THE INFLUENCE OF THE RE-MESHING IN THE SHAPE OPTIMIZATION ALGORITHM

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Abstract. An adaptive process using gradients or densities of the deformation energy can lead to a good model of finite elements, identifying in on easy way which the regions require or not mesh refinement. The model must contain an adaptable strategy to prevent the formation of distorted elements, mainly when the adaptive method of nodal relocation is used, which possess this unfavourable feature. In an heuristic way the coupling of the concept of optimized shapes are delineate, passing for regridings, that has the function to attenuate in the iterative process the problem of the distortion in the use of r method. Examples of application are used for evaluation of the considered model, that is to say: study of the influency in the shape optimization, if using the intermediary modulus of regriding, which equilibrate the desbalancing areas of elements in the discretization model.

Key words: hierarchical finite element, adaptive method, shape optimization

1. Introduction

Shape optimization has attracted great attention from the scientific community and many available techniques have been developed and successfully used in engineering analysis and design. Generally, this tecniques consist on varying some boundaries of the model to be designed in order to improve its mechanical behavior, as, for example, to reduce high stress concentrations which normally occur at corners locations or holes near to them. This process is usually done by imposing restrictions and by using the selected optimization method where aspects such as geometric definitions, mesh generation, analysis and displaying of results are usually involved. Furthermore, other elements play a decisive role in the optimization process, such as sensitivity analysis and numerical optimization programming.

The first step is to define the geometric and the analytical models. In the geometric model the design variables are imposed and it allows an explicit integration with other design tools, such as CAD or CAM systems. On the other hand, the analytical model is used to obtain the response of the structural part, under external actions. Then, a sensitivity analysis must be done to obtain a solution of the problem; and finally, an appropriate optimization algorithm must be selected to solve the optimization problem in an effective and reliable way.

Zienkiewicz and Campbell [1973] initiated the numerical treatment of shape optimization problems, a large number of publications have appeared in the field of shape sensitivity analysis. Haug et al. [1986] presented an unified theory of continuum shape design on the same line using a variational formulation for the governing equations of structural mechanics and material derivative concepts. Several researches, see for instance Yang and Botkin [1986], and Yao and Choi [1989] who have investigated the accuracy of the shape design sensitivity theory utilizing the finite element method (FEM), Choi and Kwak [1990], Baron and Yang [1988], Kane and Saigal [1988], Mellings and Aliabadi [1995], and Mellings [1994], have presented numerical results in the optimization of potential and two-dimensional (2D) elasticity problems. Also, Meric [1995] presented a sensitivity analysis of 2D shape optimization with BEM, comparing integral and differential formulations in heat conduction problems. Another work by Parvizian and Fenner [1997] optimizes 2D boundary element models using mathematical programming. The recent paper of Kita and Tanie [1997] proposed an approach based on Genetic Algorithms (GAs) and BEM, where the initial mesh is set up with a rectangular mesh of nodes. These nodes move their positions, following genetic optimization, until the optimal shape is reached. The results are encouraging although the user cannot define an initial real mesh.

Initially, many authors such as Zienkiewicz and Campbell [1973], Ramakrishnan and Francavilla [1975] among others, did not use geometric modeling in the shape optimization problems addressed by them. Instead, they defined the nodal coordinates of the discrete finite element model as design variables. This approach requires a large number of variables and tends to produce jagged edges shapes. In order to overcome this problem a large number of constraints must be added, which complicates the design task. Moreover, the lack of an associated geometric model does not allow the integration with powerful design tools like CAD or CAM systems.

The success of any optimization methodology hinges on its ability to deal with complex problems, as is the case of shape optimization design. Solving non-linear optimization problems efficiently is a challenge. Furthermore, it is quite

common in practice that methods are modified, combined and extended in order to construct an algorithm that matches best the features of the particular problem at hand.
With the objective to attenuate in the iterative process the problem of the distortion in the use of r method, this work presents the development of a technique for automatic remeshing of the finite element model. A linear triangular finite element is used in order to balance the elements area of the mesh being obtained a geometric method of nodal reallocation. A shape optimization is based on the mesh optimization method, Clapis (1999), where the formulation is based on the homogenization of the strain energy density per finite elements, and the stop criterion is always based on the maximum distortion energy value, so that the elements with smaller energy migrate to the areas where the densities of energy are larger. A FORTRAN computer code is implemented allowing to combine shape optimization with remeshing in a automatic iterative procedure.

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2 - Proposed shape optimization algorithm

Considering U_D^e, deformation energy per distortion of the element, as a negative pressure acting in a finite element of the geometric model of the structure, Clapis (1999), the nodal equivalent force vector, $\{D\}_e$ for a lineal triangular element is given for,

$$\{D\}_e = U_D^e \begin{Bmatrix} y_{32} \\ -x_{32} \\ -y_{31} \\ x_{31} \\ y_{21} \\ -x_{21} \end{Bmatrix} \tag{1}$$

where:

$$x_{ij} = x_j - x_i$$

$$y_{ij} = y_j - y_i$$

The overlap of the vectors $\{D\}_e$ for all the elements is obtained a global vector of equivalent nodal loads, $\{D\}_G$. The direction $\{d\}_0$ of the movement of the nodes can be obtained from:

$$[K]_0 \{d\}_0 = \{D\}_G \tag{2}$$

where:
$[K]_0$ it is stiffness matrix obtained from the geometric model, $\{d\}_0$ it is the "search direction" where one tries to homogenize the distribution of the strain energy density and $\{D\}_G$ it is global vector of equivalent nodal loads.

Through one-dimensional search, using the secant method, the scale factor is calculated, which will minimize the norm of the unbalance vector:

$$\underset{a}{\text{MIN}} \; \| \{D(\{x\} + a\{d\}_0)\} \| \tag{3}$$

where $\{x\}$ is the vector of coordinates of the movable nodes.

3 - Proposed Regriding Algorithm

The numeric code of regriding allows remeshing finite element mesh in two-dimensional problem. Automatically, a hierarchical triangular finite element is used with a quadratic function to balance the area of the elements of the mesh using the geometric method of nodal reallocation, Cheng (1993). It is verified the existence of a homogeneity of areas of the elements mesh in the considered area, but if it doesn 't happen, the prior established free nodes would be move in function of the resultant of the loads, Figure 1, and new x cordinate worth:

$$x_n^{\,m} = x_n^{\,m-1} + \frac{\sum A^*\left(x^* - x_n^{\,m-1}\right)}{\sum A^*} \tag{4}$$

where m indicates the iteration number, x_n and x^* are, respectively, position of the node considered and centroids of the surrounding elements, and A^* is the area of each finite element.

Similarly, for y cordinate, one has:

$$y_n^{\ m} = y_n^{\ m-1} + \frac{\sum A^*\left(y^* - y_n^{\ m-1}\right)}{\sum A^*} \tag{5}$$

This is an iterative process where the nodes move until reducing the distortion between the areas of the elements or the reallocation of the nodes value $\left(x_n^m - x_n^{m-1}\right)$ become sufficiently small.

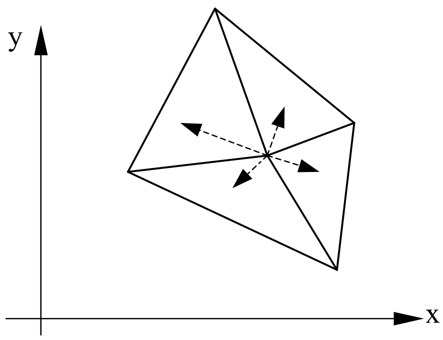


Figure 1 –Nodal reallocation diagram

4 – Automation of the algorithms of shape optimization and regriding

With the objective of correcting elements distortion after nodal reallocation in the process of the shape optimization, a numeric code was made coupling shape optimization and regriding of automatic and iterative form. Figure 2 displays the flow chart of the joining.

Application example 1 - Column of solemnity highway

The united optimization project of shape and regriding of highway columns is considered in this example. Oda (1977), Kikuchi (1986), Rossow (1976) and Clápis (1999) used of this same structure type for evaluate your proposals in the search for the ideal way or gotten better, of the structural element considered.

The Figure 3 displays the project characteristics, considering just to evaluate the proposed method, that the structure is made of steel instead of armed concrete what it is usually employed for making a structural component of this load. Other used data: module of elasticity 205000 MPa, coefficient of Poisson 0.30, constant thickness of the structure of 1000 mm and traction yield limit equal to 7 x 10^7 N/m^2.

The choice of the material bases on the distortion energy criterion (von Mises) taking into account the yield, what is not appropriate for the case when a fragile material is used as the concrete, where the most appropriate would be to use a rupture criterion.

Due to symmetry in relation to the vertical axis, only the half of the physical model is used in the united optimization project of shape with regriding, Figure 4 (a). The Figure 4 (b) shows the geometric model used in the shape optimization with 156 lineal triangular elements and the geometric model used in the regriding is presented in the Figure 4 (c).

After 4 iterations, being considered the plane state of tension, the shape optimization is obtained using the shape optimization program and the criterion to stop prior established, Figure 5 (a). It is observed, in this case, that the volume of the traverse section changed from an initial volume of 48,5 m^3 to a final volume of 38,9 m^3 (reduction of 19,80 %), as well as a better agreement of the contour is observed in the area where a high concentration of tensions exists. It is noticed in the region of concentration of tensions a great distortion of the finite elements caused by the nodal reallocation in the balance of the distortion energy, what is a non favorable characteristic for the nodal reallocation method.

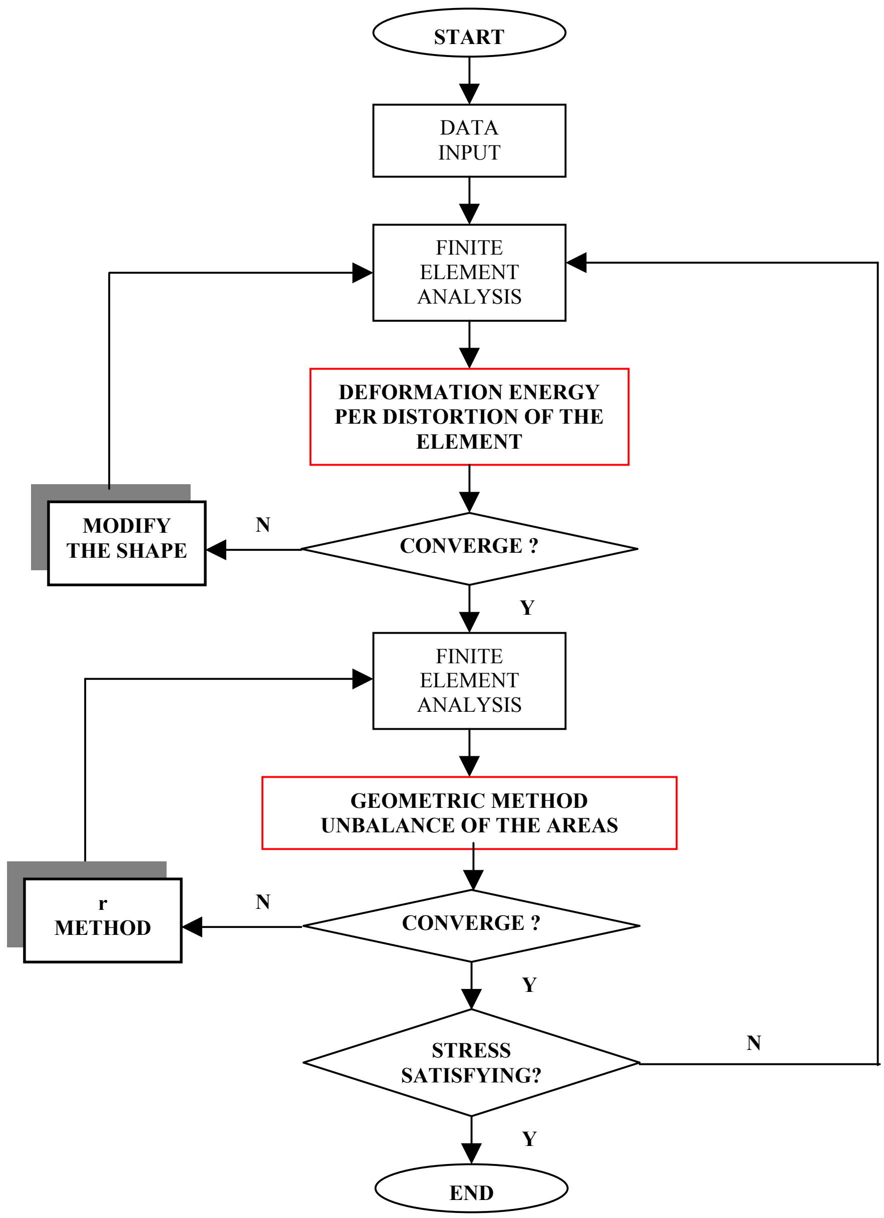


Figure 2 – Flow chart of joining of shape optimization algorithm with regriding

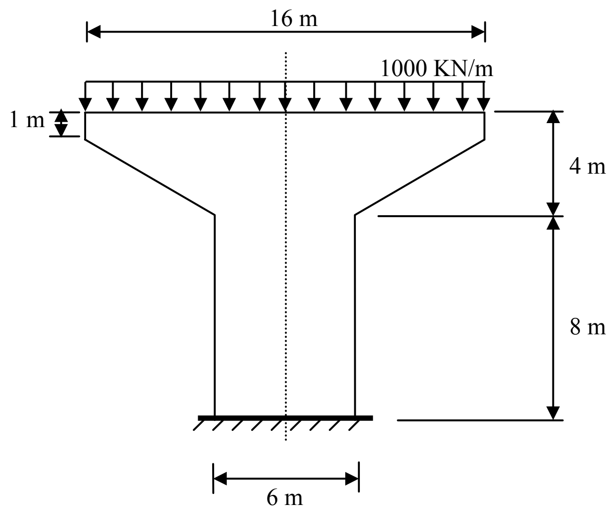
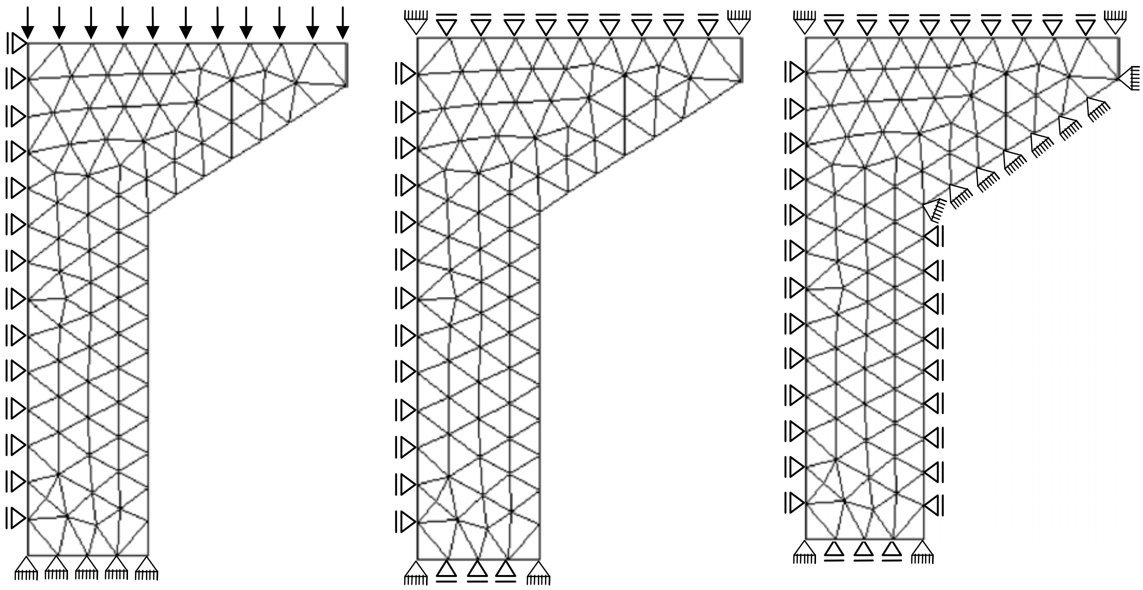


Figure 3 – Transverse section of the highway column.

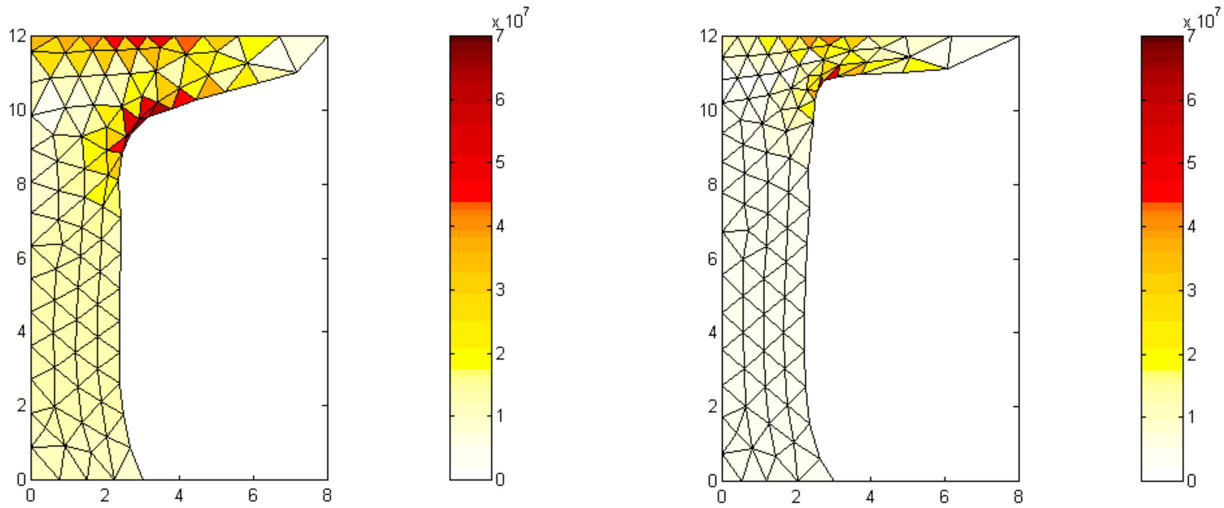


(a) (b) (c)

Figure 4 – Discretization of the column (a) Physical model, (b) Geometric model used in shape optimization, (c) Geometric model used in regriding

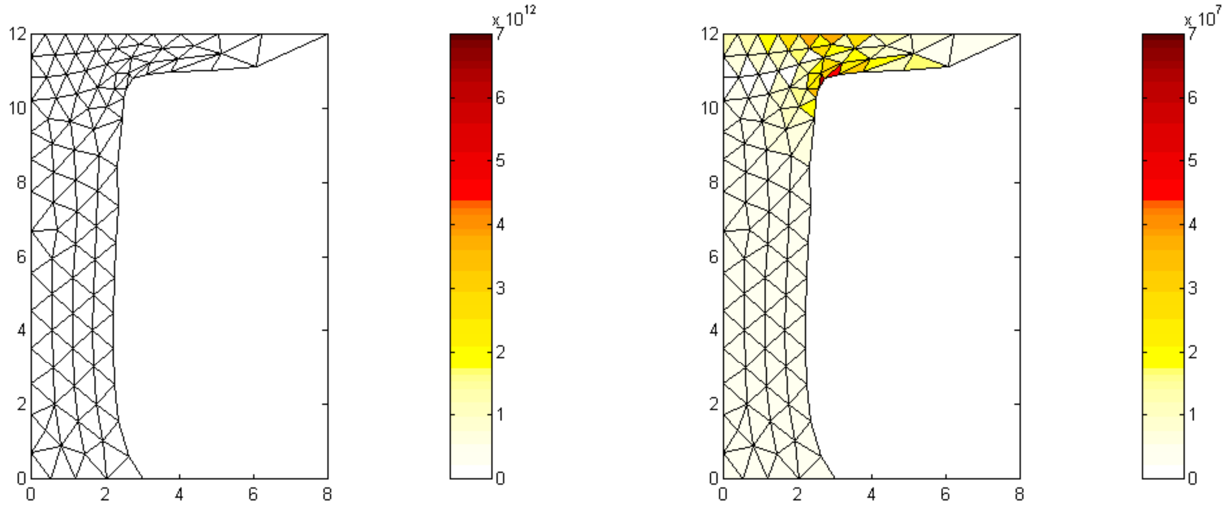
Being made the shape change with the collation of the regriding module in the numeric processing and considering for this analysis the number of iterations for each module: 1 iteration for shape, Figure 5 (b) and 5 iterations for the regriding, Figure 6 (a). The Figure 6(b) shows the equivalent tensions after the regriding. For this case the process is re-done 7 times until the stop criterion is reached.

It is observed that in the shape optimization process being inserted the regriding, the reduction of the volume of the transverse section is larger and the distortion of the finite elements, in the considered region, is smaller than in the case where only the shape optimization was used. After that was obtained the final discretization of qualities physics and geometric improved that is due the influence of the introduction of the regriding module. Being used the united optimization project of shape and regriding , the volume changed from an initial value of 48,5 m^3 to a final volume of 32,89 m^3 (reduction of 32,20%).



(a) (b)

Figure 5 - (a) Equivalent tensions after the shape optimization (4 iterations), (b) equivalent tensions after the optimization united project of shape/ regriding (1 iteration, 7th time).



(a) (b)

Figure 6 - (a) Regriding (5 iterations, 7th time), (b) Equivalent tensions after regriding (5 iterations, 7th time)

Application example 2 - Connecting rod

The optimization project of a connecting rod is shown in this example. Due to its couple symmetry, Figure 7 displays the geometry, dimensions, contour conditions and the loading of a fourth model. Loading varies lineally from zero to a maximum value equal to 500 N/mm. The objective of this problem is to obtain the optimal connecting rod contour so that achieving the smallest possible structure volume and the equivalent tension does not surpass 1200 N/mm^2. Other used data: module of elasticity 210000 N/mm^2, coefficient of Poisson 0,0 and constant thickness of the structure 1,0 mm.

Kimmich (1990) and Sienz, J. And E. Hinton (1997) also analyzed this problem to evaluate your works in the search of the optimal shape.

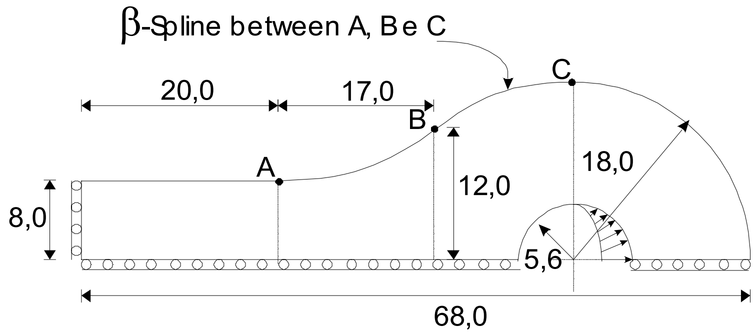
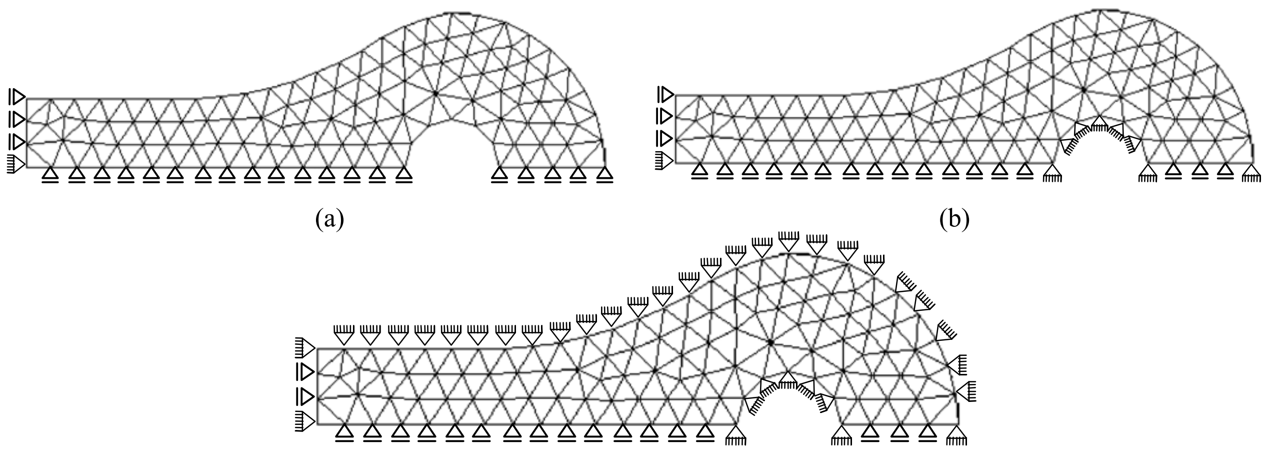


Figure 7 - Connecting rod (measures in mm).



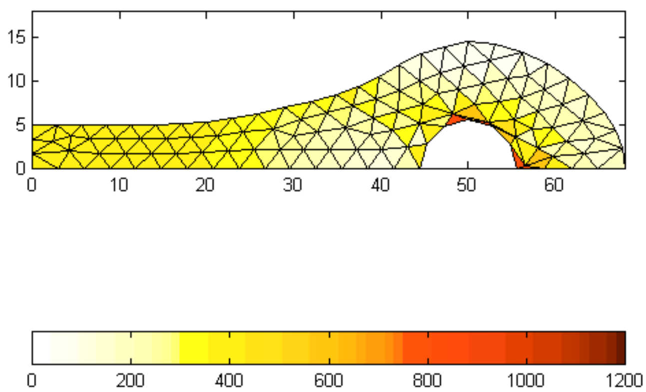
(c)

Figure 8 - Connecting rod discretization: (a) Physical model, (b) Geometric model used in shape optimization (c) Geometric model used in regriding.

The discretization for physical model, shape optimization and regriding, using 185 lineal triangular elements is presented in the Figure 8.

After 2 iterations, being considered the plane state of tension, the shape optimization is obtained only using the shape optimization program and the criterion to stop prior established, Figure 9 (a). It is observed, in this case, that the volume of the traverse section changed from an initial volume of 732,68 mm^3 to a final volume of 528,38 mm^3 (reduction of 27,90%). With the presence of a great distortion of the finite elements, in the hole area, caused by the use of the r method.

(a)



(b)

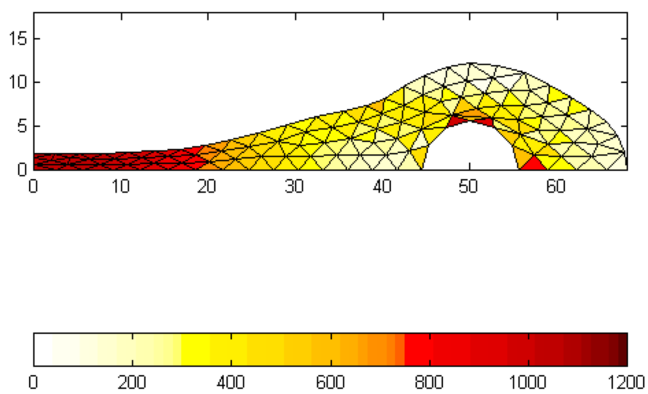


Figure 9 - (a) Equivalent tensions after the shape optimization (2 iterations), (b) equivalent tensions after the optimization united project of shape/regriding (1 iteration, 5th time).

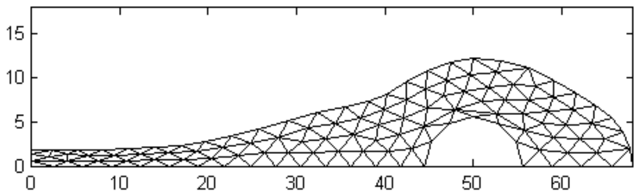
Being made the shape change with the collation of the regriding module in the numeric processing and being considered for this analysis the number of iterations for each module: 1 iteration for shape, Figure 9 (b) and 5 iterations for the regriding, Figure 10 (a). The Figure 10 (b) shows the equivalent tensions after the regriding. For this case the process is re-done 5 times until the stop criterion is reached.

It is observed that in the shape optimization process being inserted the regriding, the reduction of the volume of the transverse section is larger and the distortion of the finite elements is smaller than in the case where only the shape optimization was used. Being used the united optimization project of shape and regriding, the volume changed from an initial value of 732,68 mm^3 to a final volume of 398,05 mm^3 (reduction of 45,70%).

6. Conclusions

The shape optimization problem was formulated using an iterative consistent method based on a principle of good sense (heuristic method), not in a formulation purely mathematics, coupled to the finite elements method, in which MEF was main tool which carry us to reach the objectives previously established, that is the joining of the shape optimization with regriding of automatic and iterative form.

(a)



(b)

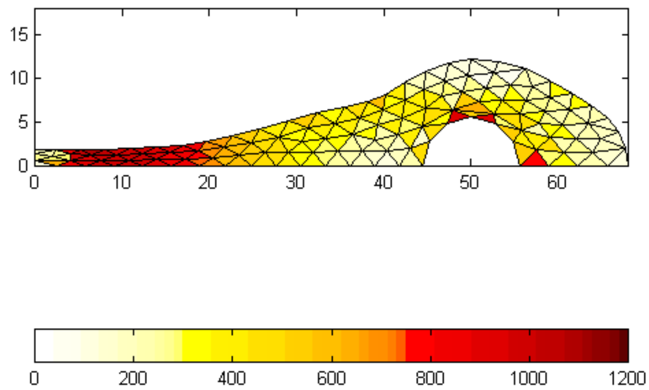


Figura 10 – (a) Regriding (5 iterations, 5th time), (b) Equivalent tensions after regriding (5 iterations, 5th time)

As main contribution a finite elements program was developed to optimize the shape of a structural element based on the homogenization of the distortion energy among the elements, adapted iteratively with a regriding, to each iteration or starting from a certain number of iterations fastened previously.

The program uses finite elements method, that is an efficient technique to obtain approximate solutions for problems where the exact solution is difficult of being obtained, the initial mesh discretization, the type and the elements number influence in the final results to the optimum shape.

It is ended that the shape optimization should be coupled to the regriding module so that the distortion of the elements is corrected after the otimization shape and the convergence of the system is reached.

The use of the program in structures where there was high stress concentration in the corners, the technique of inserting the shape optimization with regriding carried us to an optimun shape where was observed a decrease in the value of the equivalent stress and to a better distribution of stress along the movable contour when only compared with the use of the shape optimization program.

It can be observed although, while the procedure here presented it tends to soften the distribution of stress in the contour, the same can result in a material economy, because the final volume is smaller than the initial, what opens the perspective to use it in the optimal project of structural components.

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