DAMAGE DETECTION IN A 2 DOF NONLINEAR SYSTEM USING VOLTERRA SERIES AND KAUTZ FILTER

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Abstract. *The presence of non-linear behavior of structures when damages or failures are presented can mask the classical linear features indexes used for damage detection based on time series analysis from vibration data. In this sense, the present paper proposes a novelty approach to detect damage using a nonlinear index based on prediction error from expansion of discrete-time Wiener models by using Kautz filter. Once constructed a reference nonlinear model based on healthy measures, the representative kernels of this model can be employed to predict the data in other unknown conditins (healthy or damaged). A numerical example is simulated to evaluated the methodology by using a 2 degrees of freedom (DOF) nonlinear benchmark. The results have shown the applicability, drawbacks and steps necessaries to improve the procedure in order to implement in real-world engineering system.*

Keywords*: Volterra series, Kautz filter, nonlinear mechanical system, damage detection.*

1. INTRODUCTION

One of the greatest limitations to develop structural health monitoring (SHM) techniques is that the most part of systems have non-linear behavior when damages or failures are present, as for instance in the presence of cracks (Farrar and Sohn, 2000). Even if the structure can be considered linear in healthy condition, the presence of damages can become the system nonlinear. In this case, the relationships between input and output are essentially nonlinear and the conventional linear feature indexes for SHM can be overlooked. Moreover, the system can excite nonlinear components due to environmental, operation or other input variations. Up to now, SHM is case dependent and the application of index can lead to false-positives of damage.

In the last years, several authors propose nonlinear feature indexes to SHM purposes in order to overcome the above considerations. Some classical and new nonlinear mechanical system identification methods can be employed and well adapted to damage detection purposes. A complete review of this subject and future trends can be found in Kerschen et al. (2006). Among these procedures, time-domain approaches have received much attention because there is well-established the non-linear system identification using discrete-time models of stochastic processes. In this sense, Wei et al. (2005) investigated the nonlinear autoregressive moving average with exogenous input (NARMAX) models for vibrating multi-layer composite plates to assess internal delaminating in plates. The NARMAX coefficient variations are employed as nonlinear index to detect and assess severity and to localize the delaminating in the plate.

Fassois and Sakellariou (2007) used a nonlinear ARX (NARX) model to predict the time series in a simple nonlinear simulated structure. In order to detect the structural changes, it was used the functional model-based method and hypothesis tests. Basically, the method consists in reparametrize the variance magnitude, called fault magnitude. The results reached were satisfactory. The structural changes were induced in the linear stiffness of the numerical nonlinear model. However, it will be interesting to evaluate the changes occurred directly in the nonlinear parameters in futures works. Adams and Allemang (2001) proposed a modified ARX model in the frequency domain to use in nonlinear identification. Rutherford et al. (2007) combined this approach with piezoelectric actuators/sensors bonded in a frame structure to propose a non-linear feature identification technique for structural damage detection.

However, in the above nonlinear output regressive models, as NARX or NARMAX, have problems to decide the mathematical structure of the nonlinear regressor and to choose the time-lag orders (Billings, 1980). Classical Wiener/Volterra series should be more appropriated for purpose of prediction of non-linear and nonparametric model (Worden and Manson, 1998). Unfortunately, numerous problems of convergence are found in the Volterra models. In order to overcome this inconvenience, the present paper applies an approach based on discrete-time Wiener/Volterra models expanded with orthogonal Kautz filter (Van den Hof et al., 1995), called here by Kautz-Volterra model. Once a reference Kautz-Volterra model is identified in the healthy condition, these undamaged kernels are employed to monitor the input-output data in unknown conditions. If a structural variation is induced, the nonlinear prediction error will be modified, similarly as the linear counterpart using time series analysis proposed in the works Sohn et al. (2000) and Sohn and Farrar (2001).

This paper is divided in three main parts. The first one is devoted to a basic review involving the well-known Wiener/Volterra models and the Kautz filter. The second one describes the main contribution of the paper, i.e., the proposed of Kautz-Volterra model for damage detection. The last part shows an illustrative example in an academic 2 DOF nonlinear benchmark in order to evaluate the applicability and limitation of the proposed methodology, and, also, pointed out future directions for improvement of this technique.

2. VOLTERRA SERIES WITH KAUTZ FILTER

The basic procedure consists of several stages. It is first reviewed the Volterra/Wiener series and in the following the Kautz filter with the method to set the poles in expansions using the Kautz functions. The methodology is summarized in this section.

2.1. Volterra model

The discrete-time Volterra series is represented by the following expression (Rugh, 1991):

$$x(k)=\sum_{m=1}^{+\infty}\sum_{n_1=-\infty}^{+\infty}\cdots\sum_{n_m=-\infty}^{+\infty}h_m(n_1,n_2,\cdots,n_m)\prod_{i=1}^{m}u(k-n_i) \quad (1)$$

where $x(k)$ is the output signal, $u(k)$ is the input signal and the Volterra kernels are described by $h_m(n_1,n_2,\cdots,n_m)$.

A practical drawback in Volterra series is the necessity of a great number of terms in the infinity power series in Eq. (1). So, to truncate the Volterra series is necessary a high number of samples. A reason for this is relative to the fact that nonlinearities in Volterra series truncated has been modeled using only regression in the input signal (in the mechanical system are excitation forces) $u(k)$. So, it is not used feedback of the output times-series signals $x(k)$, as made in NARMAX model. The regressions in the output signal $x(k)$ represent dynamic, which is reflected by complex poles.

2.2. Wiener model

The Wiener series is an orthogonal expansion of the classical Volterra series. This mathematical approach reduces the number of parameters necessaries to compute the kernels. The basic idea is to describe the kernels $h_m(n_1,n_2,\cdots,n_m)$ by employing orthogonal basis $\psi_{i_j}(n_j)$, where $\psi_{i_j}(n_j)$ is the impulse response function of an orthogonal discrete-time filter $\boldsymbol{\psi}(z)$, where z is the complex variable. Then:

$$h_m(n_1,n_2,\cdots,n_m)\approx\sum_{i_1=1}^{M_1}\cdots\sum_{i_m=1}^{M_m}\alpha(i_1,\cdots,i_m)\prod_{j=1}^{m}\psi_{i_j}(n_j) \quad (2)$$

where $M_1,\cdots,M_m$ are the numbers of orthogonal filters used to approach the kernels and the coefficients $\alpha(i_1,\cdots,i_m)$ are projections of the kernels into an orthogonal basis $\psi_{i_j}(n_j)$, namely Wiener kernel.

The general expression of the Volterra series from Eq. (1) is replaced by the counterpart using the orthogonal functions, by ignoring the dynamics from higher-order kernels:

$$x(k)=\sum_{m=1}^{M}\sum_{i_1=1}^{N_1}\cdots\sum_{i_m=1}^{N_m}\alpha(i_1,\cdots,i_m)\prod_{j=1}^{m}l_{i_j}(k) \quad (3)$$

where M is the order of the nonlinearities, $N_1,\cdots,N_m$ are the numbers of time samples to identify each Wiener kernel, $l_{i_j}(k)$ is the input signal (excitation force) $u(k)$ filtered by orthogonal basis $\psi_{i_j}(n_j)$:

$$l_{i_j}(k)=\sum_{\tau_j=0}^{\varepsilon}\psi_{i_j}(\tau_j)u(k-\tau_j) \quad (4)$$

where ε is the quantity of lag memory. In the present paper only the two first kernels were used to describe the input-output data ($M=2$). Taking into account the symmetry property of kernels yields:

$$h_1(n_1)=\sum_{i_1=1}^{M_1}\alpha(i_i)\psi_{i_1}(n_1) \quad (5)$$

$$h_2(n_1,n_2)=\sum_{i_1=1}^{M_2}\sum_{i_2=1}^{i_1}\alpha(i_1,i_2)\psi_{i_1}(n_1)\psi_{i_2}(n_2) \quad (6)$$

where M_1 and M_2 are the number of orthogonal filters to identify the first $\alpha(i_1)$ and second $\alpha(i_1,i_2)$ orthogonal kernels, respectively. The Wiener model considered for truncating the system with N_1 and N_2 time samples to identify the first and second order kernels, respectively, is given by:

$$x(k)=\sum_{i_1=1}^{M_1}\alpha(i_1)l_{i_1}(k)+\sum_{i_1=1}^{M_2}\sum_{i_2=1}^{i_1}\alpha(i_1,i_2)l_{i_1}(k)l_{i_2}(k) \tag{7}$$

If it is known the input $u(k)$ and the output $x(k)$, the kernels can be found through the least square (LS) problem:

$$x(k)=\boldsymbol{\theta}\boldsymbol{\lambda}(k) \tag{8}$$

The vector of parameters is given by:

$$\boldsymbol{\theta}=\left[\alpha(1)\quad\cdots\quad\alpha(M_1)\quad\alpha(1,1)\quad\alpha(2,1)\quad\alpha(2,2)\quad\cdots\quad\alpha(M_2,M_2)\right]^T \tag{9}$$

The regressive vector $\boldsymbol{\lambda}(k)$ is written by considering the input signals $u(k)$ filtered by the orthogonal filters:

$$\begin{aligned}\lambda(k)=\big[&l_1(k)\quad l_2(k)\quad\cdots\quad l_{M_1}(k)\quad l_1(k)^2\quad l_2(k)l_1(k)\\ &l_2(k)^2\quad l_3(k)l_1(k)\quad l_3(k)l_2(k)\quad l_3(k)^2\quad\cdots\quad l_{M_2}(k)^2\big]\end{aligned} \tag{10}$$

Once the orthogonal kernels $\alpha(i_1)$ and $\alpha(i_1,i_2)$ are identified, the physical kernels $h_1(n_1)$ and $h_2(n_1,n_2)$ are found by projection using the orthogonal functions $\psi_{i_j}(n_j)$, as written in Eqs. (5) and (6).

2.3. Kautz filter

Several types of orthogonal functions $\psi_{i_j}(n_j)$ can be used to represent the kernels $h_1(n_1)$ and $h_2(n_1,n_2)$ in Eqs. (5) and (6), as for instance the polynomials of Chebyshev or Legendre. However, those functions are not directly linked with the dynamic system by differences equations. Thus, the number of expansions in Eq. (5) and (6) is elevated.

Fortunately, the Kautz filters are very effectives to develop the orthogonal kernels. The Kautz functions contain poles with information of the dominant dynamics of the system, Wahlberg (1994) and Van den Hof et al. (1995). The poles in a Kautz basis are able to decrease drastically the number of parameters to estimate the kernels $\alpha(i_1)$ and $\alpha(i_1,i_2)$.

The Kautz filter $\boldsymbol{\psi}_n(z)$ has a pair of complex pole in the discrete-time domain that are chose by $\beta=\sigma+j\omega$ and $\bar{\beta}=\sigma-j\omega$. These poles are given by two scalar real-values b and c, where $|b|<1$ and $|c|<1$ for stable poles. This filter constitutes a good generalization of a second-order dynamic of vibration system.

The elements in a set of Kautz filters are given by (Wahlberg, 1995), (Van den Hof, 1995) and (Rosa et al., 2007):

$$\psi_{2n}(z)=\frac{\sqrt{(1-c^2)(1-b^2)}\,z}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{n-1} \tag{11}$$

$$\psi_{2n-1}(z)=\frac{\sqrt{1-c^2}\,z(z-b)}{z^2+b(c-1)z-c}\left[\frac{-cz^2+b(c-1)z+1}{z^2+b(c-1)z-c}\right]^{n-1} \tag{12}$$

where the constants b and c are relatives to the poles β and $\bar{\beta}$ through the relationships:

$$b=\frac{(\beta+\bar{\beta})}{(1+\beta\bar{\beta})}\ \text{and}\ c=-\beta\bar{\beta} \tag{13}$$

Equations (11) and (12) are used for filtering the signals of input $u(k)$, and, then they are substituted in Eq. (4) to obtain the signals $l_{i_1}(k)$ and $l_{i_2}(k)$. The impulse response functions of Eqs. (11) and (12) are recorded to estimate the kernels. From here, the Wiener/Volterra model constructed by a Kautz filter is called as Kautz-Volterra model.

2.4. Choice of poles in expansions using Kautz functions

An important and difficulty task to overcome is the definition of poles β and $\overline{\beta}$ used to build the filters. A procedure for estimating the poles and the kernels $\alpha(i_1)$ and $\alpha(i_1,i_2)$ simultaneously in an iterative way was described in Rosa et al. (2007, 2008). Meanwhile, in structural dynamic is currently available the linear matrix of mass $\mathbf{M}$, damping $\mathbf{C}$, and stiffness $\mathbf{K}$ by using a numerical method, for example the finite element method (FEM), or by using a linear classical modal analysis to identify the natural frequencies, damping ratio and modal shapes. So, this information can be well utilized to estimate the Kautz poles in the z plan. The dominant complex pole of the linear model can be chosen as the Kautz parameter. However, the choice of the Kautz poles based on the linear portion of the dynamic system should only be done if the nonlinear system was very simple. In summary the first kernel $\alpha(i_1)$ can be estimated by using a pole identified directly using the linear model assumed known by using some classical modal method or procedure of modeling via finite elements. This pole can be given in continuous domain s. The discrete-time conversion can be made using bilinear transformation or other procedure considering the time sampling known.

In the present paper, the approach to choice the Kautz parameter used to identify the second kernel $\alpha(i_1,i_2)$ consists to minimize the following objective function:

$$\min_{s_2} \quad e(s_2) = \frac{\left\| x - \tilde{x}(s_2) \right\|^2}{\left\| x \right\|^2} \tag{14}$$

where s_2 is the complex Kautz pole in s plan (it is necessary to convert to z plan) and it is the optimization parameter, x is the experimental value of output signal and $\tilde{x}$ is the output signal computed by eq. (7) considering the second kernel $\alpha(i_1,i_2)$ expanded by using the Kautz function with poles in s_2. The constraints imposed in eq. (14) are relatives to limit the real and imaginary part of s_2 in a feasible range, normally chosen close to first Kautz parameter.

This optimization problem can be solved by using the sequential quadratic programming (SQP) algorithm. The optimal value found in this problem has not assurance that is the real global optimal, as made with the method proposed in Rosa et al. (2007,2008), where it was obtained an analytical solution for a sub-optimal expansion of Wiener series using a set of Kautz functions. However, in order to obtain the non-linear prediction model in the present application, the minimization of eq. (14) is enough.

3. DAMAGE DETECTION PROCEDURE USING WIENER SERIES AND KAUTZ FILTER

The first step in the proposed methodology to damage detection purposes is to identify the kernels $h_1(n_1)$ and $h_2(n_1,n_2)$ in healthy condition based on the undamaged experimental signals $u(k)$ and $x(k)$ by using the procedures described in section 2.

The Volterra kernels and the input signal $u(k)$ are used to predict the signal $x(k)$. This predicted signal is named as $\hat{x}(k)$ and obtained by:

$$\hat{x}(k) = \sum_{n_1=0}^{N_1} h_1(n_1) u(k-n_1) + \sum_{n_1=0}^{N_2} \sum_{n_2=0}^{n_1} h_2(n_1,n_2) u(k-n_1) u(k-n_2) \tag{15}$$

Equation (15) show that the output signal is given by $\hat{x}(k) = \hat{x}_{lin}(k) + \hat{x}_{nl}(k)$, where $\hat{x}_{lin}(k)$ is the linear contribution provided by the first kernel $h_1(n_1)$ and $\hat{x}_{nl}(k)$ is the nonlinear contribution from the second kernel $h_2(n_1,n_2)$.

Thus, the linear prediction error $e_x^{lin}(k)$ and the nonlinear prediction error $e_x^{nl}(k)$ in the reference baseline is defined as:

$$e_x^{lin}(k) = x(k) - \hat{x}_{lin}(k) \tag{16}$$

$$e_x^{nl}(k) = x(k) - \hat{x}(k) \tag{17}$$

If the reference Kautz-Volterra model does not drive a reasonable prediction to the experimental signal in unknown condition, named $y(k)$, the prediction error will have different statistics features when comparing with the reference error. Particularly, if the damages change the nonlinear parameters, the linear error $e_y^{lin}(k)$ could not be sensitive to variations in the relation of the linear reference baseline $e_x^{lin}(k)$. In order to overcome this point two feature indexes are proposed:

$$\gamma_{lin} = \frac{\sigma\left(e_y^{lin}(k)\right)}{\sigma\left(e_x^{lin}(k)\right)} \tag{18}$$

$$\gamma_{nl} = \frac{\sigma\left(e_y^{nl}(k)\right)}{\sigma\left(e_x^{nl}(k)\right)} \tag{19}$$

where $\sigma(\ )$ is the standard deviation operator. Theoretically γ_{nl} can evaluate structural changes occurred in nonlinear parameters relatives to the second order kernel.

4. APPLICATION EXAMPLE USING A BENCHMARK

In order to illustrate the approach proposed, a nonlinear two degree-of-freedom system composed by cubic stiffness in spring, namely $k_{nl} = 900$ N/m^3, is simulated, see fig. (1). This benchmark model is the same used by Sakellariou and Fassois (2002).

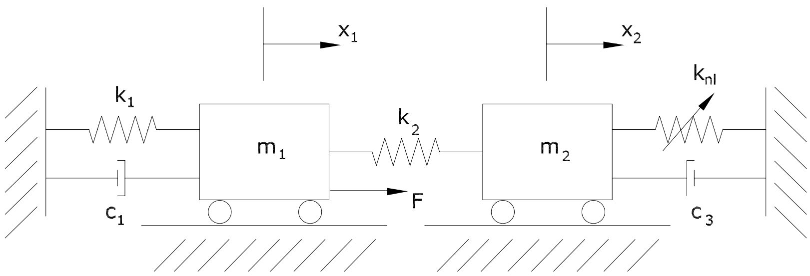


Figure 1. Nonlinear mechanical system with cubic stiffness.

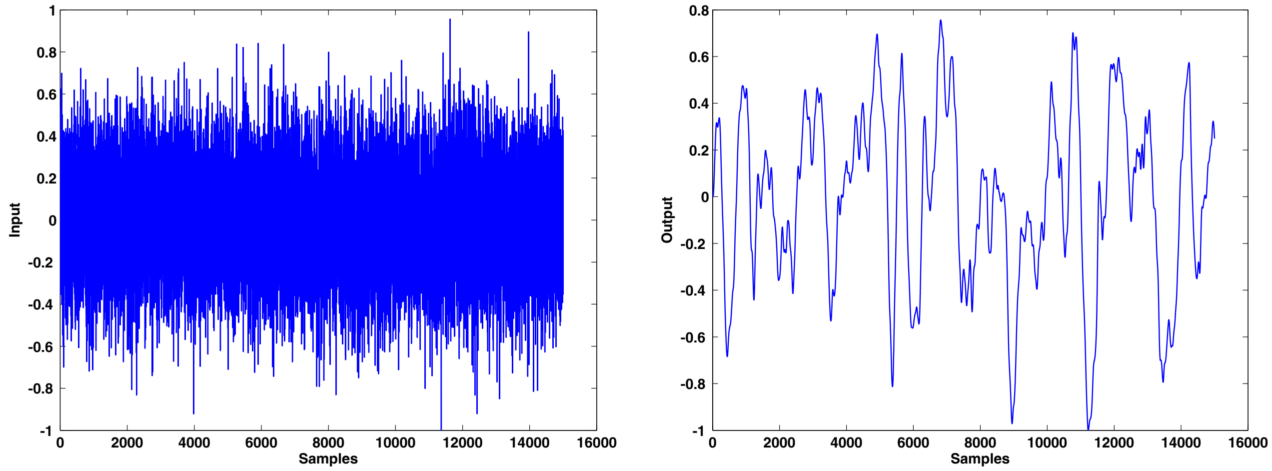
The motion equation of the nonlinear system is described by:

$$\begin{aligned} m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 + c_1\dot{x}_1 &= F \\ m_2\ddot{x}_2 - k_2x_1 + k_2x_2 + k_{nl}x_2^{\ 3} + c_3\dot{x}_2 &= 0 \end{aligned} \tag{20}$$

where the linear parameters are the mass $m_1 = 10$ kg and $m_2 = 55$ kg, stiffness $k_1 = 10$ kN/m and $k_2 = 90$ kN/m, viscous damping $c_1 = 750$ N.s/m and $c_3 = 300$ N.s/m. The random force F is applied with normal distribution at the first mass and with amplitude of 5000 N.rms.

In order to obtain the "experimental" responses, the eq. (20) was integrated using the Newmark and Newton-Raphson method considering the exact parameter values. A sampling frequency of 1500 Hz and $N = 15000$ samples were used to generate the data time. The input-output data were normalized in the interval [-1,1] in order to avoid numerical ill-posedness. Figures (2a) and (2b) show the results considering the healthy system. These data was used as baseline (reference). Only the discrete-time displacement $x_2(k)$ and the input $u(k) = F(k)$ are employed to estimate the kernels $h_1(n_1)$ and $h_2(n_1, n_2)$ that correspondent for the undamaged system (baseline).

It was assumed that the linear portion of the system is well-known. Thus, the natural frequency ω_{ni} and the damping ratio ζ_i are analytically evaluated. So, the Kautz pole to estimate the first kernel $h_1(n_1)$ can be considered in the continuous domain as function of the first mode relative to the first natural frequency ω_{n1} and the first damping ratio ζ_1. Thus, $s_1 = -\varsigma_1\omega_{n1} \pm j\omega_{n1}\sqrt{1-\varsigma_1^{\ 2}}$. The Kautz parameter for the first kernel was assumed as $s_1 = -7.687 \pm 9.54j$. The conversion of continuous-time parameter s_1 to the discrete-time parameter (β, $\overline{\beta}$) can be effected by a bilinear approximation. The number of Kautz filter used to estimate the first kernel was $M_1 = 2$. It is also considered that the number of samples in the $h_1(n_1)$ was $N = 2000$ samples. Figure (3) presents the first kernel estimated by comparing the analytical impulse response function of the linear equivalent model, considering $k_{nl} = 0$.



(a) Input signal applied $u(k)$. (b) Output signal displacement $x_2(k)$.

Figure 2. Normalized input-output data in healthy conditions.

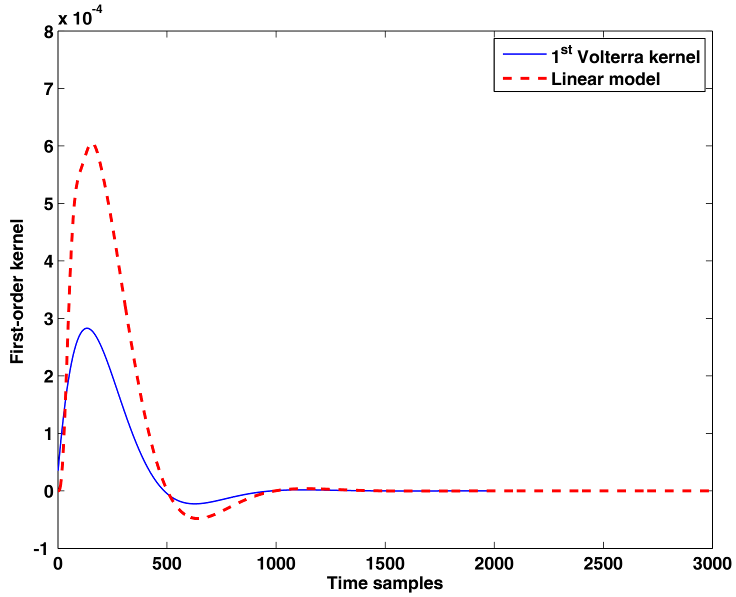


Figure 3. First order kernel $h_1(n_1)$ estimated and the impulse response function of a linear equivalent model – Baseline condition (Reference model).

The first Volterra kernel can be well identified by using the linear portion of the system, case this information is available. However, the second kernel $h_2(n_1, n_2)$ is function of an unknown Kautz parameter in the filter. In the present paper, it is considered to minimize eq. (14) in order to find an approximation to this value. The constraint imposed in the second Kautz parameter was to limit the values in the interval $[-20 \quad -5]$ in the real part and $[-30 \quad -5]$ in the imaginary part. The initial value in the optimization procedure was to consider the pole associated with the first pole by $2s_1$. A procedure by using the sequential quadratic programming (SQP) was solved to find the optimal value with these conditions. The result found was $s_2 = -12.9 + 14.1j$ with the value of the objective function as $e(s_2) = 0.027$. Figure (4) illustrates the objective function in these intervals with a mesh considering the limits. Since the kernel is estimated by using the discrete-time Kautz filter, the pole is converted to z plan. This pole is used to estimate the second Volterra kernel $h_2(n_1, n_2)$ by using $M_2 = 6$. Kautz functions. The number of Kautz functions used is chosen based on increasing the number of filter and considering the stabilization of the system. $M_2 = 6$ is an enough number to estimate the second kernel in this example. The long memory terms of the second kernel – longer than 1000 samples – are considered to be null. Figure (5) illustrates the kernel $h_2(n_1,n_2)$ approximated by Kautz filter using the information described before.

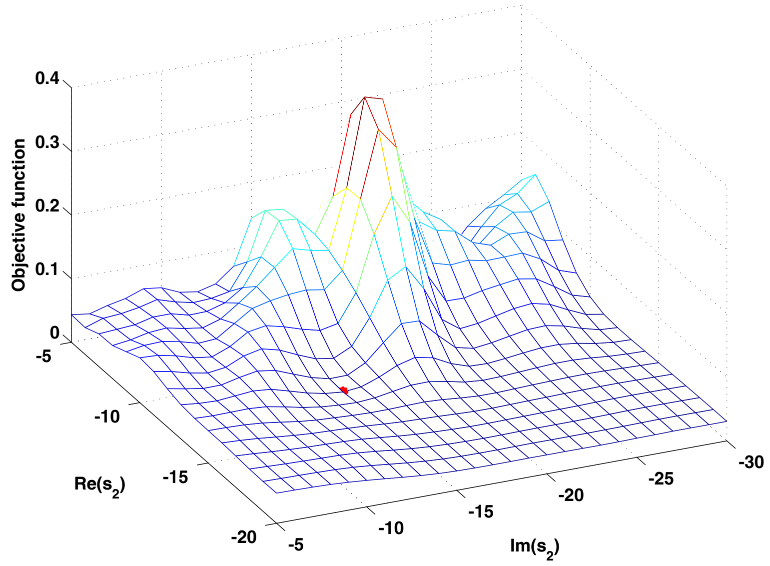


Figure 4. Objective function used to find the optimal Kautz parameter to identify the second kernel $h_2(n_1,n_2)$. The optimal value in this range is shown as a red circle.

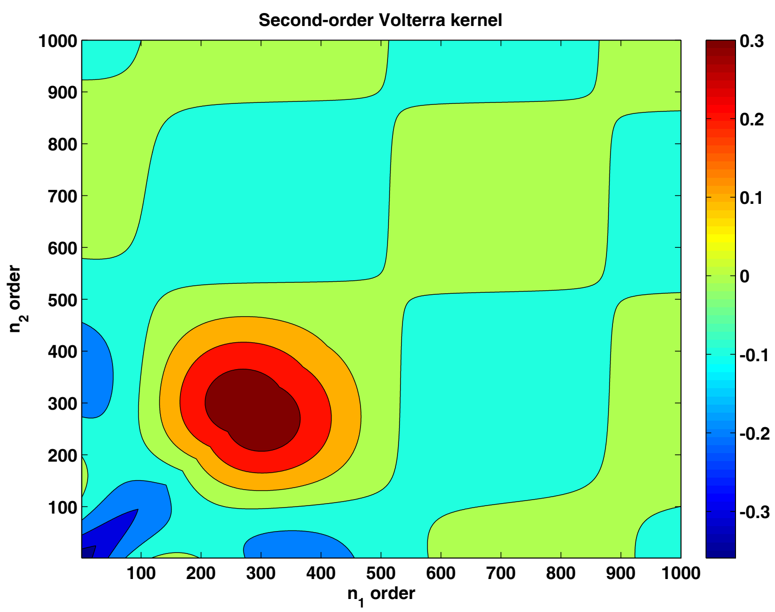


Figure 5. Second order kernel estimated $h_2(n_1,n_2)$ – Baseline condition (Reference model).

Six cases in unknown structural conditions are simulated with the model considering changes in the stiffness parameters (linear and nonlinear parts). The tab. (1) describes each condition, where the case (1) is an undamaged situation to evaluate false-positive tests and cases (2) to (6) correspond to different structural changes in the system. Figures (6) and (7) show a comparison between the reference output normalized and the output from case (1) and case (6), respectively. It worth to observe that is very difficult to detect structural changes by analyzing only these figures.

The nonlinear prediction by using the baseline Volterra model, eq. (15), is computed with the data in each unknown conditions. The linear and nonlinear residual error using eqs. (16) and (17) were computed from all structural cases. It was compared the probability density function (PDF) estimated through the kernel smoothing method (Bowman and Azzalini, 1997). If there is damage in the nonlinear mechanical system, then the PDF should change. Figures (8) and (9) illustrate this by comparing the PDF of the linear and nonlinear residual error from case 1 (healthy), case 6 (with damage) and with the reference signal error (baseline). Both PDFs were able to detect changes related failures, because the damages were simulated high-intensity (average of 50% in case 6).

Table 1. Structural conditions.

Case	Description
(1)	Undamaged – false-positive test.
(2)	Damage – reduction of 30 % in stiffness parameter k_1
(3)	Damage – reduction of 30 % in stiffness parameter k_2
(4)	Damage – reduction of 5 % in stiffness parameter k_{nl}
(5)	Damage – reduction of 20 % in stiffness parameter k_{nl}
(6)	Damage – reduction of 50 % in stiffness parameter k_{nl}

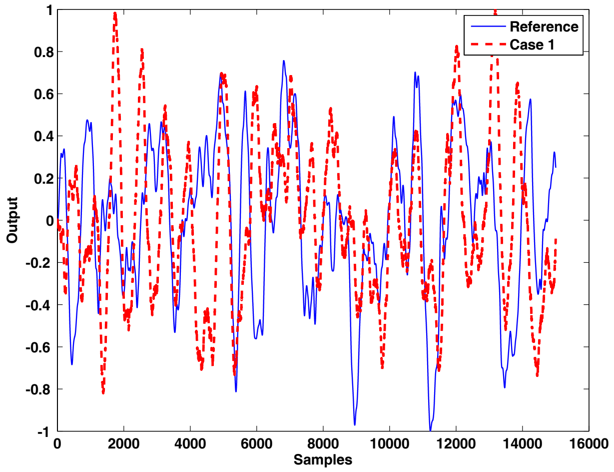


Figure 6. Comparison between the reference output and case 1.

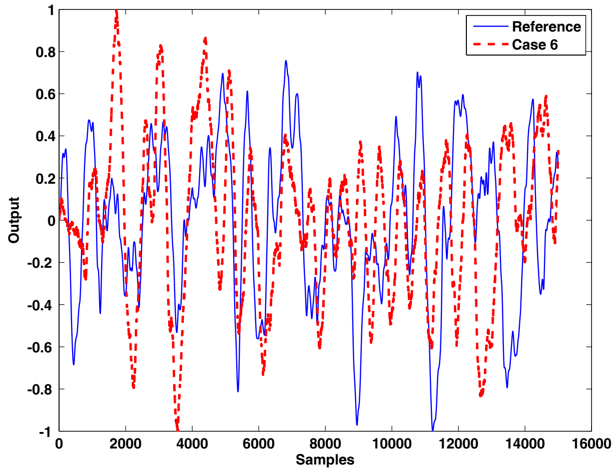


Figure 7. Comparison between the reference output and case 6.

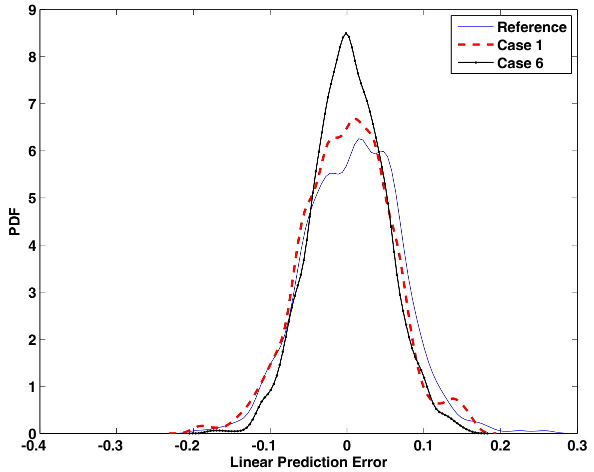


Figure 8. Comparison between the probability density function (PDF) of the linear prediction error from reference, case 1 and case 6.

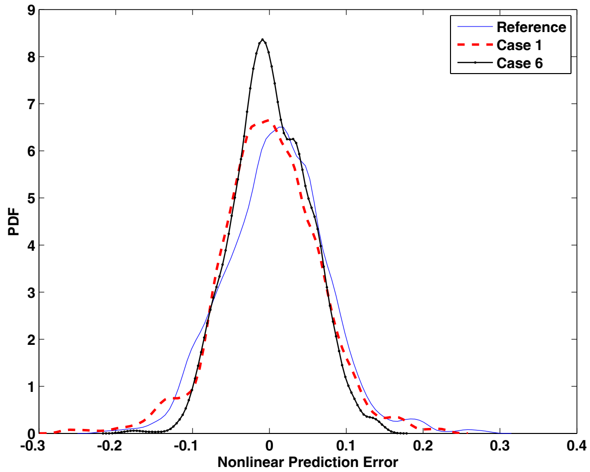


Figure 9. Comparison between the probability density function (PDF) of the nonlinear prediction error from reference, case 1 and case 6.

In order to classify the structural states based on the dataset with a more rigorous statistical criterion was employed the eqs. (18) and (19). The linear and nonlinear feature, γ_{lin} and γ_{nl}, respectively, were computed. Figure (10) presents these ratios for all six structural conditions. The approach presents suitable detection ability. It is also worth to note that the index feature increased when a structural variation was introduced and becomes the severity becomes larger. But, this result is not conclusive due to low number of data considered. Tests with larger amount of data must be performed in order to take information about the level of severity. If the index γ_{lin} and γ_{nl} are close to unity it means that the system is healthy.

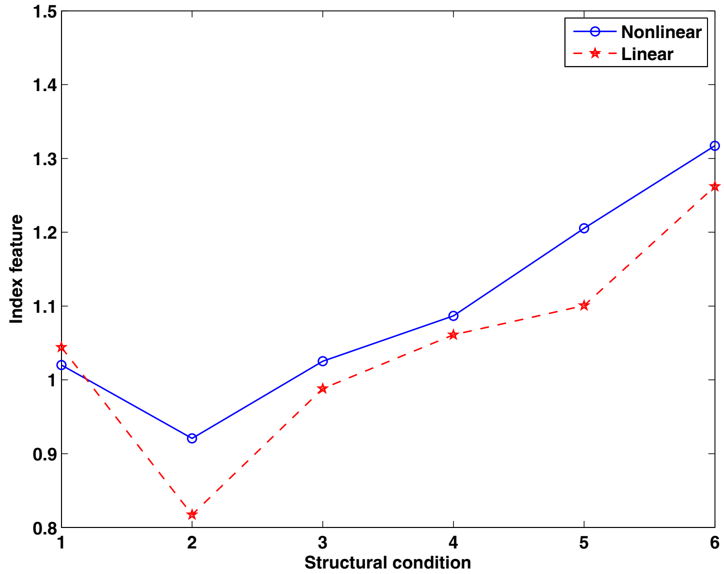


Figure 10. Linear and nonlinear index features, γ_{lin} and γ_{nl}, respectively, for all structural condition studied.

Both indexes gives a clear indicative to the presence of faults or condition undamaged of the system. In the false-positive test (case 1) the nonlinear index is nearest to unity value. In all other cases the nonlinear feature is bigger than the linear index, even when the damage reaches the variation of parameters of linear stiffness. When damages occur in the simulated parameter of nonlinear stiffness and its severity is high, for example in cases (5) and (6), the index difference between the linear and nonlinear indicator is even greater.

5. FINAL REMARKS

A novelty approach to structural health monitoring of structures with non-linear behavior was presented in this paper. The methodology was based on non-linear error prediction extracted from discrete-time Wiener/Volterra models. In order to overcome the difficulties found in Volterra theory, the Kautz basis are employed to identify the reference Wiener kernels in the healthy conditions (baseline). The Kautz parameters in these filters were estimated by using the linear structural matrix considered known to identify the first kernel and an optimization procedure to identify the poles relatives to the second order kernel. For the numerical application evaluated, the non-linear feature index γ_{nl} proposed was more representative to structural changes than the linear index γ_{lin}, even if the damages affect only linear parameters. Despite the simplifications in this application, it is possible to deal with more complex structures with bolted joints and other local non-linearities. Future studies are needed in order to evaluate the capabilities and effectiveness of this method involving real applications in engineering structural system, especially by using smart structures and impedance signals in time domain. In this sense, a nonlinear prediction error extracted by Volterra model could be used to structural health monitoring in smart structures, enhancing the results reached by the author in Silva et al. (2008).

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