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VARIABLE STRUCTURE CONTROL WITH MODEL FOLLOWING APPLIED TO A QUARTER CAR MODEL SUBJECTED TO RANDOM EXCITATION

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Abstract. The present work considers the variable structure control with model following for the design of an active suspension system. A quarter car model subjected to stationary random excitations is utilized to assess the performance of the proposed active suspension with respect to the bounce mode. A reference model is utilized to specify the desired performance of the active system. An ideal single degree of freedom (SDOF) Skyhook Damping System was chosen as the reference model and the controller was designed based on a SDOF model related to the sprung mass. The proposed active suspension system does not require an estimation of the road disturbance and, despite the presence of parameters uncertainties, improved performances with respect to ride quality, stroke and tyre deflection was obtained when compared with the passive suspension. Numerical simulations considering different levels of road disturbances and vehicle velocities are considered to asses the performance of the proposed controller.

Keywords: Vibration isolation, active suspension, variable structure control, quarter-car model and model-following .

1. Introduction

Conventional vehicle suspension systems comprises passive elements such as springs and dampers. However, conflicting trade-offs between different performance measures, such as ride quality, which is related with the vertical acceleration of the sprung mass, and handling, which is related with the contact forces between the tyres and the road surface and is assumed to be linearly dependent on the tyre deflection, are extremely difficult to achieve with passive suspensions (Sharp and Hassan, 1986). These conflicting trade-offs are specially difficult when one considers the very different dynamic modes that influence the ride quality and handling, namely, the bounce, pitch and roll modes. The use of active suspensions (Hrovat, 1997; Elbeheiry et al., 1995; Goodall and Kortum, 1983) and semi-active suspensions (Tseng and Hedrick, 1994; and Karnopp, 1985) to overcome these conflicts between ride and handling is worthwhile.

The majority of active suspension systems, presented in the literature, are derived from optimal control techniques (Hrovat, 1997; Elbeheiry and Karnopp, 1996; Narayanan and Senthil, 1998) and most of them aim at improving the ride quality. Hence, in the performance index to be minimized, the handling criterion is weighted much less than the ride quality criterion and, as a consequence, the active suspension may yield a better ride quality at the expense of a greater tyre deflection relative to the passive suspension (Narayanan, 1998). The road disturbance are often required to be estimated in the optimal control techniques.

In the present work a variable structure control with model following (Edwards and Spurgeon, 1998) is proposed for the design of the active suspension system. The variable structure approach was chosen because of its robustness properties and for its simplicity in incorporating a reference model. The robustness of the variable structure control is required for dealing with the parameters uncertainties commonly present in the vehicle and a reference model is used to specify the desired performance of the active system (Yokoyama et al., 2001). An ideal SDOF *Skyhook Damping System* was chosen as the reference model and the controller was designed based on a SDOF model related to the sprung mass. All the state variables are considered as measured in the controller design, but the road disturbance is not required to be estimated. An active suspension using the concept of sliding mode control was presented in Yoshimura et al. (2001), however, the approach used differs from the present one on attempt to force all the state variables to be in sliding mode and, further, an estimation of the road disturbance is also required.

In order to assess the performance of the proposed active suspension, a quarter car model subjected to stationary random excitation was considered. Simulations were carried out considering different levels of road disturbances and vehicle velocities.

2. Mathematical modelling

2.1. System model

The dynamics of a vehicle is very complex, it encompasses some very different modes. The principal dynamic modes which influence the ride comfort and the handling are the bounce, pitch and roll modes. The present work considers a quarter car model with parameters uncertainties to assess the performance of the proposed active suspension with respect to the bounce mode (Hrovat, 1997). A variable structure control with model following is considered in the design of the present active suspension.

The quarter car model consists of two masses, m_1, representing the quarter car mass and m_2, representing the masses of the wheel and the effective mass of the suspension system. The mass m_1 is isolated from m_2 by passive elements comprised by a spring k_1 and by a damper d_1, along with an active one, represented by the control force u, in parallel with the formers. The mass m_2 is connected with the ground by the tyre spring k_2 and damping d_2. The road disturbance is described by the elevation of the road z and its time derivative. The quarter car model is depicted in Fig. 1.

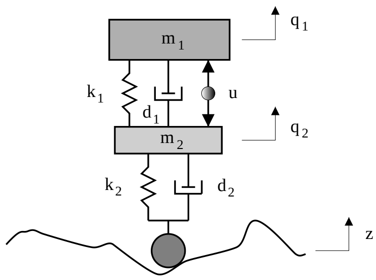

Figure 1: Quarter car model.

The dynamic behaviour of the quarter car is governed by the equations

$$\begin{cases} \bar{m}_1\ddot{q}_1 + \bar{d}_1(\dot{q}_1 - \dot{q}_2) + \bar{k}(q_1 - q_2) = u \\ \\ \bar{m}_2\ddot{q}_2 + \bar{d}_2(\dot{q}_2 - \dot{z}) + \bar{d}_1(\dot{q}_2 - \dot{q}_1) + \bar{k}_2(q_2 - z) + \bar{k}_1(q_2 - q_1) = -u \end{cases} \tag{1}$$

where $\bar{m}_1 = (m_1 + \Delta m_1)$ represents the real mass of the quarter car, composed by a nominal parameter m_1 and an uncertain one Δm_1. The same organization applies to the other parameters.

The majority of the active suspension approaches aim at improving the ride quality, which is direct related with the vertical acceleration $\ddot{q}_1$, without degrading other criteria like the tyre deflection $(\dot{q}_2 - z)$ and the suspension stroke $(q_1 - q_2)$. Further, the active suspension must provide tracking of the low frequency components of the road.

The present controller design will be based on the SDOF model related to the mass m_1, which is obtained from the suspension depicted in Fig.1, when one considers the dynamics of mass m_2 as an input to the mass m_1. Hence, the SDOF model is obtained from considering only the first equation in Eq.(1), which may be rewriten as follows

$$\ddot{q}_1 + 2\zeta_1\omega_{n1}\dot{q}_1 + \omega_{n1}^2 q_1 = u_1 + f_2 + f_p \tag{2}$$

where $\omega_{n1} = \sqrt{k_1/m_1}$ and $\zeta_1 = d_1/2m_1\omega_{n1}$ are the nominal natural frequency and damping ratio associated with the mass m_1. The signals f_2 and f_p are defined as

$$f_2 = 2\zeta_1\omega_{n1}\dot{q}_2 + \omega_{n1}^2 q_2$$
$$f_p = -\tfrac{1}{m}\left[\Delta m_1\ddot{q}_1 + \Delta k_1(q_1 - q_2) + \Delta d_1(\dot{q}_1 - \dot{q}_2)\right] \tag{3}$$

Therefore, the signal f_2 is a known one, since it is defined from the nominal system parameters and from the state $(q_2, \dot{q}_2)$, which is considered as known. The signal f_p, on the other hand, is associated with the parameters uncertainties and then it represents an unknown disturbance. The state-space representation of Eq.(2) can be written as

$$\mathbf{x}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1[u_1 + f_2 + f_p] \tag{4}$$

2.2. The reference model

In order to specify the desired performance of the system, an ideal *Skyhooh Damping System*, which is depicted in Fig.2, is chosen as the reference model. The model consists of a mass m_m isolated from a moving base by a spring k_m and a damper d_m and a *skyhook* damper d_s connecting the mass to an inertial reference. Hence, the behavior of the reference model is given by

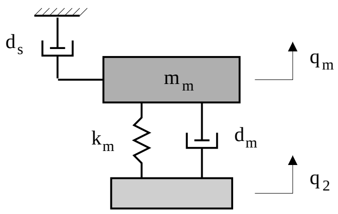

Figure 2: Reference model.

$$\ddot{q}_m + 2(\zeta_m + \zeta_s)\omega_{nm}\dot{q}_m + \omega_{nm}^2 q_1 = 2\zeta_m\omega_{nm}\dot{q}_2 + \omega_{nm}^2 q_2 \tag{5}$$

where ζ_m and ζ_s represent the damping ratios associated with d_m and d_s, respectively, ω_n is the natural frequency of the reference model. It is worth noting that the base disturbance, that provides the excitation to the reference model, is governed by the actual dynamics of the mass m_2.

Defining $r = 2\zeta_m\omega_{nm}\dot{q}_2 + \omega_{nm}^2 q_2$, Eq.(5) may be rewritten in state-space form a as

$$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m r \tag{6}$$

Therefore, the dynamic behaviour governed by Eq.(6) represents the one which must be followed by the dynamics of the SDOF governed by Eq.(4).

3. The controller design

Defining the tracking error between the system and reference model responses as follows

$$\mathbf{e} = \mathbf{x}_1 - \mathbf{x}_m \tag{7}$$

and considering Eqs.(4) and (6), the dynamics of the error is governed by

$$\dot{\mathbf{e}} = \mathbf{A}_m\mathbf{e} + \mathbf{B}_1 u_1 + \big(\mathbf{A}_1 - \mathbf{A}_m\big)\mathbf{x}_1 + \mathbf{B}_1 f_2 - \mathbf{B}_m r + \mathbf{B}_1 f_p \tag{8}$$

Consider the following sliding surface

$$\mathcal{S} = \{\mathbf{e} \in I\!R^n /\ s(\mathbf{e}) = 0\} \tag{9}$$

where $s(\mathbf{e})$ is the switching function and it is defined as

$$s(\mathbf{e}) = \mathbf{S}\mathbf{e} \tag{10}$$

where $\mathbf{S} = [S_1 \;\; S_2]$. Without loss of generality, from now on, one makes $S_2 = 1$.

If after a finite time t_s, an sliding mode takes place, one has

$$s = \mathbf{S}_1\mathbf{e}_1 + \mathbf{S}_2\mathbf{e}_2 = 0 \; ; \qquad \forall t \geq t_s \tag{11}$$

and, therefore, the dynamics during sliding mode is governed by

$$e_1(t) = exp(-S_1 t)e_1(t_s) \; ; \qquad \forall t \geq t_s \tag{12}$$

where $e_1 = (q_1 - q_m)$ is the first component of the error vector. Therefore, during sliding mode the dynamics is insensitive to parameters uncertainties, which corresponds to matched disturbances, and it is governed by the parameter S_1, which defines the sliding surface $\mathcal{S}$.

Once the dynamics of the sliding mode is determined after judicious choice of the sliding surface $\mathcal{S}$, one has to determine a control law which gives sufficient conditions to induce and maintain the sliding mode. The proposed control law comprises two components, a linear one, determined to stabilize the nominal system, and a nonlinear component, which must be designed to compensate the disturbances present in the system. Specifically

$$u(t) = u_l(t) + u_n(t) \tag{13}$$

The linear component is given by

$$u_l(t) = -\big(\mathbf{S}\mathbf{B}_1\big)^{-1}\mathbf{S}\left[\mathbf{A}_m\mathbf{e} + \big(\mathbf{A}_1 - \mathbf{A}_m\big)\mathbf{x} - \mathbf{B}_m r + f_2\right] + \phi s \tag{14}$$

where ϕ is any negative constant and it is related with the transient of the dynamics before the trajectory reach the sliding surface. The nonlinear component u_n is defined as

$$u_n(t) = -\rho(t, \mathbf{e})sgn(s); \qquad \text{para} \quad s(t) \neq 0 \tag{15}$$

where $\rho(t, \mathbf{e})$ is a modulation function to be chosen in order to give sufficient conditions to induce the sliding mode despite the presence of parameter uncertainties. The function sgn represents the signum function defined as

$$sgn(s) = \begin{cases} 1 & , \quad s > 0 \\ -1 & , \quad s < 0 \end{cases} \tag{16}$$

For the design of the modulation function ρ, one considers the following Lyapunov function candidate

$$V(s) = \frac{1}{2}s^2 \tag{17}$$

whose the time derivative along the trajectories given by Eqs.(10) and (8), after substitution of Eqs.(14) and (15), can be written as

$$\dot{V}(s) = \phi s^2 - \rho sgn(s) + s f_p \tag{18}$$

and the following inequality holds

$$\dot{V}(s) \leq -|s|^2 - |s|(\rho - f_p) \tag{19}$$

Therefore, if the modulation function is chosen to satisfy the inequality

$$\rho(t, \mathbf{e}) \geq |f_p| + \eta \tag{20}$$

Table 1: Quarter car model parameters in SI units

m_1	1000	Δm_1	$0.2m_1$
d_1	1398	Δd_1	0
k_1	36000	Δk_1	$-0.1k_1$
m_2	100	Δm_2	0
d_2	0	Δd_2	0
k_2	360000	Δk_2	$0.1k_2$

where η is any positive constant, one can prove that the sliding mode takes place in $\mathcal{S}$ in a finite time irrespective the presence of parameter uncertainties f_p. Hence, if the modulation function is properly chosen according to Eq.(20), the error dynamics is asymptotically stable with respect to the origin. One may chose a time varying modulation function, according to Eq.(3), using the known signals $(q_1, \dot{q}_1, \ddot{q}_1)$ and $(q_2, \dot{q}_2)$ and some upper bounds on the parameter uncertainties Δm_1, Δd_1 and Δk_1. The modulation function ρ may also be chosen as a constant, however, the stability of the overall system can be only locally proven. It is worthwhile noting that the variable structure control may present robustness with respect to others matched disturbances not considered in the controller design. For example, even though the modulation function may be chosen considering only the parameter uncertainties, according to Eq.(20), the controller may present robustness with respect to other disturbances acting in the system if an inequality as the one in Eq.(20) is satisfied.

The practical implementation of variable structure control presents the well-known phenomenon of chattering, which is characterized by a high frequency switching by the actuators. The chattering represents an undesirable phenomenon once it may degrade the actuator and, further, excite the high-frequency unmodelled dynamics of the system. In order to reduce the chattering, one may consider one of the suavization techniques presented in the literature (Peixoto et al., 2001; Song and Mukherjee, 1998).

4. Active suspension analysis

In order to assess the performance of the proposed active suspension, numerical simulations considering the quarter car model subjected to stationary random excitation were carried out. Different levels of road disturbances and vehicle velocities were considered. The parameter values used in the simulations are given in Table 1. Several other uncertainty values were also considered but due to space limitation were not reported here.

The road disturbance was modelled in a such way that the disturbance velocity input consists of a white-noise process specified by

$$E[\dot{z}(t)] = 0 \quad , \quad E[\dot{z}(t)\dot{z}(t-\tau)] = W\delta(\tau) \tag{21}$$

where E denotes the expectation operator and δ represents the Dirac function, with $W = 2\pi G_z V$ being the intensity of the white-noise, which depends on the road roughness G_z and the vehicle velocity V. The results presented in the sequel were carried out with $G_z = 3.8850 \cdot 10^{-4}$, which corresponds to a poor road quality. Simulations with other road data were also performed but the results were omitted due to the same qualitative behavior of the controlled system.

The parameters of the reference model are chosen in such a way that the suspension parameters are made equal the quarter car ones, hence, $m_m = m_1$, $dm = d_1$ and $k_m = k_1$. The damping ratio of the skyhook damper is chosen as $\zeta_s = 1$.

The sliding surface $\mathcal{S}$ is defined by making $S_1 = 6$. The modulation function is made constant, namely, $\rho = 200N$ and in order to reduce the numerical chattering a saturation function $sat(s/0.0001)$ was used instead of the signum function $sgn(s)$ in Eq.(15).

The root mean square (rms) acceleration of mass m_1, which is a measure of ride quality, is plotted in Fig.3 for the passive suspension and for the active one, for different vehicle velocities. It is clear from Fig. 3, that the active suspension performs much better than the corresponding passive one, with respect to this performance measure.

The performance of the active system is also better than the corresponding passive one when one considers the rms value of the stroke, suspension deflection, as can be seen in Fig.4.

Most of active suspension systems provide improved vehicle ride, but the effective control of wheel resonance is an open area of research (Elbeheiry et al., 1995). This is somewhat depicted in Fig.5, where the active suspension yielded a smaller power spectral density (PSD) of the acceleration of mass m_1 than the passive counterpart in the low frequency range, i.e., around the natural frequency $\omega_{n1} = 1$ Hz. For frequencies greater than ω_{n1}, the PSD is almost the same for the passive and active suspensions, even in the second natural frequency, which represents the wheel resonance. The result depicted in Fig. 5 was derived with a vehicle

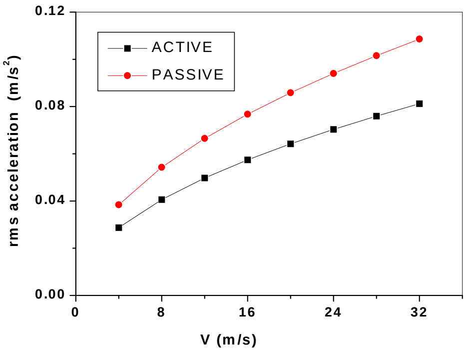

Figure 3: Root mean square acceleration of mass m_1.

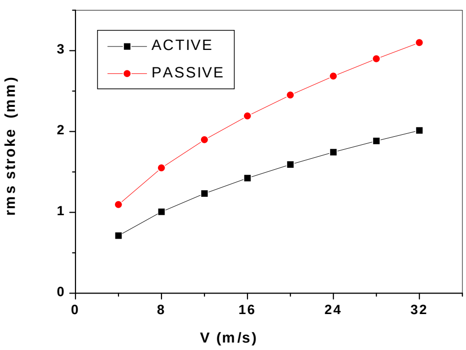

Figure 4: Root mean square stroke.

velocity $V = 20$ m/s.

The active suspensions are required to improve the compromise between conflicting performance measures, as, for instance, the ride quality and the handling. However, the majority of active suspension systems, presented in the literature, are derived from optimal control techniques and most of them aim at improving the ride quality. Hence, in the performance index to be minimized, the handling criterion, which is related with the tyre deflection, is weighted much less than the ride quality criterion and, as a consequence, the active suspension may yield a better ride quality at the expense of a greater tyre deflection relative to the passive suspension (Narayanan, 1998). The rms tyre deflection for the passive and active suspensions is depicted in Fig. 6 and the proposed active suspension also performed better than the passive one, which would result in improvement of both traction and lateral controllability.

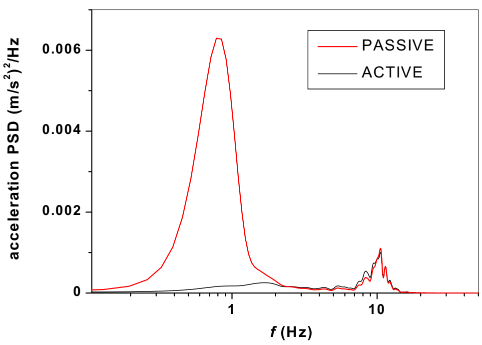

Figure 5: Power spectral density of acceleration of mass m_1.

Finally, the desired tracking of the low frequency components of the road profile provided by the passive and active suspension are depicted in Fig. 7. It is clear from Fig. 7 that the active suspension effectively isolate the vehicle from the undesired road disturbances represented by the high frequencies components.

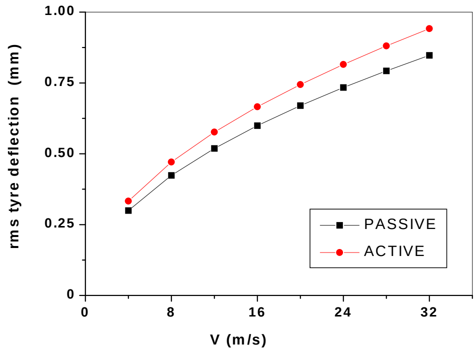

Figure 6: Root mean square tyre deflection.

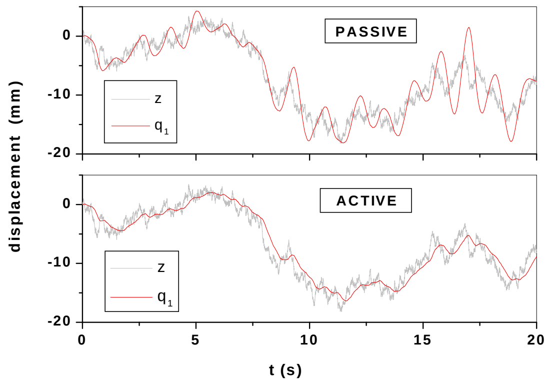

Figure 7: Tracking of the low frequency components of the road profile.

5. Concluding Remarks

The present work considered the variable structure control with model following for the design of an active suspension system. The proposed active suspension does not require an estimation of the road disturbance. Despite the presence of parameters uncertainties, improved performances with respect to ride quality, stroke and tyre deflection was obtained when compared with the passive suspension in a quarter car model. Numerical simulations considering different levels of road disturbances and vehicle velocities were considered to asses the performance of the proposed controller. The present work presents an initial effort of the Solid Mechanics Laboratory in the study of active vibration isolation systems, particularly the suspension systems. Parallel studies of active and semi-active magneto-rheological suspensions in a half car model with flexible body are under consideration. Experiment verification of the proposed active suspension will be assessed in an quarter car prototype system.

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