NATURAL CONVECTION IN MULTILAYERED POROUS MEDIA

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Abstract. *This work presents numerical solutions for flow and heat transfer in square cavity with two layers of porous material. The microscopic flow and energy equations are integrated in a representative elementary volume in order to obtain a set of equations valid in both the clear flow region and in the porous matrix. A unique set of equations is discretized with the control volume method and solved with the SIMPLE algorithim. The results show that, for the invetigated range of parameters, the flow structure and the heat transfer could be different than what Darcy model predicts. Futher, experimental results show that the Nusselt number depends not only on the Ra_m and the aspect ratio, but also on the fluid Rayleigh number, Ra_f, Darcy number, Da, Prandtl number, Pr and the thermal conductivity ratio between the fluid and solid phases*
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Keywords. *Natural Convection, Turbulence Model, Porous Media, Heat Transfer*

1. Introduction

The study of natural convection in composite enclosures having layers of distinct materials, have several applications of great engineering relevance. Insulating systems in engineering equipment often use layers of porous material with the aim of controlling heat transfer rates across heated surfaces. In addition, advanced nuclear reactor systems consisting of a reactor core embedded in a large coolant pool are considered to be inherently safe since the heat generated in the event of an accident if passively removed the action of gravity. In many of these systems, materials and components can be modeled as a porous structure in an enclosure subjected to natural convection currents.

When treating turbulent flow in porous media, recent works in the literature propose a macroscopic treatment of the properties of interest, integrating these quantities in a representative elementary volume so that macroscopic equations for the flow arise (Anthohe and Lage (1997), Pedras and de Lemos (2001)). With respect to cavity flows in clear and in porous media, subjected to a temperature gradient across the layer, the literature is vast and a great number of solutions can be found, so much for clear cavities (de Vahl Davis (1983), de Lemos (2000)) as for porous enclosures (Charrier-Mojtabi (1997)). Also for this geometry, the work of Braga and de Lemos (2002a) presents results for laminar convection in square cavities heated on the sides. Later, Braga and de Lemos (2002b) extended their results to horizontal annuli. Turbulent flow in eccentric and concentric annuli was also investigated Braga and de Lemos (2002c). Also, a study on natural convection in cavities completely filled with porous material was presented in Braga and de Lemos (2002d). In this last work, the two geometries previously analyzed, namely square and annular region, were considered. More recently, Braga and de Lemos (2002e) presented results for laminar and turbulent flow in square cavity for clear and porous media. The modeling of the turbulent natural convection in porous enclosures is fully documented in de Lemos and Braga (2003). The turbulence model employed therein is the standard k-ε turbulence model with wall function.

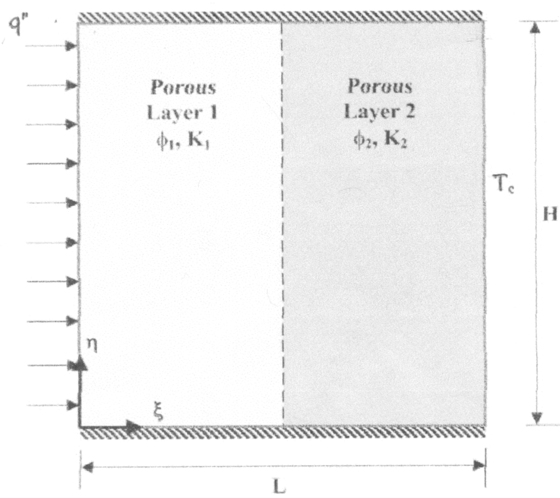


Figure 1 – Multilayered cavity under consideration.

In all the results mentioned above, the considered cavity was either totally clear or totally filled with a porous substrate. In this work, the considered problem is shown schematically in Fig. (1). Here, the situation considered treats two-dimensional flow of an incompressible fluid in a square cavity filled with two layers of porous material of height H and width L. Further, for the cavity of the illustration one considers constant heat flux on the left face, q'', and temperature constant on the right, T_C. The other two walls are maintained insulated.

In Magro and de Lemos (2002a) laminar flow and heat transfer in the cavity of Figure 1 were investigated. In that work, the effect of the number of Rayleigh and the treatment of the interface, located at $x=L/2$, were the objective of the analysis. There, the interface treatment used was the one proposed in Ochoa-Tapia & Whitaker (1995). Later (Magro and de Lemos (2002b)), the previous investigation was complemented, taken then into account the effects of porosity and permeability of the porous region. In both works, the analysis was made for flow in laminar regime. Recently, Magro and de Lemos (2002b) and de Lemos and Magro (2003) considered turbulent flow in a cavity having 50% of its volume occupied by a porous material.

In this work laminar natural convection in an enclosure filled with two layers of porous media is investigated numerically and compared with the work of Merrikh and Mohamad (2002).

2. Mathematical Model

The mathematical model here employed has its origin in the works of Pedras and de Lemos (2001) for the hydrodynamic field and e Rocamora and de Lemos (2000) for the thermal field. The consideration of buoyancy forces was taken in the works of Braga and de Lemos (2002a-e) and the implementation of the jump condition at the interface was considered in Silva and de Lemos (2002) based on the theory proposed in Ochoa-Tapia & Whitaker (1995). Therefore, these equations will be here just reproduced and details about their derivations can be obtained in the mentioned works. These equations are:

2.1 Macroscopic continuity equation:

$$\nabla \mathbf{u}_D = 0 \tag{1}$$

where, $\mathbf{u}_D$ is the average surface velocity ('seepage' or Darcy velocity). The equation (1) represents the macroscopic continuity equation for an incompressible fluid.

2.2 Macroscopic transport equations:

$$\rho\left[\nabla\left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)\right] = -\nabla\left(\phi\langle p\rangle^i\right) + \mu\nabla^2\mathbf{u}_D - \rho\beta_\phi\mathbf{g}\phi\left(\langle T\rangle^i - T_{ref}\right) - \left[\frac{\mu\phi}{K}\mathbf{u}_D + \frac{c_F\phi\rho|\mathbf{u}_D|\mathbf{u}_D}{\sqrt{K}}\right] \tag{2}$$

where the last two terms in equation (2), represent the Darcy-Forchheimer contribution. The symbol K is the porous medium permeability, c_F is the form drag coefficient (Forchheimer coefficient), $\langle p\rangle^i$ is the intrinsic average pressure of the fluid, ρ is the fluid density, μ represents the fluid viscosity and ϕ is the porosity of the porous medium.

In a similar way, applying volume-average operator to the microscopic energy equation, for the fluid and for the porous phases, two equations appear. Assuming then the hypothesis of **Local Thermal Equilibrium**, which considers $\langle T_f\rangle^i = \langle T_s\rangle^i = \langle T\rangle^i$, and adding up the two obtained equations, one has (see Braga and de Lemos (2002a-e) and de Lemos and Braga (2003) for details),

$$\left(\rho c_p\right)_f \nabla\left(\mathbf{u}_D\langle T\rangle^i\right) = \nabla\left\{\left[k_f\phi + k_s(1-\phi)\right]\nabla\langle T\rangle^i\right\} \tag{3}$$

where k_f and k_s are the fluid and solid conductivities, respectively.

At the interface, the conditions of continuity of velocity and pressure in addition to their respective diffusive fluxes, are given by,

$$\mathbf{u}_D\big|_{\phi<1} = \mathbf{u}_D\big|_{\phi=1} \tag{4}$$

$$\langle p\rangle^i\big|_{\phi<1} = \langle p\rangle^i\big|_{\phi=1} \tag{5}$$

The jump condition at the interface is given by,

$$\mu_{eff}\frac{\partial \mathbf{u}_{D_p}}{\partial y}\bigg|_{0<\phi<1} - \mu\frac{\partial \mathbf{u}_{D_p}}{\partial y}\bigg|_{\phi=1} = \mu\frac{\beta}{\sqrt{K}}\mathbf{u}_{D_i}\Big|_{\text{interface}} \tag{6}$$

where $\mu_{eff} = \mu/\phi$, the non-slip condition for velocity is applied to all of the four walls.

3. Numerical Method

The numerical method used in the resolution of the equations above was the Finite Volumes technique and the *SIMPLE* algorithm of Patankar (1980). The interface is positioned to coincide with the border between two control volumes, generating, in such a way, only volumes of the same type of material. The flow and energy equations are resolved then in the two set of porous layers, being respected the interface conditions mentioned earlier.

4. Results and Discussion

Laminar natural convection in an enclosure filled with two layers of porous media is investigated numerically and compared with the work of Merrikh and Mohamad (2002).

The problem considered is showed schematically in Figure 1. Constant heat flux is imposed on the left vertical wall and right wall is assumed to be at a low temperature the other two walls are assumed to be adiabatic.

Calculation are performed for a modified Rayleigh number defined as,

$$Ra_m = \underbrace{\frac{g\beta q H^4}{\alpha_{eff}\nu_f k_{eff}}}_{Ra_f}\cdot\underbrace{\frac{K_2}{H^2}}_{Da}, \text{ with } k_{eff} = \phi k_f + (1-\phi)k_s,\ \alpha_{eff} = \frac{k_{eff}}{(\rho c_p)_f} \text{ and } K_r = K_1/K_2 . \tag{7}$$

The results show that, for the investigated range of parameters, the flow structure and the heat transfer could be different than what Darcy model predicts. Further, experimental results show that the Nusselt number depends not only on the Ra_m and the aspect ratio, but also on the fluid Rayleigh number, Ra_f, Darcy number, Da, Prandtl number, Pr and the thermal conductivity ratio between the fluid and solid phases.

The Prandtl number is assumed to be a unity. For a fixed Ra_m , different Ra_f (or different Da) result different flow structures and heat transfer for non-Darcy regime.

Figure (2) shows the streamlines and isotherms for Ra_m=1000 and K_r=0.01. Figure. (2) clearly shows that increasing the Ra_f decreases the flow circulation at layer 1. Since the Ra_m is fixed, a lower Ra_f is associated with a more permeable media (i.e. higher Darcy number). Thus, the strength of the flow in the layer 1 increases by decreasing Ra_f. The stronger flow at the layer 1 results flow stratification and parallel isotherms in such region.

For Ra_f=10^6, isotherms are located down to mid-height of the enclosure as a result of high penetration from the left layer. But for Darcy flow, almost no temperature gradient exists in layer 1 due to high mixing mechanism in the right layer, Fig (2)

Figures (3) and (4) show the simulated streamlines and isotherms, respectively, for Ra_m=1000 and K_r=0.01. The flow patterns of Fig. (3) and (4) are slightly different of those from the Fig. (2). This is probably due to the different parameters adopted by the authors, since the work of Merrikh and Mohamad (2002) is unclear in some aspects. However, the numerical code reproduces the basic features of the flow, namely, the decrease of flow circulation at layer 1 due to the increase of Ra_f.

Table 1 – Average Nusselt numbers for a vertical square cavity with two layers of porous medium for Ra_m=1000, K_1/K_2 =0.01 for Ra_f ranging from 10^6 to 10^8.

Ra_f	ϕ_1	ϕ_2	K_1	K_2	$D_{p1,2}$	β_{calc}	β_{corr}	k_s/k_f	$\overline{Nu}$	Merrikh and Mohamad (2002)
10^6	0.908	0.989	10^{-5}	10^{-3}	4×10^{-3}	1.21×10^{-4}	1.21×10^{-3}	10	3.65	4.93
10^7	0.773	0.968	10^{-6}	10^{-4}	4×10^{-3}	1.69×10^{-3}	1.69×10^{-2}	10	5.14	6.55
10^8	0.559	0.908	10^{-7}	10^{-5}	4×10^{-3}	3.00×10^{-2}	3.00×10^{-1}	10	7.69	7.28
Darcy	-	-	10^{-7}	10^{-5}	-	-	-	-	-	8.05

Table (1) shows the average Nusselt number for a vertical square cavity with two layers of porous medium for Ra_m=1000, K_1/K_2 =0.01 for Ra_f ranging from 10^6 to 10^8. Due to the absence of better information, the authors fixed the particle diameter and the permeability to obtain the porosity for each porous layer and Ra_f. After that, the parameter β_{calc} was calculated for each Ra_f .and then corrected in order to yield values of the Nusselt number close to those obtained by the work of Merrikh and Mohamad (2002). It is clearly seen from the Table (1) that the enhancement of the Nusselt number due to the increase of the Ra_f was captured.

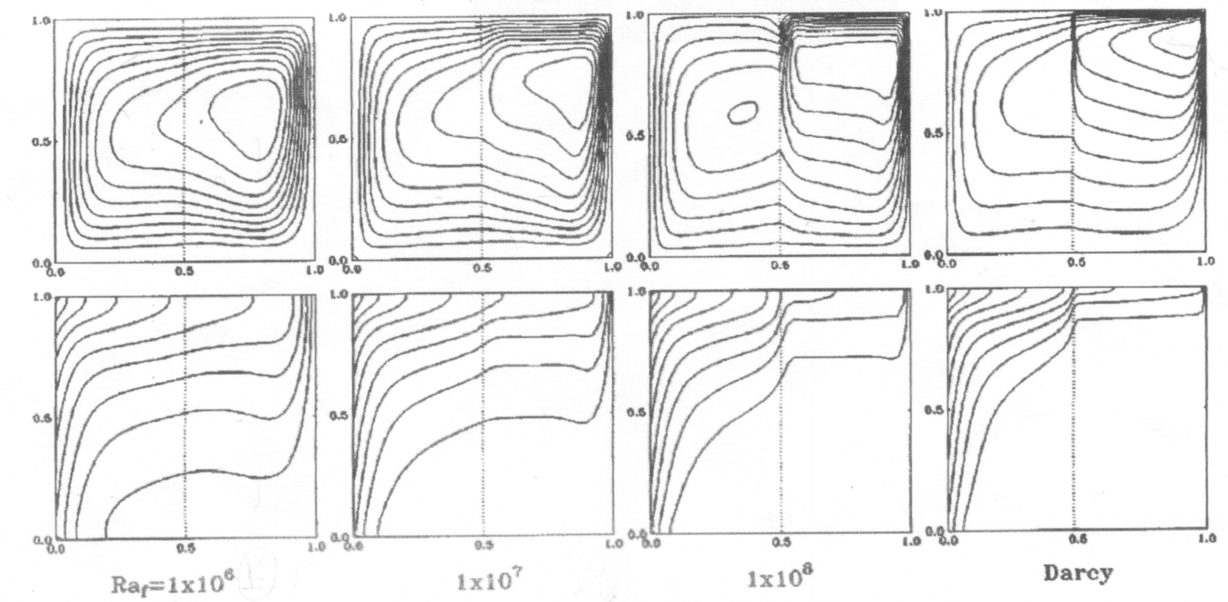


Figure 2 - Laminar streamlines (top) and isotherms (bottom) for Ra_m=1000, K_1/K_2 =0.01, from Merrikh and Mohamad (2002).

According to Merrikh and Mohamad (2002), for a fixed Ra_m, higher Nusselt numbers are obtained for higher fluid Rayleigh numbers compared to lower fluid Rayleigh numbers and this shows the role of the fluid Rayleigh number in such phenomena. The Darcy model can be a limiting case for such a problem, since at very low permeabilities, the viscous terms are negligible compared to the Darcy term in the momentum equations. Brinkman term includes an additional diffusion into the transport equations and results in a decrease in heat transfer predictions of the general model when compared to the Darcy model.

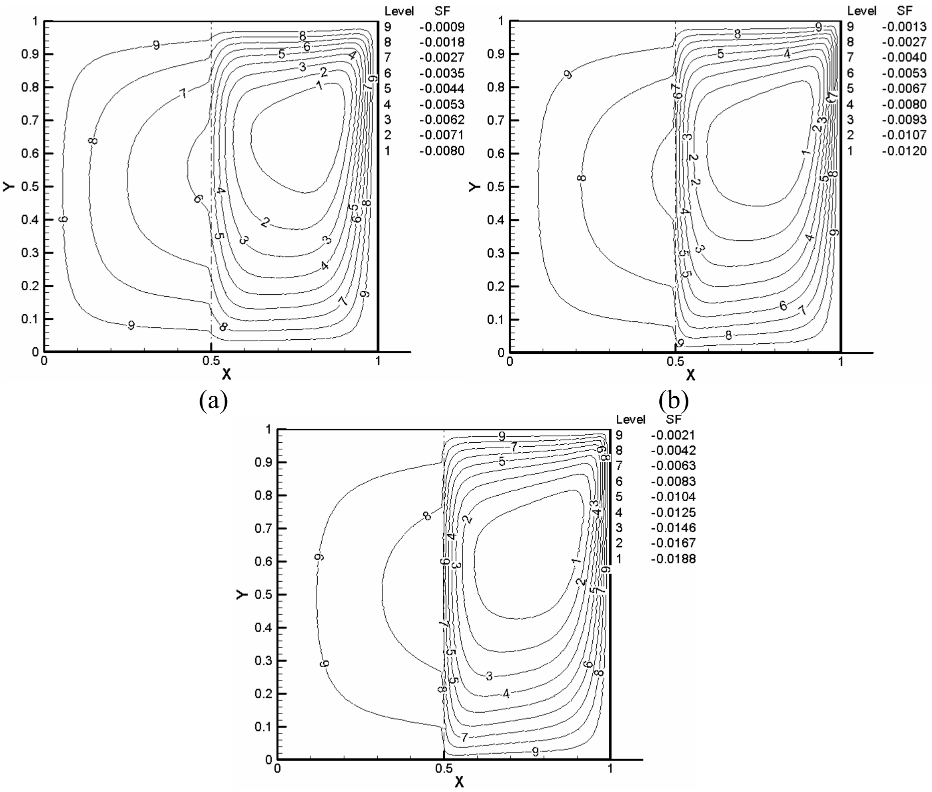
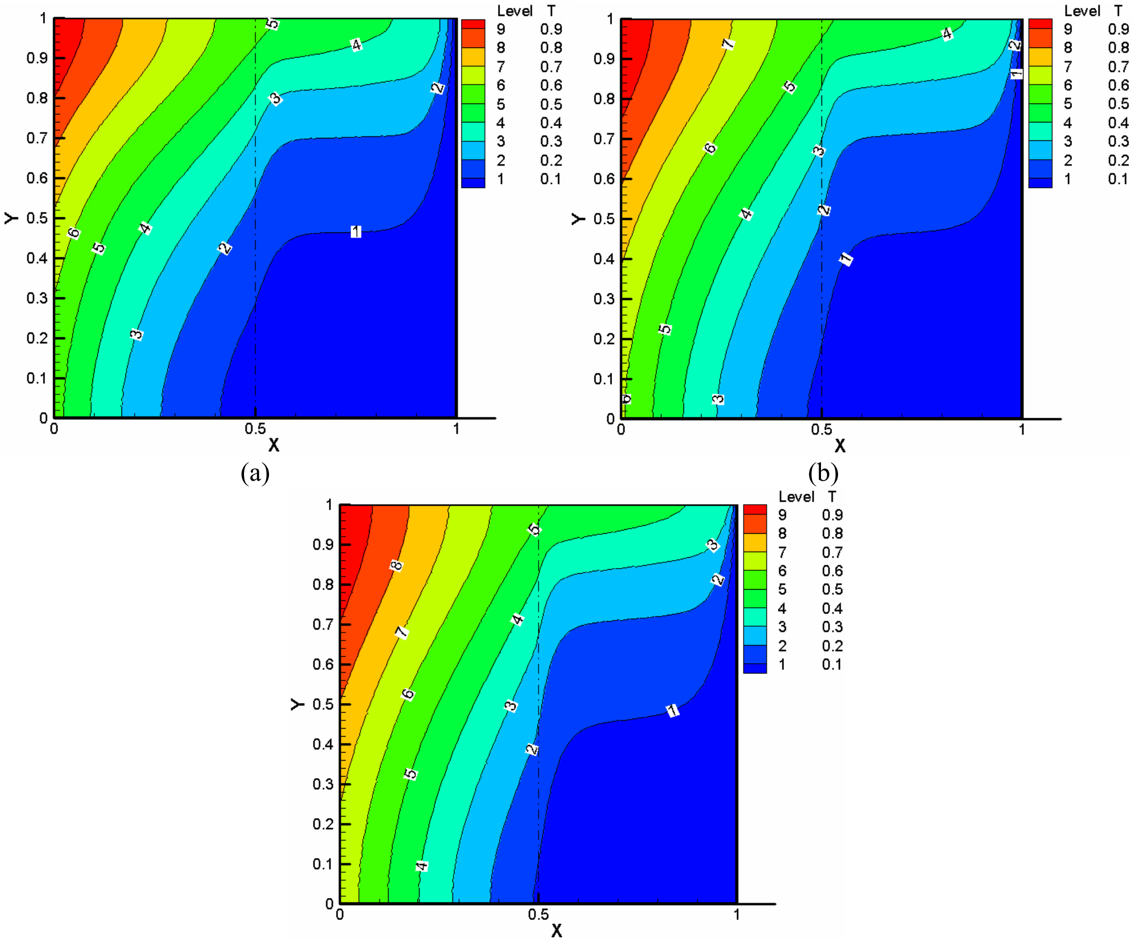


Figure 3 - Laminar streamlines for Ra_m=1000, K_1/K_2 =0.01; (a) Ra_f=10^6, (b) Ra_f=10^7, (c) Ra_f=10^8.



(a) (b) (c)

Figure 4 - Laminar isotherms for Ra_m=1000, K_1/K_2 =0.01; (a) Ra_f=10^6, (b) Ra_f=10^7, (c) Ra_f=10^8.

5. Concluding Remarks

In this work, numerical results were presented for laminar flows in hybrid domain with heat transfer, which involved an interface between the two porous layers. The used numerical method made possible the simultaneous treatment of the two porous layers with a single calculation domain, naturally considering the interface conditions between the two media. The fluid Raleigh number was varied and the change in global heat transfer across the cavity was obtained. The results show that, for the investigated range of parameters, the flow structure and the heat transfer could be different than what Darcy model predicts. Further, experimental results show that the Nusselt number depends not only on the Ra_m and the aspect ratio, but also on the fluid Rayleigh number, Ra_f, Darcy number, Da, Prandtl number, Pr and the thermal conductivity ratio between the fluid and solid phases. Ultimately, the authors would to emphasize that other runs will made to yields more accurate results.

6. Acknowledgments

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7. References

Antohe, B.V.; Lage, J.L., 1997, “A general two-equation macroscopic turbulence model for incompressible flow in porous media”, *Int. J. Heat Mass Transfer*, vol. 40, pp. 3013-3024.

Braga, E.J., de Lemos, M.J.S., 2002a, FREE CONVECTION IN SQUARE AND RECTANGULAR CAVITIES HEATED FROM BELOW OR ON THE LEFT, *Proc. CONEM2002, 3rd Congresso Nacional de Engenharia Mecânica*, João Pessoa, PB, Brazil, August 12-16.

Braga, E.J., de Lemos, M.J.S., 2002b, LAMINAR NATURAL CONVECTION IN CONCENTRIC AND ECCENTRIC ANNULI, *Proceedings of ENCIT2002, 9th Brazilian Congress of Thermal Engineering and Sciences* (on CD-ROM), Caxambu, MG, Brazil, October 13-17.

Braga, E.J., de Lemos, M.J.S., 2002c, NATURAL CONVECTION IN TURBULENT REGIME IN CONCENTRIC AND ECCENTRIC HORIZONTAL ANNULAR REGIONS, **Paper AIAA-2002-3316**, *Proc. of 8th AIAA/ASME, Joint Thermophysics and Heat Transfer Conference*, St Louis, Missouri, U.S.A, June 23-27.

Braga, E.J., de Lemos, M.J.S., 2002d, NATURAL CONVECTION IN CAVITIES COMPLETELY FILLED WITH POROUS MATERIAL, *Proceedings of APM2002, 1st International Conference on Applications of Porous Media*, **Paper APM-164**, vol. 1, pp. 551-560, Jerba, Tunisia, June 2-8.

Braga, E.J., de Lemos, M.J.S., 2002e, Turbulent Natural Convection in Enclosures Completely Filled With Porous Material, **Paper IMECE2002-34403**, *2002 ASME International Mechanical Engineering Congress*, New Orleans, LA, USA, November 17-22, 2002.

Charrier-Mojtabi, M.C., 1997, NUMERICAL SIMULATION OF TWO –AND –THREEDIMENSIONAL FREE CONVECTION FLOWS IN A HORIZONTAL POROUS ANNULUS USING A PRESSURE AND TEMPERATURA FORMULATION, Int. J. Heat Mass Transfer, vol, 40(7), pp. 1521-1533.

de Lemos, M.J.S., 2000, Flow And Heat Transfer In Rectangular Enclosures Using A New Block-Implicit Numerical Method, *Numerical Heat Transfer Part B – Fundamentals*, vol. 37(4), pp. 489–508.

de Lemos, M.J.S., Braga, E.J., 2003, Modeling of Turbulent Natural Convection in Saturated Rigid Porous Media, Int. Comm. Heat and Mass Transfer, (accepted, in press).

de Lemos, M.J.S., Magro, V.T., 2003, Turbulent Free Convection in a Composite Enclosure, accepted for presentation at 2003 ASME Heat Transfer Conference, Las Vegas, NV, USA, July 21-23.

de Vahl Davis, G., 1983, Natural convection in a square cavity: A benchmark numerical solution, *Int. J. Num. Methods in Fluids*, vol 3, pp. 249-264.

Magro, V.T., de Lemos, M.J.S., 2002a, "Convecção Natural em Regime Laminar em Cavidade Contendo Material Poroso" (In Portuguese), Proc. CONEM 2002 - II Congresso Nacional de Engenharia Mecânica, João Pessoa, PA, 23-28 Agosto.

Magro, V.T., de Lemos, M.J.S., 2002b, "Efeito da Permeabilidade e Porosidade na Convecção Natural em Cavidade contendo Material Poroso" (In Portuguese), Proc ENCIT 2002 - Congresso Brasileiro de Engenharia e Ciências Térmicas, Caxambu, MG, Brazil, 15-18 October.

Magro, V.T., de Lemos, M.J.S., 2002c, "Convecção Natural em Regime Turbulento em Cavidade Contendo Material Poroso", ETT 2002 - Escola de Transição e Turbulência, Florianópolis, SC, 23 a 27 de Setembro.

Merrikh, A.A., Mohamad, A.A., 2002, Non Darcy Effects in Buoyancy Driven Flows in an Enclosure Filled with Vertically Layered Porous Media, Intern. J. Heat and Mass Transfer, Vol 45, pp. 4305-4313.

Ochoa-Tapia, J.A.; Whitaker, S., 1995, "Momentum transfer at the boundary between a porous medium and a homogeneous fluid-I. Theoretical development", Int. J. Heat Mass Transfer, vol. 38, pp. 2635-2646.

Patankar, S.V., 1980, NUMERICAL HEAT TRANSFER AND FLUID FLOW, Mc-Graw Hill.

Pedras. M.H.J., de Lemos, M.J.S., 2001, Macroscopic Turbulence Modeling For Incompressible Flow Through Undeformable Porous Media, Int. J. Heat Transfer, vol. 44(6), pp. 1081-1093.

Rocamora, F.D.J., de Lemos, M.J.S., 2000, Analysis Of Convective Heat Transfer For Turbulent Flow In Satured Porous Media, Int. Comm. Heat Mass Transfer, vol. 27(6), pp. 825-834

Silva, R.A., de Lemos, M.J.S., 2002, "Numerical Treatment of the Stress Jump Interface Condition for Laminar Flow in a Channel Containing a Porous Layer", *Numerical Heat Transfer – Part A*, vol. 43(6), pp. 603-617.