represents the potential solution of the optimization problem.

At each iteration, the basic swarm parameters position and velocity are updated using the following equations:

$$
\begin{aligned}
v_{i,j}^{k+1} &= \chi[\omega v_i^{k+1} + c_1 r_1 (xlbest_{i,j}^k - x_{i,j}^k) + c_2 r_2 (xgbest_j^k - x_{i,j}^k)] \\
x_{i,j}^{k+1} &= x_{i,j}^k + v_{i,j}^{k+1}
\end{aligned}
\tag{1}
$$

where ω is the inertia weight for velocities (previously set between 0 and 1), $x_{i,j}^k$ is the current value (*k*) of design variable j of particle *I*, $v_{i,j}^{k+1}$ is the updated velocity of design variable j of particle i, is the best design variable *j* ever found by particle *i*, $xlbest_{i,j}^k$ is the best design variable *j* ever found by the swarm, r_1 and r_2 are uniform random numbers in the [0,1] range, c_1 means the cognitive component (self confidence of the particle) and c_2 means the social component (swarm confidence) and they are constants that influence how each particle is directed towards good positions taking into account personal best and global best information , respectively. They usually are set as $c_1 = c_2 = 1.5$. The role of the inertia weight ω is crucial for the P.S.O. convergence. It is employed to control the impact of previous velocities on the current particle velocity. A general rule of thumb indicates to set a large value initially to make the algorithm explore the search space and than gradually reduce it in order to get refined solutions. In this paper it is initially set as $\omega = 0.8$ and updated based on coefficient of variation ($cov = \sigma / \mu$) of the swarm objective function accordingly to $\omega = 0.4[1 + min(\mathrm{cov}, 0.6)]$.

The χ parameter is used to avoid divergence behavior in the algorithm and it is given by the following expression, which was developed based on convergence assumptions for the algorithm, as indicated by Bergh and Engelbrecht (2006).

$$\chi = \frac{1.6}{\left|2-(c_1+c_2)-\sqrt{(c_1+c_2)^2-4(c_1+c_2)}\right|} \tag{2}$$

This coefficient is crucial to keep the algorithm stable and avoid divergence in the iteration process. There are variations in the algorithm that add a third term in the previous velocity update that accounts for neighborhood information. This requires the user to set an influence region to define the neighborhood. In this paper the just the simple algorithm was used in order to reduce the number of heuristic parameters. For the generation of initial particles of swarm it is common to set randomly distributed particles across the design space, so:

$$x_{i,j}^0 = x_{j\,min} + r(x_{j\,max} - x_{j\,min}) \tag{3}$$

$$v_{i,j}^0 = 0 \tag{4}$$

where $x_{i,j}^0$ means the initial position for design variable *j* of particle *i*, *r* means an uniformly random generated number in the *[0,1]* range, and means the lower and upper bounds for design variable *j*. It is implicit in this formulation that the iterations mean the time step of the process.

A simple way to understand this updating procedure is depicted by Hassan *et al.* (2004) and indicated in the following Fig. 1.

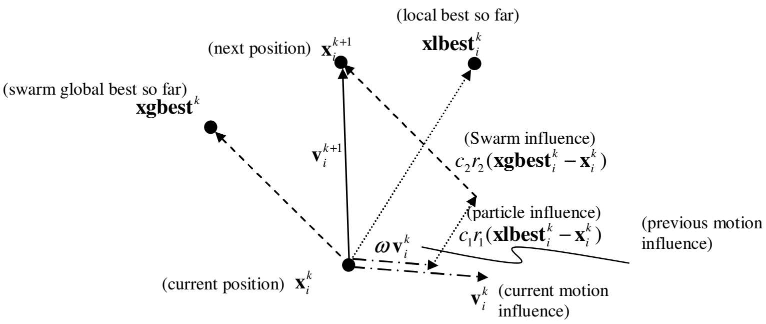

Figure 1. Vector representation of velocity and position updates in Particle Swarm Optimization Algorithm (Hassan *et al.*, 2004).

The easier way to set a convergence criterion for the algorithm is monitoring the differences in the global best design variables between iteration or even the global best objective function. However a more effectively one can be built based on the Coefficient of Variation of objective function in the swarm. In this paper a combination of the three criteria was simultaneously employed.

In the following a pseudo-code for the implemented Swarm Optimization Algorithm is depicted in the Fig. 2. So, in terms of Truss Optimization the problem can be mathematically stated as:

$$\begin{aligned}
&\textit{minimize } Mass = \sum_{i=1}^{n} L_i \rho_i A_i \quad i = 1,...,n \textit{ for all bars}\\
&\textit{Subjected to } \omega_j \geq \omega_j^* \textit{ for some eigenvalues } j\\
&\qquad\qquad \omega_k \leq \omega_k^* \textit{ for some eigenvalues } k\\
&\textit{and}\\
&A_{l\min} \leq A_l \qquad \textit{for some bar cross sectional areas } l\\
&\mathbf{x}_{q\min} \leq \mathbf{x}_q \leq \mathbf{x}_{q\max} \quad \textit{for some node coordinates } q
\end{aligned} \tag{5}$$

In this paper the constraints violations will be treated with the penalty function technique so the objective function to be minimized is modified to:

$$Mass = (\sum_{i=1}^{n} L_i \rho_i A_i)(1 + PF) \qquad \textit{for all bars} \tag{6}$$

where the Penalization Factor (PF) is defined as the sum of all active constraints violations as indicated.

$$PF = \sum_{i=1}^{nc} \left| \frac{\omega_i}{\omega_i^*} - 1 \right| \qquad \textit{for all active constraints} \tag{7}$$

This formulation allows that for solutions with violated constraints, the objective function is always greater than the non-violated one.

Set the algorithms parameters: number of particles *n*, number of design variables *m*, cognitive parameter c_1, social parameter c_2, velocity momentum ω, coefficient to avoid divergence χ, minimum coefficient of variation cov_{min}, upper and lower bound for design variables $\mathbf{x}_{min}$ and $\mathbf{x}_{max}$.
Create initial random Swarm and initialize the local best values
For each particle *i* in the swarm
  For each design variable *j*
    r=*uniform*[0,1]
    $x_{i,j}^0 = x_{j\,min} + r(x_{j\,max} - x_{j\,min})$
    $v_{i,j}^0 = 0$
    Set the local best design variable as the current one
    $xlbest_{i,j} = x_{i,j}$
  End
  Set the local best objective function as the current one
  $f_i(\mathbf{xlbest}_i) = f(\mathbf{x}_i)$
End
Iterates with the Swarm to find particle with design variables that lead to a minimum objective function
Loop until convergence criterion of Coefficient of Variation($cov < cov_{min}$), global best objective function ($f(\mathbf{xgbest}_{i+1})$-$f(\mathbf{xgbest}_i)$ <tolerance) or global best design variable ($|\mathbf{xlbest}_{i+1} - \mathbf{xlbest}_i|$<tolerance) of the Swarm is met
  Evaluate for each particle the objective function $f_i(\mathbf{x}_i)$
  Update the local best and their objective function
  For each particle *i*
    If $f(\mathbf{x}_i) < f_i(\mathbf{xlbest}_i)$ then $f_i(\mathbf{xlbest}_i) = f(\mathbf{x}_i)$ and $\mathbf{xlbest}_i = \mathbf{x}_i$
  End
  Find the minimum particle objective function *min*($f(\mathbf{x}_i)$)
  If *min*($f(\mathbf{x}_i)$) < $f(\mathbf{xgbest}_i)$ then $f(\mathbf{xgbest}_i)$ = *min*($f(\mathbf{x}_i)$) and $\mathbf{xgbest}_i = \mathbf{x}_{index[\min(f_i(\mathbf{x}_i))]}$
  For each particle *i* in the swarm
    r_1= *uniform*[0,1]
    r_2= *uniform*[0,1]
    $\mathbf{v}_i^{k+1} = \omega \mathbf{v}_i^k + c_1 r_1 (\mathbf{xlbest}_i^k - \mathbf{x}_i^k) + c_2 r_2 (\mathbf{xgbest}_i^k - \mathbf{x}_i^k)$
    $\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \chi \mathbf{v}_i^{k+1}$
  End
End

Figure 2. Pseudo-code for the simple Particle Swarm Optimization.

4. NUMERICAL EXAMPLES

4.1. Ten bar Truss

This example was first solved by Grandhi and Venkayya (1988) using the optimality algorithm. Sedeghati *at al.* (2002) used a Sequential Quadratic Programming (SQP) with conjunction with finite element force method to solve the problem. Wang *et al.* (2004) used a evolutionary node shift method and Lingyum *et al.* (2005) used Niche Hybrid Genetic Algorithm. This paper address this problem using the particle Swarm Algorithm previously described. It is a simple 1-bar truss with fixed shape and variable continuous bar sizes. At each free node it is attached a non-structural mass of 454.0 kg as depicted by Fig. 3.The material properties as design variable ranges are listed in Tab. 1. So this is a truss optimization on size with three frequency constraints and ten design variables.

Tab. 2 shows the design variables results and the final mass for the optimized truss. It should be highlighted the good results obtained with the PSO algorithm. The truss mass obtained by the PSO was a little worse than Sedaghati *et al.* (2002) results.

Tab. 3 shows the optimized frequencies (Hz) obtained by several authors in the literature and the results obtained by the present work. It is clear that none of the frequency constraints were violated. This was only true for the Sedaghati *et al.* (2002) results, which presented the lighter result so far. Since none of the papers presented the number of function evaluations, it is not performed here this type of comparison.

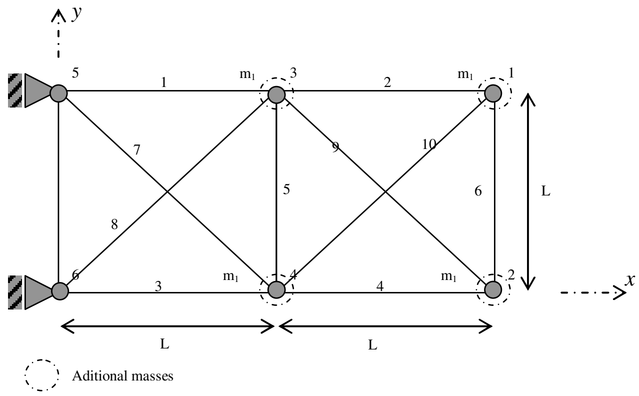

Figure 3. 10-bar truss structure with added masses.

Table 1. Material properties and frequency constraints for 10-bar truss structure.

Property	Value	Unit
E (Young Modulus)	6.98×10^{10}	N/m^2
ρ (Material density)	2770.0	kg/m^3
Added Mass	454.0	kg
Design Variable Lower Bound	0.645×10^{-4}	m^2
Main bar's Dimension	9.144	m
Constraints on first 3 frequencies	$\omega_1 \geq 7$, $\omega_2 \geq 15$, $\omega_3 \geq 20$	Hz

Table 2. Optimal design cross sections (cm^2) for several methods (Weight does not consider added masses).

Element No.	Wang (2004)	Grandhi (1993)	Sedaghati (2002)	Lingyum (2005)	Present Work
1	32.456	36.584	38.245	42.234	37.712
2	16.577	24.658	9.916	18.555	9.959
3	32.456	36.584	38.619	38.851	40.265
4	16.577	24.658	18.232	11.222	16.788
5	2.115	4.167	4.419	4.783	11.576
6	4.467	2.070	4.419	4.451	3.955
7	22.810	27.032	20.097	21.049	25.308
8	22.810	27.032	24.097	20.949	21.613
9	17.490	10.346	13.890	10.257	11.576
10	17.490	10.346	11.452	14.342	11.186
Weight(kg)	553.8	594.0	537.01	542.75	537.98

The statistical results of 5 independent runs are shown in Tab. 4. It can be noticed a little deviation from the mean value of the independent runs. The result used for comparisons are the best one obtained.

Fig. 4 shows the convergence iterations of the PSO algorithm for the 10-bar truss structure with added masses.

Table 3. Optimized frequencies (Hz) with several methods for the 10-bar truss structure.

Frequency No.	Wang (2004)	Grandhi (1993)	Sedaghati (2002)	Lingyum (2005)	Present Work
1	7.011	7.059	6.992	7.008	7.000
2	17.302	15.895	17.599	18.148	17.786
3	20.001	20.425	19.973	20.000	20.000
4	20.100	21.528	19.977	20.508	20.063
5	30.869	28.978	28. 173	27.797	27.776
6	32.666	30.189	31.029	31.281	30.939
7	48.282	54.286	47.628	48.304	47.297
8	52.306	56.546	52.292	53.306	52.286

Table 4. Statistical results for the 5 independent runs of swarm optimizations for 10-bar truss structure

Mean Mass of Swarm (kg)	Standard Deviation	No. of Particles	Social Constant	Cognitive Constant	Velocity Momentum	Mean No. of Iterations	Tolerance for Convergence
540.89	6.84	50	1.5	1.5	0.5	40	10^{-3}

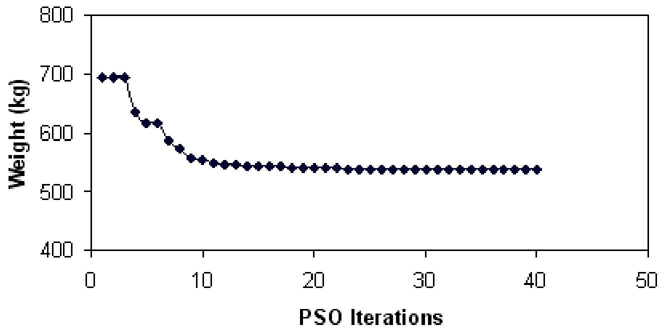

Figure 4 – PSO iterations for 10-bar truss structure with added masses.

4.2 52-bar space truss

In this example a hemispherical space truss (like a dome) is optimized on shape and size with constraints in the first two natural frequencies. The space truss has 52 bars and non-structural masses of m=50 kg are added to the free nodes. The cross-sectional areas are permitted to vary between 0.0001 m^2 and 0.001 m^2. The shape optimization is performed taking into account that the symmetry should be kept in the design process. Each movable node is allowed to vary ± 2 m. For the frequency constraint it is set that $\omega_1 \leq 15.916$ Hz and $\omega_2 \geq 28.649$ Hz. A sketch of the initial design is shown in Fig. 12 and Fig. 13. This example is considered to be a truss optimization problem with two natural frequency constraints and thirteen design variables (five shape variables plus eight size variables).

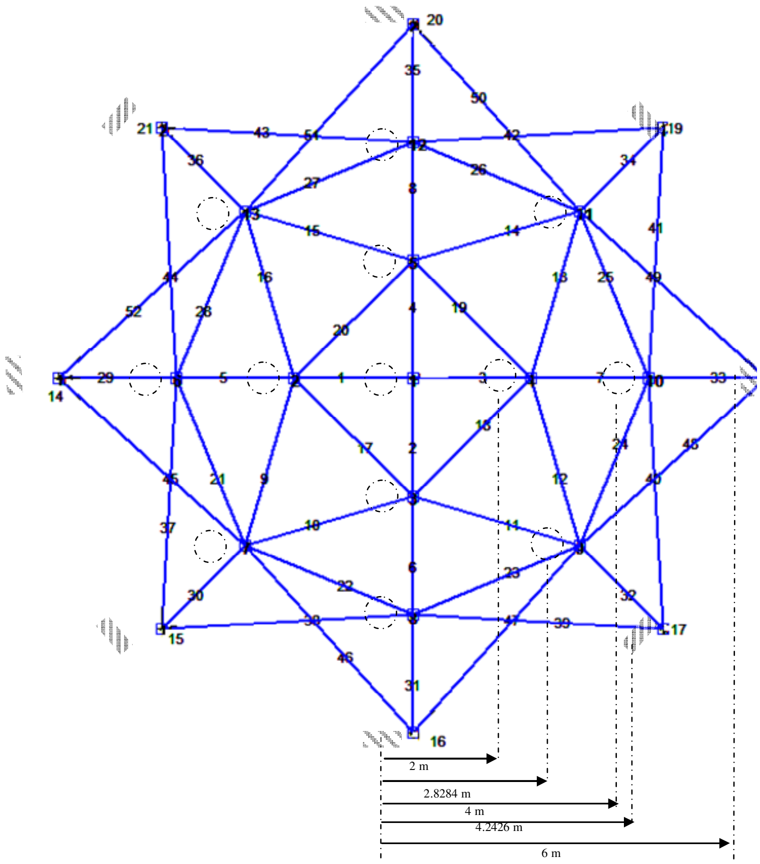

Figure 12. Initial design of a 52-bar dome structure (top view).

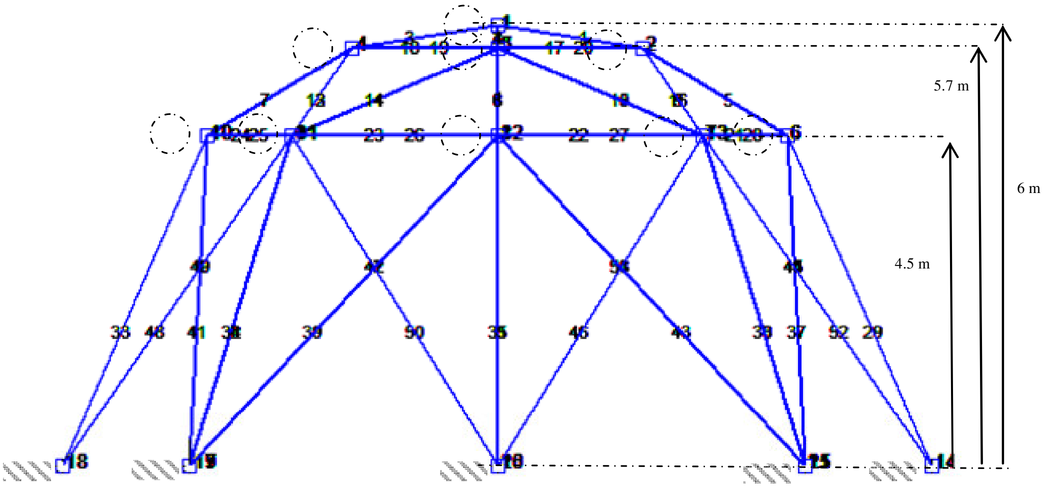

Figure 13. Initial design of a 52-bar dome structure (lateral view).

Table 12 shows the initial and final optimized coordinates and cross sectional areas and final mass, as well. It can be noticed that the PSO perform better than the other methods. PSO mass optimum is about 3.36% lighter than Lingyum *at al.* (2005) optimum.

In Tab. 13 it is shown the final optimized frequencies (Hz) for the methods compared. It is noticed that any of the frequency constraints were violated.

Table 14 shows the statistics of 5 independent runs for the 52-bars truss example and the parameters used for the PSO algorithm.

Table 12. Optimal design cross section for several methods for the 52-bar space truss (weights does not consider added masses).

Variable No.	Initial Design	Lin (1982)	Lingyum (2005)	Present Work
Z_A (m)	6.000	4.3201	5.8851	5.5344
X_B(m)	2.000	1.3153	1.7623	2.0885
Z_B (m)	5.700	4.1740	4.4091	3.9283
X_F (m)	2.828	2.9169	3.4406	4.0255
Z_F(m)	4.500	3.2676	3.1874	2.4575
A_1(cm^2)	2.000	1.00	1.0000	0.3696
A_2(cm^2)	2.000	1.33	2.1417	4.1912
A_3(cm^2)	2.000	1.58	1.4858	1.5123
A_4(cm^2)	2.000	1.00	1.4018	1.5620
A_5(cm^2)	2.000	1.71	1.911	1.9154
A_6(cm^2)	2.000	1.54	1.0109	1.1315
A_7(cm^2)	2.000	2.65	1.4693	1.8233
A_8(cm^2)	2.000	2.87	2.1411	1.0904
Weight(kg)	338.69	298.0	236.046	228.381

Table 13. Optimized frequencies (Hz) for several methods for the 52-bar space truss.

Frequency No.	Initial Design	Lin (1982)	Lingyum (2005)	Present work
1	22.69	15.22	12.81	12.751
2	25.17	29.28	28.65	28.649
3	25.17	29.28	28.65	28.649
4	31.52	31.68	29.54	28.803
5	33.80	33.15	30.24	29.230

Table 14. Statistical results for 5 independent runs of Swarm Optimization for the 52-bar space truss.

Mean Mass of Swarm	Standard Deviation	No. of Particles	Social Constant	Cognitive Constant	Velocity Momentum	Mean No. of Iterations	Tolerance for Convergence
234.3	5.22	70	1.5	1.5	0.5	161	10^{-3}

In Fig. 14, Fig. 15 and Fig. 16, the shape of initial design and the optimized solutions found by the literature are compared with that obtained in the present work. Again, the similar shapes of the final truss of the present work and that presented by Lingyum *et al* (2005) is noticed. Since none of the papers presented the number of function evaluations, it is not performed here this type of comparison.

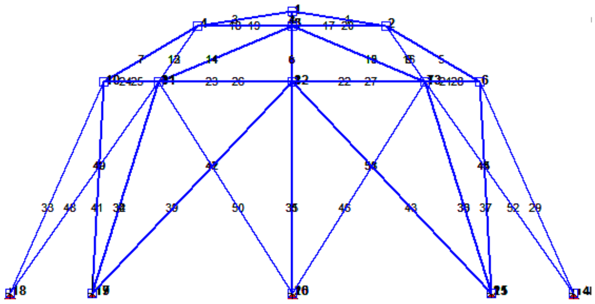

Figure 14. Initial Design of a 52-bar dome structure.

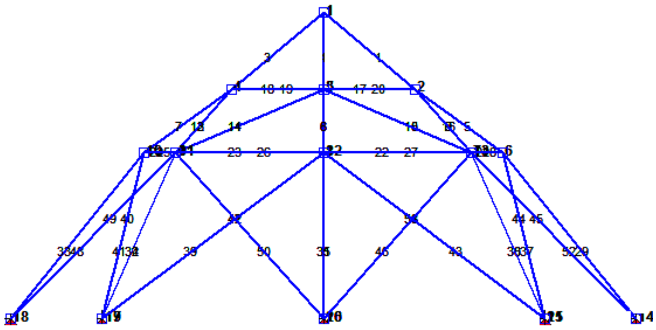

Figure 15. Optimized design of a 52-bar dome structure by Lingyum (2005).

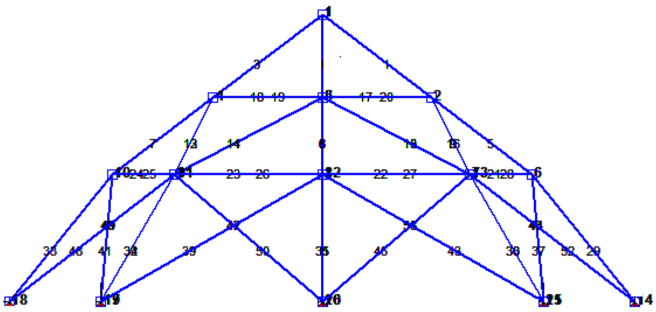

Figure 16. Optimized design of a 52-bar dome structure by present work.

5. FINAL REMARKS

In this paper the problem of truss design optimization with frequency constraints was addressed. The constraints were treated as usual with penalty functions. It is well-known that this kind of optimization problem has high-nonlinear behavior regarding the frequency constraints especially for shape optimization, since eigenvalues are very sensitive to shape modifications. In the literature it was reported several methods that treat this problem using a gradient based formulation, however initial feasible design points are necessary to start the process. In this paper a new methodology is proposed based on a heuristic algorithm. The Particle Swarm Optimization Algorithm is referred in the literature as a global optimizer with advantages in relation to other heuristic algorithms like Genetic ones. Some capabilities that make this Heuristic Algorithm attractive are its lower number of parameter necessary to set previously and its floating point treatment for the design variables. Another common feature to heuristic algorithms is that it requires just objective functions evaluations (it is not required objective function gradients), which allows the method to deal with this kind of problem (symmetrical trusses with equal eigenvalues) without any modifications. Another important feature is the fact that the algorithm works with a population and random parameters which allows exploration/exploitation capabilities and escape from local minima in the search process. It was present two examples of increasing difficulty which were compared with results presented in the literature. In an engineering point of view, the method performed well in the four cases, showing to be promising. Besides, the method got better results than that reported in the literature in the last example. In this last example, compared with the NHGA algorithm (Lingyum *et al.*, 2005), the method needed less function evaluations (about 11270 compared to 13519 from Lingyum *et al.*, 2005).

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4. REFERENCES

Bellagamba, L., Yang, T. Y., 1981, ”Minimum mass truss structures with constraints on fundamental natural frequency”, AIAA Journal, Vol. 19, No.11, pp.1452.

Bergh, F., Engelbrecht, A. P., 2006, “A study of particle swarm optimization particle trajectories”, Information Sciences, Vol. 176, pp. 937-971.

Gholizadeh, S., Salajegheh, E., Torkazadeh, P. , 2008, “Structural optimization with frequency constraints by genetic algorithm using wavelet radial basis function neural network”, Journal of Sound and Vibration, Vol. 312, pp.316-331.

Grandhi, R. V., 1993, “Structural optimization with frequency constraints – a review”, AIAA Journal, Vol. 31, No. 12, pp.2296-2303.

Grandhi, R. V., Venkayya, V. B., 1988, “Structural optimization with frequency constraints”, AIAA Journal, Vol. 26, No.7, pp.858-866.

Hassan, R., Cohanim, B., Weck, O. , 2005, “A comparison of particle swarm optimization and the genetic algorithm”, In: Proceedings of the 46th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Material Conference, No. 1897, 18-21 April, Austin, Texas.

Kennedy, J., Everhart, R. C., 1995, “Particle Swarm Optimization”, In: Proceedings of the IEEE Int. Conference on Neural Networks, V.4, pp.1942-1948.

Li, L.J., Huang, Z. B., Wu, Q. H. , 2007, “A heuristic particle swarm optimizer for optimization of pin connected structures”, Computers and Structures, Vol. 85, pp.340-349.

Lingyun, W., Mei, Z., Guangming, W., Guang, M., 2005, “Truss optimization on shape and sizing with frequency constraints based on genetic algorithm”, Journal of Computational Mechanics, Vol. 25, pp.361-368.

Lin, J.H., Chen, W. Y., Yu, Y. S., 1982, “Structural optimization on geometrical configuration and element sizing with static and dynamic constraints”. Computers and Structures, Vol. 15, No. 5, pp. 507-515.

Ohsaki, M., Fujisawa, K., Katoh, N., Kanno, Y., 1999, “Semi-definite programming for topology optimization of trusses under multiple eigenvalue constraints”, Journal of Computer Methods in Applied Mechanics and Engineering, Vol.180, No.1-2, pp.203-217.

Roux, W. J., Stander, N., Haftka, R. T., 1998, “Response surface approximations for structural optimization”, International Journal for Numerical Methods in Engineering, Vol.42, pp.517-534.

Salajegheh, E., Gholizadeh,S., Torkzadeh, P. , 2007, “Optimal design of structures with frequency constraints using neural wavelet back propagation”, Asian Journal of Civil Engineering (Building and Housing), Vol. 8, No. 1, pp.97-111.

Sedaghati, R., Suleman, A., Tabarrok, B., 2002, “Structural optimization with frequency constraints using finite element force method”. AIAA Journal, Vol.40, No.2, pp.382-388.

Tong, W. H., Liu, G. R., 2001, “An optimization procedure for truss structures with discrete design variables and dynamic constraints”. Computers and Structures, Vol. 79, pp.155-162.

Tong, W. H., Jiang, J. S., Liu, G. R. , 2000, “Solution existence of the optimization problem of truss structures with frequency constraints”, International Journal of Solids and Structures, Vol. 37, pp.4043-4060.

Torkzadeh, P., Salajegheh, J., Salajegheh, E. , 2008, “Efficient methods for structural optimization with frequency constraints using higher order approximations”, International Journal of Structural Stability and Dynamics, Vol.8, No.3, pp.439-450.

Xu, B., Jiang, J., Tong, W., Wu, K., 2003, “Topology concept for truss topology optimization with frequency constraints.” Journal of Sound and Vibration, Vol. 261, pp.911-925.

Wang, D., Zhang, W. H., Jiang, J. S., 2004, "Truss optimization on shape and sizing with frequency constraints", AIAA Journal, Vol.42, No.3, pp.1452-1456.

Yang, X. Y., Xie, Y. M., Steven, G. P., Querin, O. M., 1999, "Topology optimization for frequencies using an evolutionary method", Journal of Structural Engineering, Vol. 125, No. 12, pp.1432-1438.

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