PROPORTIONAL NAVIGATION PLUS PURSUIT INTERCEPT LAW APPLYING TARGET MANEUVERING MEASURING

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Abstract. An enhanced intercept law based on proportional navigation and pursuit applying target maneuvering measuring is proposed. Observations on simulated scenarios show that proportional navigation is better suited for non-maneuvering targets while pursuit is better suited for high maneuvering targets. Therefore one defines an intermediate law that applies proportional navigation for low maneuvering targets and pursuit for high maneuvering targets. The relationship between those basic laws is supposed to be linear where the coefficients depend on the target maneuvering level. Simulated scenarios using proportional navigation plus pursuit law result in better performance than proportional navigation or pursuit law implemented independently as it is shown herein.

Keywords. *Guidance, Control, Intercept, Aerospace.*

1. Introduction

Intercept algorithms has being developed mostly for guided missiles in which an air target shall be engaged under a large set of scenarios possibilities. Interception of a maneuvering target is a non-trivial task even to the modern intercept techniques. The most common approach for this problem is the proportional navigation. However, when the target randomly maneuvers the proportional navigation becomes expensive in energy and time. In this case, the pursuit law ensures the intercept requirements whether the interceptor lateral acceleration limits are considerably large.

Since the level of target maneuvering may be measured by means of computing the standard deviation of target velocity variation modulus, the proportional navigation and pursuit laws are proposed to be jointed into a unique law by a linear relationship. Simulations give results with more interceptors' path stability even though target maneuvers.

2. The Known Intercept Laws

The most known intercept laws are the line-of-sight, constant bearing, pursuit and proportional navigation. Each one is intended to a specific application. For convenience we will discuss those laws applicable to the study herein.

2.1. Pursuit

The purpose is to keep the interceptor aligned with the line-of-sight which is the straight line passing through interceptor and target positions. The dynamics is similar to a dog hunting a rabbit that runs before the victim position.

There are three pursuit types: *attitude*, *velocity* and *deviated*. In attitude, the angle defined between the interceptor body axis and the line-of-sight is aimed to zero. Otherwise, in velocity, the angle defined between the velocity vector and the line-of-sight is aimed to zero. When the pursuit is deviated, the angle is aimed to a constant value.

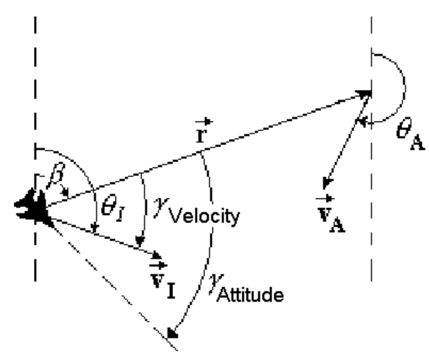


Figure 1: pursuit notation in an intercept scenario

The pursuit lateral acceleration a_P is shown by equation (1), where v is the interceptor speed, κ is a constant, and γ is γ_V or γ_A depending on the pursuit type.

$$a_P = \kappa v \gamma \quad (1)$$

Figure below shows a simulated scenario using the velocity pursuit law. One can see that the trajectory tangent line is always aimed to target.

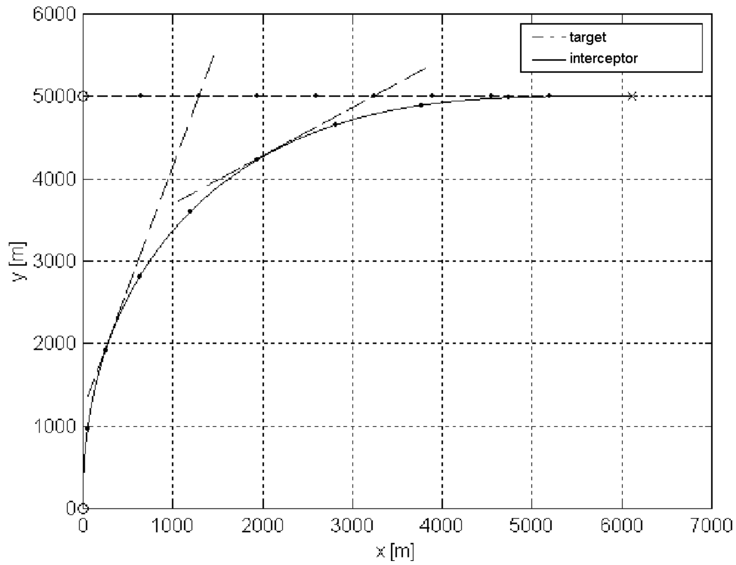


Figure 2: target and interceptor trajectories for a velocity pursuit intercept

Pursuit is better suited for intercept scenarios in which the intercept must be accomplished and the target is pretty unpredictable. For instance, the interception is hard to be successful for maneuvering targets when interceptors are guided by proportional navigation with large constants, but it is not hard for pursuit-guided interceptors.

2.2. Proportional Navigation

Unlike pursuit the main purpose of the proportional navigation is to command the lateral acceleration to a value proportional to the line-of-sight rotation. Therefore the interceptor will find a constant bearing path in order to minimize the rotation. The main formula to obtain the lateral acceleration is shown below where β is the line-of-sight angle, κ is the proportional navigation constant and v is the interceptor speed.

$$a_p = \kappa v \dot{\beta} \tag{2}$$

When κ is 1, the proportional navigation is worthy to the pursuit law if the interceptor tracking is initialized aimed to the target. In this case the interceptor is going to be aimed to target during the whole trajectory. In other hand, when κ is big the proportional navigation approximates to the constant bearing law. Those conclusions may be seen in figure 3.

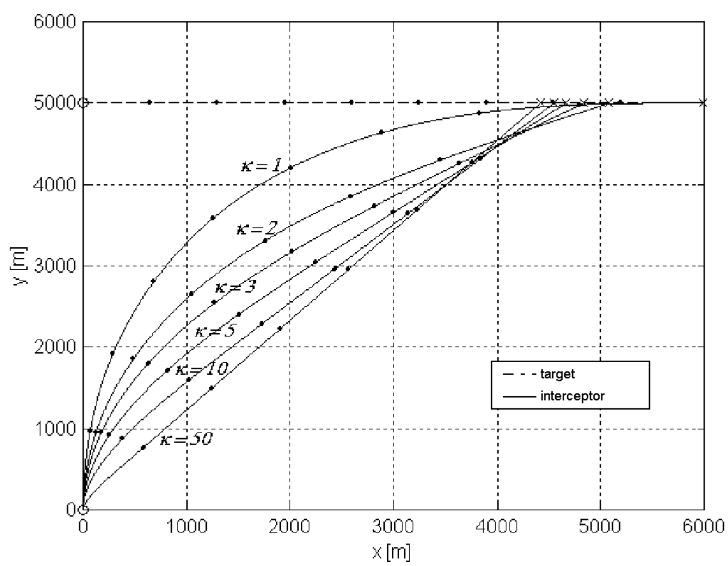


Figure 3: target and interceptor trajectories for several navigation constants ruled by PNG

3. Target Maneuvering Level

In order to implement in enhanced intercept laws, the target maneuvering level may be obtained by means of computing a single standard deviation of target velocity as shown below:

$$\vec{\mathbf{v}}_{M,k} = \frac{1}{N}\sum_{i=k-N}^{k}\vec{\mathbf{v}}_i \tag{2}$$

$$\sigma^2_{|\vec{v}|,k} = \frac{1}{N-1}\sum_{i=k-N}^{k}\left|\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_{M,i}\right|^2 \tag{3}$$

where $\sigma_{|\vec{v}|,k}$ is the standard deviation of target velocity for the last N steps.

The amount of computing operations may be reduced by use of equations (2) and (3) in a differential form as described below:

$$\vec{\mathbf{v}}_{M,k} = \vec{\mathbf{v}}_{M,k-1} + \vec{\mathbf{V}}_k - \vec{\mathbf{V}}_{k-N} \tag{4}$$

where:

$$\vec{\mathbf{V}}_k = \frac{\vec{\mathbf{v}}_k}{N} \tag{5}$$

The variance may be computed by:

$$\sigma^2_{|\vec{\mathbf{v}}|,k} = \sigma^2_{|\vec{\mathbf{v}}|,k-1} + \Gamma_k - \Gamma_{k-N} \tag{6}$$

where:

$$\Gamma_i = \frac{\left|\vec{\mathbf{v}}_k - \vec{\mathbf{v}}_{M,k}\right|^2}{N-1} \tag{7}$$

The values of $\vec{\mathbf{V}}_i$ e Γ_i, $i = k\text{-}N,\ldots,N$ shall be stored in a circular memory for use on equations (4) and (6).

4. Proportional Navigation Plus Pursuit Law

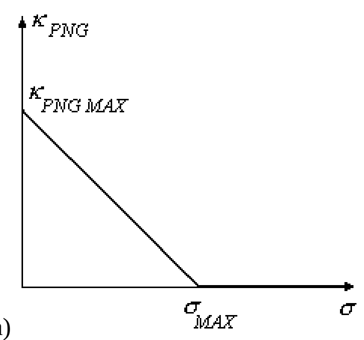
The proportional navigation plus pursuit law is represented by a linear relationship between the two basic laws in which the lateral acceleration is represented as a function of the line of sight angle and the target maneuvering level as follows:

$$a_P = v\left[\kappa_{PNG}(\sigma)\dot{\beta} + \kappa_{PER}(\sigma)(\theta_I - \beta)\right] \tag{8}$$

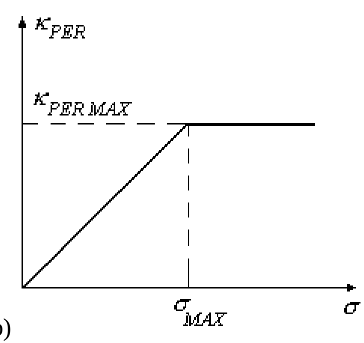
where the navigation coefficients κ_{PNG} e κ_{PER} are functions of the target maneuvering level. A linear relationship was defined in a way that the proportional navigation is effective when the target maneuvers and the pursuit is effective when the target follows a constant bearing path.

$$\kappa_{PNG}(\sigma) = \begin{cases} \kappa_{PNGMAX}\left(1 - \dfrac{\sigma}{\sigma_{MAX}}\right), & 0 \le \sigma < \sigma_{MAX} \\ 0, & \sigma \ge \sigma_{MAX} \end{cases} \tag{9}$$

$$\kappa_{PER}(\sigma) = \begin{cases} \dfrac{\kappa_{PERMAX}}{\sigma_{MAX}}\sigma, & 0 \le \sigma < \sigma_{MAX} \\ \kappa_{PERMAX}, & \sigma \ge \sigma_{MAX} \end{cases} \tag{10}$$



(a)



(b)

Figure 4: proportional navigation (a) and pursuit (b) coefficients as functions of the target maneuvering level.

5. Simulation Scenario Suppositions

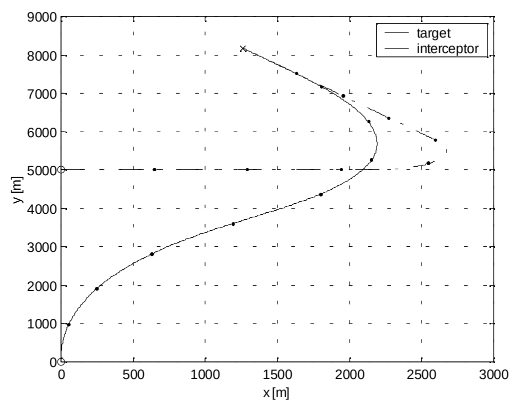
On simulations the intercept dynamic was considered being lied on the horizontal plane. Target and interceptor speed are supposed to be constant in the whole trajectory and the interceptor commanded lateral acceleration was supposed to be unlimited. The intercept is considered effective when the distance between target and interceptor is below a fixed value and is increasing.

6. Results

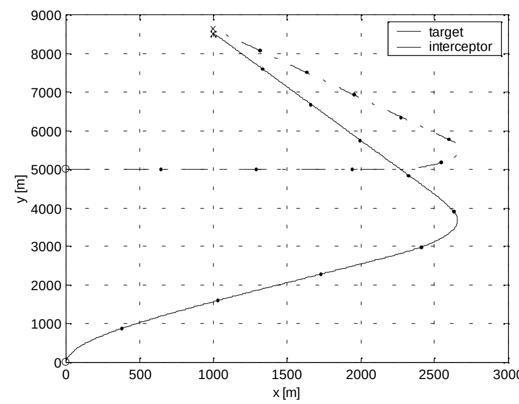
After simulating an intercept scenario by use of a dynamic intercept model in a horizontal plan, it was observed that the interceptor responds with more stability to target maneuverings than the proportional navigational implemented alone. The simulations are shown on figures 5a, 5b, 5c and 5d for target performing a “v” maneuver and on figures 6a, 6b, 6c and 6d for target performing an “s” maneuver.

The pursuit law is not the optimal control for the interceptor lateral acceleration but ensures the intercept mission if interceptor is faster than target. The figures 5a and 6a show the pursuit path. In other hand, the proportional navigation law is the optimal control for constant bearing moving targets since the time is minimized but it may become instable for maneuvering targets. The figures 5b and 6b show the proportional navigation path.

As one can see on figures 5c, 5d, 6c and 6d, pursuit rules the interceptor lateral acceleration when target maneuvers and proportional navigation rules the interceptor lateral acceleration when target follows a constant velocity flight. It gives stability to interceptor even though interceptor maneuvers.



(a)



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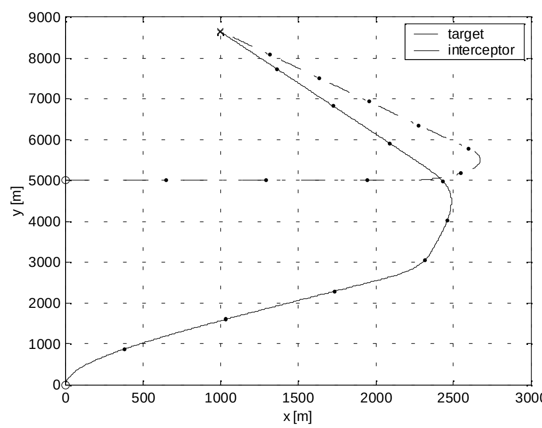
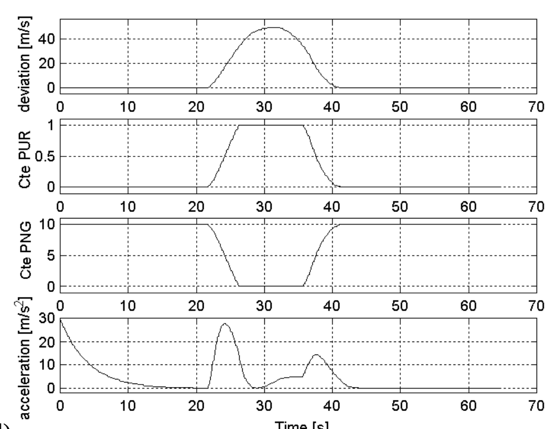


(c)



(d)

Figure 5: pursuit simulation (a), PNG simulation (b), PNG plus pursuit simulation path (c) and parameters (d) in “V” trajectory

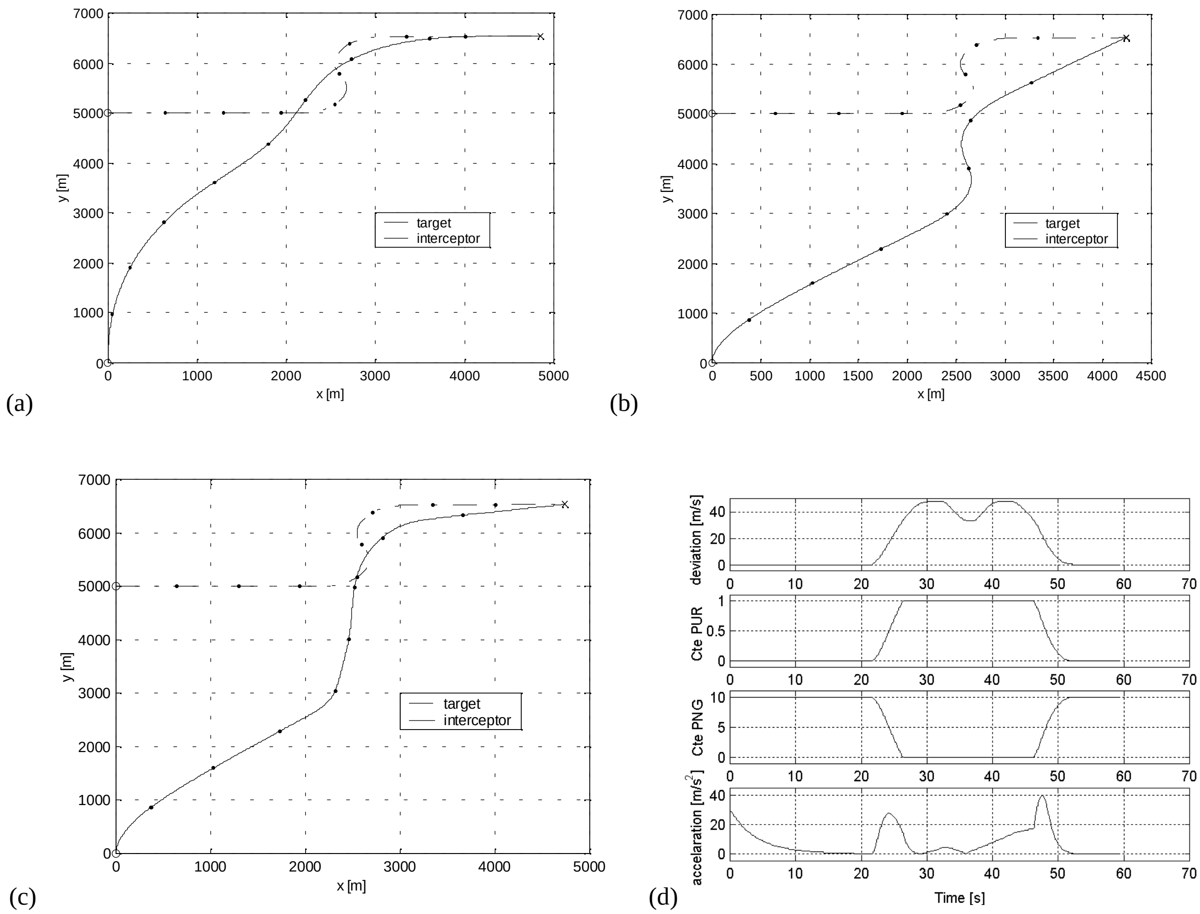


Figure 6: pursuit simulation (a), PNG simulation (b), PNG plus pursuit simulation path (c) and parameters (d) in “S” trajectory

7. Conclusions

The proportional navigation plus pursuit law was proposed herein for the target intercept task. It was shown by simulations that this technique joins each known law feature in a unique one, stable for maneuvering targets and effective for non-maneuvering targets. Pursuit rules the interceptor guidance when target maneuvers. In other hand proportional navigation rules the interceptor guidance when target follows a constant velocity flight.

The pursuit guidance increases the intercept goal probability and proportional navigation decreases time-to-intercept. It means that the pursuit implementation ensures the interceptor to reach target no matters how intercept costs. And proportional navigation ensures minimal intercept time when target is in constant velocity motion. Such statements are considering no existence of limits on lateral acceleration.

More ideas may be developed in order to enhance known intercept laws by use of maneuvering parameters as the target maneuvering level was used herein. Stability and intercept mathematical proofs may be developed in order to ensure effectiveness of proposed laws. As it can be seen, the intercept study is a non-fully known science and still has space to be explored.

8. References

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