3D CYCLIC SYMMETRY IN THE ANALYSIS OF A STRUCTURAL – ACOUSTIC SYSTEM

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Abstract *Finite element modal analysis of structural systems could lead to large system of equations due to large physical domains. Even with the constant increasing on computer velocity and memory storage capacity, it is of great interest to have procedures that can reduce the number of equations to be solved. To achieve this goal, component mode synthesis methods has been used. These consists in performing dynamic analysis of structures by its decomposition into substructures. When applying these methods to structures with cyclic symmetry, only the model of one sector is needed to obtain the eigenfrequencies and modes of the whole structure.These techniques has been applied in the eigenanalysis of submerged structures, like turbomachinery wheels. The fluid is considered incompressible and its influence on the structure dynamic behaviour is taken into account by including an added mass matrix on the left hand side of structure's discrete equations. A more general formulation is necessary to include the effect of fluid compressibility and evaluate both the displacement and pressure fields inside the elastic and fluid domains. In this work the finite element formulation for cyclic symmetric systems with fluid - structure interaction is reviewed. The effect of fluid compressibility is included. An analysis of a 3D structural – acoustic domain is performed using these methods and the results are compared with the ones of the whole system.*

Keywords. *Cyclic Symmetry, Fluid-structure interaction, Finite element method*

1. Introduction

Structures with cyclic symmetry are commonly used in many fields of engineering: aerospace, mechanical, naval, nuclear. The numerical eigenanalysis of this kind of structures is important in the design stage to avoid excessive displacements and stresses in resonance conditions. For industrial applications, the finite element method has been used for numerical eigenanalysis of complex shape structures. Despite of the increasing in core memory and processing velocity of modern computers, the need for more accurate models make procedures for model reduction very useful.

According to the wave propagation theory in periodic systems (D.L. Thomas, 1979), the dynamic behaviour of the whole system can be computed by analyzing just one sector. This technique can be combined with modal reduction to achieve an even greater reduction in the number of degrees of freedom on the eigenanalysis (Tran, 2001 and Balasubramanian et al. , 1992). Harmonic responses could be calculated also (Omprakash, V. 1989). In the case of hydraulic turbines, the sector is composed by solid and fluid domains. The fluid - structure interaction in this case is usually included by the added mass effect and only the structure displacement field could be obtained (Pavanello,1991; Richardet &Dal-Ferro, 1995).

In this paper we present the formulation of the already known method for computing frequencies of rotationally periodic systems for domains with fluid-structure interaction taking into account the compressibility of the fluid. A finite element C++ code built by the Computational Mechanics Department (MEFLAB++) was used to analyze a sector of a structure in contact with air and compute the frequencies of the whole system. To validate the results the same entire vibro-acoustic system was analyzed by the commercial FEM code ANSYS (ANSYS, 1994).

2. Theoretical formulation of fluid-structure interaction

The governig equations of a coupled fluid – structure system are derived using a Lagrangian formulation for the structure (displacement field) and an Eulerian one for the fluid (pressure field). Small harmonic motions around initial static equilibrium are assumed and the material is considered continuous, homogeneous, isotropic and linear elastic.

The problem can be discretized using the standard finite element method. Each one of the two domains has its own variables (vector displacements $\{u\}$ for the solid and pressures p for the fluid). Using the interpolation functions for the solid ($\lfloor N_s \rfloor$) and the fluid ($\lfloor N_f \rfloor$) one can write the displacements and pressures by:

$$\{u\} = [N_s]\{u_n\}$$
$$p = [N_f]\{p_n\} \quad (1)$$

where $\{u_n\}$ and $\{p_n\}$ are respectively the nodal and pressure displacements of the coupled system. After some classical finite element calculations, the solid kinetic energy $\boldsymbol{T_s}$, strain energy $\boldsymbol{U}_s$ and the pressure virtual work $\boldsymbol{W_s}$ of the solid domain can be expressed by :

$$\boldsymbol{T_s} = \tfrac{1}{2}\{\dot{u}_n\}^T[M]\{\dot{u}_n\}$$
$$\boldsymbol{U}_s = \tfrac{1}{2}\{\dot{u}_n\}^T[K]\{\dot{u}_n\} \quad (2)$$
$$\boldsymbol{W_s} = \delta\{u_n\}^T[L]\{p_n\}$$

where [*M*], [*K*] and [*L*] are mass, stifness and fluid – structure interface matrices respectively, and δ means virtual variations.

In the absence of volume forces, the discretized equations of motion for the structure can be derived by Lagrange's equations, which gives:

$$[M]\{\ddot{u}_n\} + [K]\{u_n\} = [L]\{p_n\} \quad (3)$$

A fluid domain at rest, considered inviscid, homogeneous, with no-rotational flow is governed by the Helmholtz equation. Assuming the conditions cited above and a fluid domain limited by rigid and flexible non absorbing walls, the Helmholtz equation can be discretized in a similar way and the kinetic energy ($\boldsymbol{T_f}$), potential energy ($\boldsymbol{U_f}$) and the virtual work of the interface forces ($\boldsymbol{W_f}$) could be write in the following form :

$$\boldsymbol{T_f} = \tfrac{1}{2}\{p_n\}^T[H]\{p_n\}$$
$$\boldsymbol{U_f} = \tfrac{1}{2c^2}\{\dot{p}_n\}^T[E]\{\dot{p}_n\} \quad (4)$$
$$\boldsymbol{W_f} = -\rho_f\delta\{p_n\}^T[L]^T\{\ddot{u}_n\}$$

where [*E*], [*H*], [*L*] are the compressibility, the volumetric and the fluid – structure interface matrices respectively and ρ_f is the fluid density.

Applying Lagrange's equations leads to:

$$[E]\{\ddot{p}_n\} + [H]\{p_n\} = -\rho_f[L]^T\{\ddot{u}_n\} \quad (5)$$

The discretized equation of the coupled system for free vibrations is (Zienkiewicz & Taylor, 1994)::

$$\begin{bmatrix} [M] & [0] \\ \rho_F[L]^T & [E] \end{bmatrix}\begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{p}\} \end{Bmatrix} + \begin{bmatrix} [K] & -[L] \\ [0] & [H] \end{bmatrix}\begin{Bmatrix} \{u\} \\ \{p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \quad (6)$$

3. Wave propagation in rotationally periodic systmes

A rotationally periodic coupled system is constituted by N identical jointed fluid-structure sectors. Acording to wave propagation theory in periodic media (Thomas D. L.,1979), the displacements $\{u\}_j$ and pressures $\{p\}_j$ of the different sectors *j* are related to the associated quantities of a reference sector by the following phase relations (Jacquet-Richardet, Dal-Ferro,1996):

$$\{\delta\}_j = \{u_n^c\}\cos(j-1)\beta_n + \{u_n^s\}sin(j-1)\beta_n \quad (7)$$
$$\{p\}_j = \{p_n^c\}\cos(j-1)\beta_n + \{p_n^s\}sin(j-1)\beta_n$$

where $\beta_n = 2\pi m/N$ is the phase angle between two adjacent sectors, *N* is the total number of sectors and *n*, the Fourier number, takes the discrete values *n=0,1,2,...D/2* with *D=N* if *N* is even and *D=(N-1)* if *N* is odd.

Computing the generalized vectors $\{u_n^c\}$, $\{u_n^s\}$, $\{p_n^c\}$ and $\{p_n^s\}$, associated with the reference sector, gives the displacement and pressure fields of the entire system.

Remembering that the N sectors are identical, the matrices which characterize the finite element model of each sector are also identical, provided they are expressed in a cylindrical reference frame linked to the symmetry axis of the system.

For a structure in free vibration we have:

$$\left(\begin{bmatrix} [K_{rr}] & [K_{ri}] & [K_{re}] \\ [K_{ir}] & [K_{ii}] & [K_{il}] \\ [K_{lr}] & [K_{li}] & [K_{ll}] \end{bmatrix} - \omega^2 \begin{bmatrix} [M_{rr}] & [M_{ri}] & [M_{re}] \\ [M_{ir}] & [M_{ii}] & [M_{il}] \\ [M_{lr}] & [M_{li}] & [M_{ll}] \end{bmatrix}\right)\begin{Bmatrix} \{u_r\} \\ \{u_i\} \\ \{u_l\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{Bmatrix} \tag{8}$$

The matrices are partitioned considering the position of the degrees of freedom into the sector (Jacquet-Richardet, G.,Ferraris, G.,Rieutord, P.,1996). The vectors $\{u_r\}$, $\{u_l\}$ and $\{u_i\}$ are respectively the displacement vector of the the right contour degrees of freedom, the displacement vector of the left contour degrees of freedom and the displacement vector of the interior degrees of freedom. The same procedures are applied to the fluid sector. See Fig. (1).

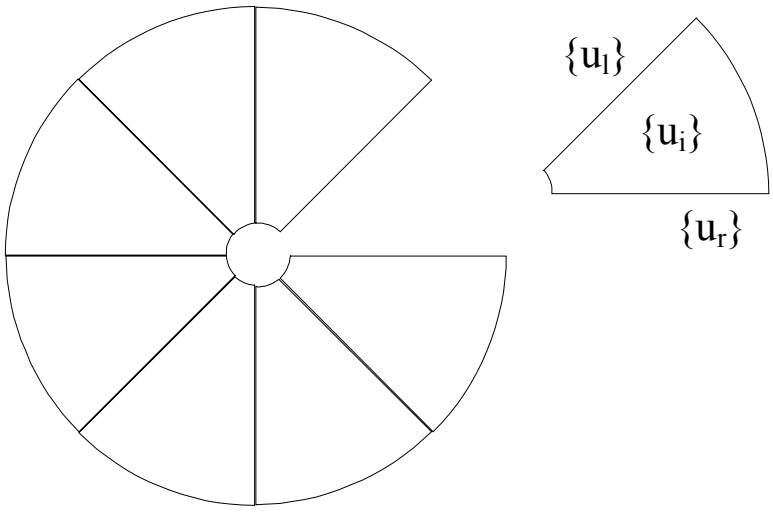

Figure 1 Cyclic symmetric structure

To consider a sector with a node on the symmetry axis some aditional procedures are necessary (Dal-Ferro, 1994).

Using the linear relations linking the right and left contours of the reference sector (equations (7)) one can eliminate one of the contour's degrees of freedom and obtain the matrtices $[M(\beta_n)]$, $[K(\beta_n)]$, $[L(\beta_n)]$, $[E(\beta_n)]$, $[H(\beta_n)]$, given by (Pavanello, 1991):

If $\beta_n \neq 0$ or $\beta_n \neq \pi$ (9)

$$[A(\beta_n)] = \begin{bmatrix} [A_{ll}]+[A_{rr}]+([A_{lr}]+[A_{rl}])\cos\beta_n & [A_{li}]+[A_{ri}]\cos\beta_n & [A_{lr}]+[A_{rl}]sin\beta_n & -[A_{ri}]sin\beta_n \\ [A_{il}]+[A_{ir}]\cos\beta_n & [A_{ii}] & [A_{ir}]sin\beta_n & [0] \\ [A_{rl}]+[A_{lr}]sin\beta_n & [A_{ri}]sin\beta_n & [A_{ll}]+[A_{rr}]+([A_{lr}]+[A_{rl}])\cos\beta_n & [A_{li}]+[A_{ri}]\cos\beta_n \\ -[A_{ir}]sin\beta_n & [0] & [A_{il}]+[A_{ir}]\cos\beta_n & [A_{ii}] \end{bmatrix}$$

If $\beta_n = 0$ or $\beta_n = \pi$

$$[A(\beta_n)]_{0\pi} = \begin{bmatrix} ([A_{lr}]-[A_{rl}])sin\beta_n & [A_{li}]sin\beta_n \\ -sin\beta_n[A_{il}] & [0] \end{bmatrix}$$

where:

$[A] = [M(\beta_n)], [K(\beta_n)], [L(\beta_n)], [E(\beta_n)], [H(\beta_n)]$ and $[A]_{0\pi} = [M(\beta_n)]_{0\pi}, [K(\beta_n)]_{0\pi}, [L(\beta_n)]_{0\pi}, [E(\beta_n)]_{0\pi}, [H(\beta_n)]_{0\pi}$.

Thus, the energies of the whole system are derived by summing the energies of each sector given by:

$$
\begin{aligned}
U_s &= N\left[\frac{1}{2}\{u\}_{c0\pi}^T\left[K(\beta_n)\right]_{0\pi}\{u\}_{c0\pi}\right]+\frac{N}{2}\left[\frac{1}{2}\begin{Bmatrix}\{u\}_{cn}\\ \{u\}_{sn}\end{Bmatrix}^T\left[K(\beta_n)\right]\begin{Bmatrix}\{u\}_{cn}\\ \{u\}_{sn}\end{Bmatrix}\right]\\
T_s &= N\left[\frac{1}{2}\{\dot{u}\}_{c0\pi}^T\left[M(\beta_n)\right]_{0\pi}\{\dot{u}\}_{c0\pi}\right]+\frac{N}{2}\left[\frac{1}{2}\begin{Bmatrix}\{\dot{u}\}_{cn}\\ \{\dot{u}\}_{sn}\end{Bmatrix}^T\left[M(\beta_n)\right]\begin{Bmatrix}\{\dot{u}\}_{cn}\\ \{\dot{u}\}_{sn}\end{Bmatrix}\right]\\
W_s &= N\left[\{\delta u\}_{c0\pi}^T\left[L(\beta_n)\right]_{0\pi}\{p\}_{c0\pi}\right]+\frac{N}{2}\left[\begin{Bmatrix}\{\delta u\}_{cn}\\ \{\delta u\}_{sn}\end{Bmatrix}^T\left[L(\beta_n)\right]\begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}\right]\\
U_f &= N\left[\frac{1}{2}\{\dot{p}\}_{c0\pi}^T\left[E(\beta_n)\right]_{0\pi}\{\dot{p}\}_{c0\pi}\right]+\frac{N}{2}\left[\frac{1}{2}\begin{Bmatrix}\{\dot{p}\}_{cn}\\ \{\dot{p}\}_{sn}\end{Bmatrix}^T\left[E(\beta_n)\right]\begin{Bmatrix}\{\dot{p}\}_{cn}\\ \{\dot{p}\}_{sn}\end{Bmatrix}\right]\\
T_f &= N\left[\frac{1}{2}\{p\}_{c0\pi}^T\left[H(\beta_n)\right]_{0\pi}\{p\}_{c0\pi}\right]+\frac{N}{2}\left[\frac{1}{2}\begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}^T\left[H(\beta_n)\right]\begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}\right]\\
W_f &= N\left[\rho_f\{\delta p\}_{c0\pi}^T\left[L(\beta_n)\right]_{0\pi}^T\{\ddot{u}\}_{c0\pi}\right]+\frac{N}{2}\left[\rho_f\begin{Bmatrix}\{\delta p\}_{cn}\\ \{\delta p\}_{sn}\end{Bmatrix}^T\left[L(\beta_n)\right]^T\begin{Bmatrix}\{\ddot{u}\}_{cn}\\ \{\ddot{u}\}_{sn}\end{Bmatrix}\right]
\end{aligned}
\tag{10}
$$

Applying Lagrange's equations to equations (10) gives the equations of motion of the coupled system:

$$
\begin{aligned}
&\left[M(\beta_n)\right]\begin{Bmatrix}\{\ddot{u}\}_{cn}\\ \{\ddot{u}\}_{sn}\end{Bmatrix}+\left[K(\beta_n)\right]\begin{Bmatrix}\{u\}_{cn}\\ \{u\}_{sn}\end{Bmatrix}=\left[L(\beta_n)\right]\begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}\\
&\left[\left[E(\beta_n)\right]\begin{Bmatrix}\{\ddot{p}\}_{cn}\\ \{\ddot{p}\}_{sn}\end{Bmatrix}+\left[H(\beta_n)\right]\begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}=-\rho_f\left[L(\beta_n)\right]^T\begin{Bmatrix}\{\ddot{u}\}_{cn}\\ \{\ddot{u}\}_{sn}\end{Bmatrix}\right]
\end{aligned}
\tag{11}
$$

which can be assembled in the same way of equation (6):

$$
\begin{bmatrix}\left[M(\beta_n)\right] & [0]\\ \rho_f\left[L(\beta_n)\right]^T & \left[E(\beta_n)\right]\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\{\ddot{u}\}_{cn}\\ \{\ddot{u}\}_{sn}\end{Bmatrix}\\ \begin{Bmatrix}\{\ddot{p}\}_{cn}\\ \{\ddot{p}\}_{sn}\end{Bmatrix}\end{Bmatrix}+\begin{bmatrix}\left[K(\beta_n)\right] & -\left[L(\beta_n)\right]\\ [0] & \left[H(\beta_n)\right]\end{bmatrix}\begin{Bmatrix}\begin{Bmatrix}\{u\}_{cn}\\ \{u\}_{sn}\end{Bmatrix}\\ \begin{Bmatrix}\{p\}_{cn}\\ \{p\}_{sn}\end{Bmatrix}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix}
\tag{12}
$$

Equations (11) and (12) are valid for $\beta_n \neq 0$ or $\beta_n \neq \pi$. Analogous equations can be written for $\beta_n = 0$ or $\beta_n = \pi$ using the matrices $\left[M(\beta_n)\right]_{0\pi}, \left[K(\beta_n)\right]_{0\pi}, \left[L(\beta_n)\right]_{0\pi}, \left[E(\beta_n)\right]_{0\pi}, \left[H(\beta_n)\right]_{0\pi}$ and the vectors $\{u\}_{c0\pi}, \{\dot{u}\}_{c0\pi}, \{\ddot{u}\}_{c0\pi}, \{p\}_{c0\pi}, \{\dot{p}\}_{c0\pi}, \{\ddot{p}\}_{c0\pi}$.

Solve the eigenvalue problem given by the equation (12) for each value of β_n gives the frequencies of the entire system. The eigenvectors of the sector could be used to obtain the mode shapes of the whole system by using the equation (7).

4. Numerical examples

The coupled system analyzed as case test is an annular structure clamped at the inner radius and free at the outer radius, topped by a cylinder of air surrounded by rigid walls at the inner and outer radius with free surface at the upper side. This coupled system is useful to evaluate the acoustic-structural coupling, despite of its simplicity. See Fig. (2).

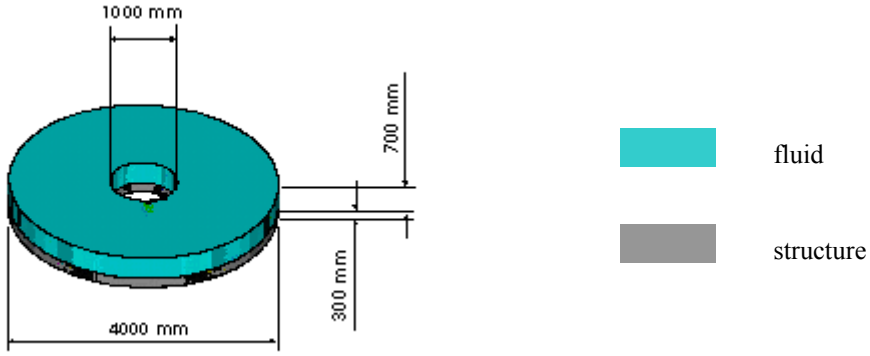

Figure 2 Coupled system analyzed

The materials properties used are:
Structure:
Aluminum: ρ= 2768 Kg/m^3 E= 0.75x10^{11} N/m^2 ν=0.3
Fluid:
Air: ρ= 1.225 Kg/m^3 c=340 m/s

Using the FEM code ANSYS we calculate the structural and acoustic uncoupled frequencies. Then we analyze the whole coupled system. For all cases only the first 13 modes are considered here. The structural frequencies varies from 30.724 Hz to 294.37 Hz and the acoustic ones varies from 121.68 Hz to 170.65 Hz. Analyzing these frequency ranges one can notice the possibility of structural-acoustic dynamic coupling. These frequencies are shown in Tab. (1) bellow. The double modes are marked with (2).

We can see that the coupling effect decreases the frequencies of all predominantly structural modes but the mode 5. The displacements for this mode occurs on radial and tangential directions, not in the Z direction (orthogonal to the plate plane). The coupling effect also increases the frequencies of all predominantly acoustic modes but the modes 11 and 12.

Table 1 Structure, fluid and coupled system frequencies

Mode number	Structure	Fluid	Coupled system	Mode type
1	30.724 (2)	121.68	30.708 (2)	Predominantly Structural
2	33.541	123.71 (2)	33.523	Predominantly Structural
3	37.479 (2)	128.36 (2)	37.461 (2)	Predominantly Structural
4	72.507 (2)	134.41 (2)	72.466 (2)	Predominantly Structural
5	99.167	135.97	99.167	Predominantly Structural
6	127.99 (2)	139.50 (2)	121.72	Predominantly Fluid
7	199.20 (2)	141.73 (2)	123.75 (2)	Predominantly Fluid
8	209.52	149.44 (2)	127.77 (2)	Predominantly Structural
9	216.40 (2)	150.27 (2)	128.41 (2)	Predominantly Fluid
10	219.15 (2)	159.93 (2)	134.48 (2)	Predominantly Fluid
11	241.71 (2)	162.55 (2)	135.97	Predominantly Fluid
12	285.33 (2)	169.97	139.49 (2)	Predominantly Fluid
13	294.37	170.65	141.95 (2)	Predominantly Fluid

Cyclic symmetric modes can be described as diameter modes (with diameter nodal lines), circle modes (with circle nodal lines) or a combination of both. In Figs. (3) and (4) we present some of these modes calculated by ANSYS for the entire coupled system. On the left column the mode frequency and type are shown. For example, the (0C,1D) is a cyclically symmetric mode with zero nodal circles and one nodal diameter. For a better visualization, structural and pressure fields were plotted on the structural and fluid domains respectively.

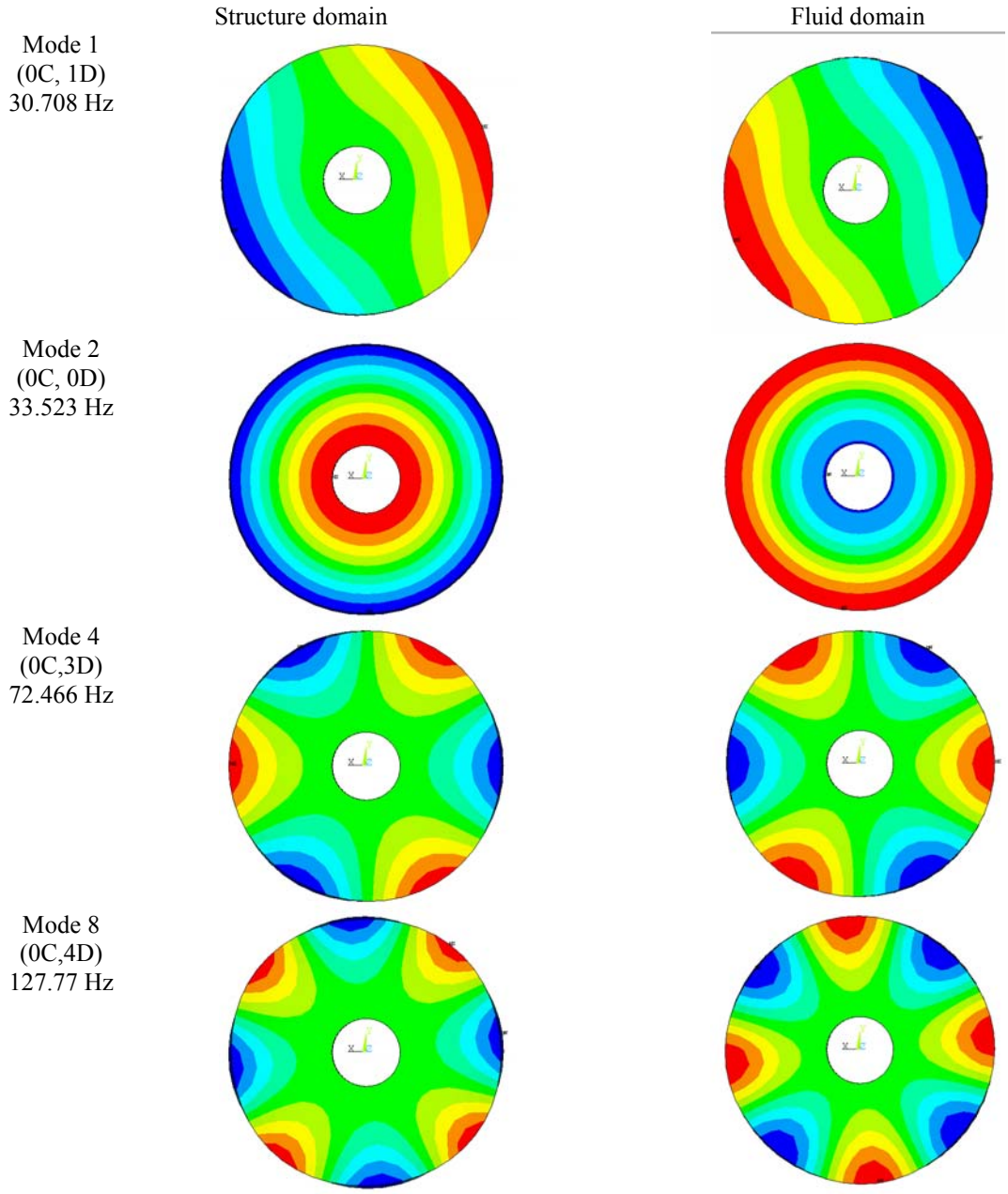

Figure 3 Coupled system modes (predominantly structural modes)

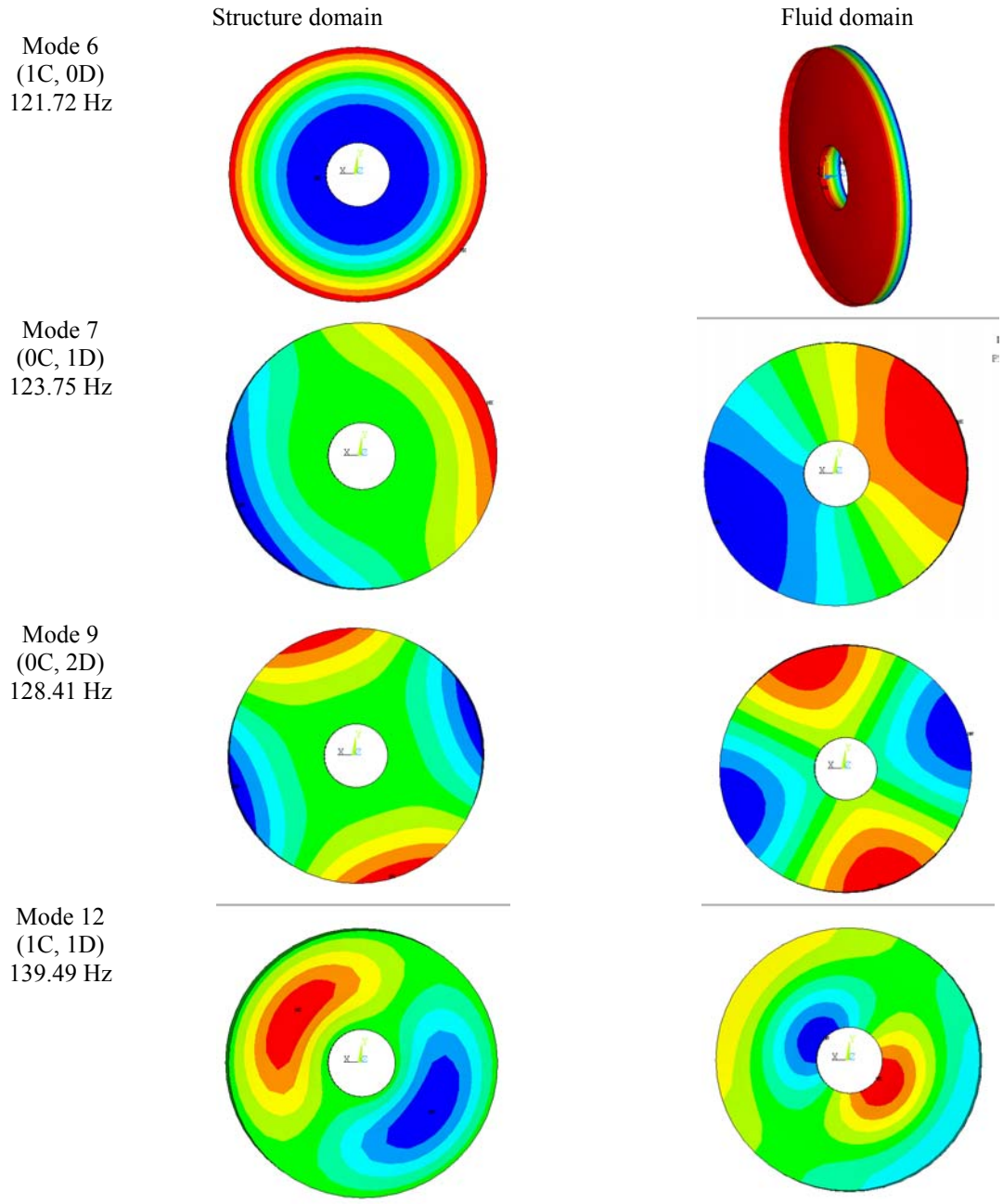

Figure 4 Coupled system modes (predominantly acoustic modes)

We can see in Fig (3) the predominantly structural modes and in Fig (4) the predominantly acoustic modes. On mode 6 the pressure field varies only on the z direction.

The mesh used by ANSYS and MEFLAB++ to build the model of the sector is shown at Fig. (5). Since the sector has an internal angle of 30^o the total number of sectors is 12. The mesh is composed by 96 elements for the structure and 224 elements for the fluid (795 degrees of freedom for the coupled system). The mesh used to analyze the whole system is composed by 1376 elements for the structure and 2464 elements for the fluid (9540 degrees of freedom for the coupled system). Eight nodes brick elements were used in both domains. The free surface condition was considered by imposing zero pressure at the upper side of the fluid cylinder.

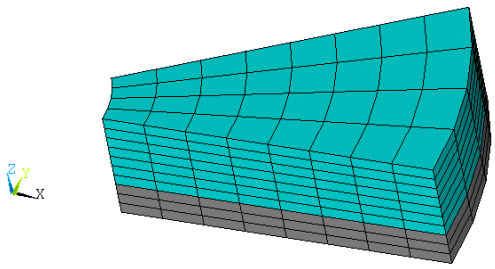

Figure 5 Sector meshed

These model was analyzed in MEFLAB++ using the cyclic symmetry relations described above (equations 7-12) and so the natural frequencies of the whole coupled system were calculated and compared with ANSYS's results. See Tab. (2).

Table 2 ANSYS and MEFLAB++ results.

Mode number	ANSYS	MEFLAB++
1	30.708 (2)	30.708 (2)
2	33.523	33.523
3	37.461 (2)	37.461 (2)
4	72.466 (2)	72.466 (2)
5	99.167	99.167
6	121.72	121.72
7	123.75 (2)	123.75 (2)
8	127.77 (2)	127.77 (2)
9	128.41 (2)	128.41 (2)
10	134.48 (2)	134.48 (2)
11	135.97	135.97
12	139.49 (2)	139.49 (2)
13	141.95 (2)	141.95 (2)

5. Conclusion

The method for computation of frequencies of repetitive fluid-structure systems described here has been used for the hydroelastic problem in which the fluid is considered incompressible. In this way, the fluid influence on the structure is taken into account by including an added mass into the structural equation and only this equation is solved.

In this work the complete coupled fluid - structure problem is solved (acoustic-structural). Therefore, not only the structural modes are available but the acoustic modes also.

Comparing computer time and storage required to solve a structural eigenvalue problem of a system exploiting or not cyclic symmetry, we should keep in mind that many eigenvalue subroutines take a central processing time proportional to the number of degrees of freedom cubed and a core storage time proportional to the number of degrees of freedom squared. Therefore, in the case of a whole structure composed by N sectors each one with J the number of degrees of freedom, the computer time will be proportional to N^3J^3. Using the cyclic symmetric approach, for an even value of N (which is the worst case) the CPU time will be proportional to $(2N+4)J^3$ (Thomas, 1979).

To store the matrices, the core storage required is proportional to N^2J^2 (the whole structure) and using cyclic symmetry the core storage needed is proportional to $2J^2$ (Thomas, 1979). Therefore, the main goal of this method is to save CPU time and computer core storage.

The numerical results of the example are in good agreement with ANSYS's results of the entire model. In a next work modal reduction will be applied to the sector analyzed.

6. Acknowledgement

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