TRANSONIC AERODYNAMIC INFLUENCE COEFFICIENTS FOR APPROXIMATE AEROELASTIC STABILITY ANALYSIS

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Abstract. *A transonic aerodynamic influence coefficients approach is presented to improve the doublet-lattice method to take into account the non-linear behavior characteristic of the transonic flow. The methodology is based on corrections of the linear aerodynamic influence coefficients matrix for a given reduced frequency and Mach number through the post multiplication by a weighting matrix. The simulated lifting surface is put to move rigidly around its span axis at the desired reduced frequency. Unsteady pressure distributions are computed by finite-difference Navier-Stokes simulation of the flow around lifting surface for the same above mentioned parameters. Based on comparisons of the linear obtained unsteady pressure and the corresponding non linear ones, the weighting matrix can be calculated solving a linear equations system. Then a conventional aeroelastic analysis is performed with the corrected doublet lattice method to take into account the transonic dynamic behavior of these flow regime.*

Keywords: *Aeroelasticity, Transonic Unsteady Flow, Doublet Lattice, Computational Fluid Dynamics.*

1. Introduction

The aeroelastic behavior of an aircraft is typically more critical in the transonic flight regime. The transonic "dip" phenomenon is due to the increase of the quasi-steady lift curve slope and the sudden variation downstream of the center of pressure position (Mabey, 1989; Landahl, 1951). Most modern aircraft fly under these conditions and methods for analyzing the aeroelastic stability are usually based on CFD computations, which are computationally expensive and, thus, inadequate for industrial development (Bennet, 1998).

Most flutter computations use commercial finite-element codes with aeroelastic modeling capability such as NASTRAN™. These codes, however, are usually based on linear aerodynamic methods and, thus, limited to subsonic or supersonic analysis. In recent years there have been a number of attempts at solving the transonic aeroelastic problem using mixed procedures based on corrections of the linear unsteady aerodynamic flow by the introduction of correction factors to modify the aerodynamic influence coefficients (Bergh and Zwaan, 1966; Giesing et all., 1976; Houwink *et al.*, 1982; McCain, 1985; Zwaan, 1985; Pitt and Goodman, 1987; Suciu *et al.*, 1990; Liu *et al.*, 1988; Baker *et al.*, 1998; Silva and Mello, 1999; Chen *et al.*, 2000). Bergh and Zwaan (1966) present a procedure to obtain the general unsteady lift distribution based on measurements of unsteady pressures associated to a single mode of vibration by the use of correction factors which multiply the aerodynamic influence coefficient matrix (AIC). Their results show some insensitivity of the correction factors to the reduced frequency, for the case of incompressible flows, and when the experimental pressure distribution is reasonably described by the theory. Giesing, Kalman and Rodden (1976) developed some procedures to correct the subsonic lifting surface theory based also on experimental results. They use pre- and post-multiplication of the AIC matrix by weighting factors to match the experimental pressure results. Their work was principally motivated by the requirement of a safe prediction of the control surface flutter, usually underestimated by the potential theory due to boundary layer thickness effects which change the effective downwash.

In the above mentioned work, the correction factor technique was applied successfully for subsonic cases. However, near the transonic flight regime, nonlinear effects are important to be considered. As shown by Ashley (1980), the shock wave movement and strength usually destabilize the single-degree-of-freedom flutter, and affects profoundly the flexure-torsion flutter. In this way, it is necessary to pay special attention to the flutter mechanism in these circumstances.

In the work of Dowell, Bland and Williams (1983) some studies in the linear behavior of the nonlinear unsteady transonic flow were made in order to identify the range of parameters, such as phase angle and shock strength, related to reduced frequency and angle of attack, over which the linear behavior occurs. The objective in this case is to understand the limits of linear behavior for aeroelastic applications, where most analyses apply linear equations and the motions are small. Some conclusions arise such as, the linearity of shock wave motion with respect to a small angle of attack change, which is the case of aeroelastic applications, and the importance of an accurate mean flow calculation to take into account the correct shock wave position and strength.

As long as the applications of interest are within the region for which the linear behavior of unsteady transonic flows can be considered, the correction factor techniques applied in the modification of unsteady potential flows are adequate tools for aeroelastic engineering applications. The papers by Houwink, Kraan and Zwaan (1982; 1985), McCain (1985), Pitt and Goodman (1987), for example, use correction factor techniques in the calculation of unsteady transonic flows. Other methods based on corrections of the linear AIC matrix include the Transonic Equivalent Strip method (TES) (Liu *et al.*, 1988), used in conjunction with a modal AIC approach (Chen *et al.*, 2000), and the correction procedure of Baker, Yuan and Goggin (1998).

The present work is based on the method proposed by Pitt and Goodman (1987), who developed modifications of the doublet-lattice influence coefficients by the post-multiplication of the AIC matrix using results from a transonic small disturbance (TSD) code. That method was capable of simulating the transonic dip phenomenon with small differences with respect to wind tunnel data. Post-multiplication allows modifying both the real and imaginary parts of the downwash, thus changing the pressures and phase angles (McCain, 1985). Some discrepancies were found and attributed by the authors to viscous effects. Indeed, viscous effects alter the strength and location of shock waves over the wing surface, which in turn, may have a significant effect on the flutter computations. Post-multiplication typically worsened the phase angles and thus, the aeroelastic stability .

In order to take viscous effects into account and obtain a more accurate mean flow prediction, the present method uses results from viscous simulations (Reynolds-averaged Navier-Stokes equations) to modify the doublet-lattice influence coefficients. In previous works (Silva and Mello, 1999), aerodynamic coefficients were corrected based on rigid wing pitch simulations, or modal displacements (Silva, Mello and Azevedo, 2001). The present work introduces a modification of the latter work, now considering the unsteady pressure coefficients for a given reduced frequency of a wing moving rigidly in pitch motion. Finite-difference Navier-Stokes simulations are performed around a lifting surface, which is deformed according to the pitch mode shape. Unsteady pressure differentials are obtained from these simulations and used to obtain correction factors to the doublet-lattice aerodynamic coefficients. The corrected coefficients are then used for aeroelastic analysis, as a post-multiplying matrix of the AIC matrix.

2. Aerodynamic Models

Aeroelastic analyses are performed for the standard aeroelastic configuration AGARD Wing 445.6. The structural properties for the AGARD wing were obtained from Yates (1988). The transonic aerodynamic approximation employs a mixed formulation, which uses pressure coefficients obtained from a nonlinear aerodynamic model based on a computational fluid dynamics (CFD) formulation. The linear steady pressure coefficient distribution, obtained by the doublet lattice method (DLM) (Albano and Rodden, 1969), is then corrected considering the non linear baseline pressure distribution.

2.1. Linear Model

The equations of motion can be written in a non-homogeneous form by the inclusion of the aerodynamic loading terms which are also dependent of the state variables of the structure:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F(u,\dot{u})\} \quad , \tag{1}$$

where u is the physical displacement vector, $u(x,y,z)$, M and K are the mass and stiffness matrices respectively, and F is the aerodynamic load vector.

The aerodynamic model for the right hand side of Eq. (1) has been based on a standard version of the doublet lattice method (DLM) (Albano and Rodden, 1969). All the lifting surfaces of the aircraft have been discretized in terms of interfering panels which contain singular solutions of the unsteady acceleration potential equation for a given value of reduced frequency. These solutions are based on the Küssner relation between the acceleration potential (pressure) and the normalwash on two distinct points. The individual solution of each panel, as well as the interference of one panel onto others, can be represented by an algebraic form as

$$\left\{\frac{P}{q_\infty}\right\} = [AIC]\left\{\frac{w}{U_\infty}\right\} \quad , \tag{2}$$

where $[AIC]$ is the aerodynamic influence coefficient matrix, w the induced downwash, U_∞ the free-stream flow speed, and q_∞ the associated dynamic pressure. Equation (2) relates the pressure coefficient to the non-dimensional downwash on all surface panels.

For the determination of the pressure coefficient vector in Eq. (2) it is necessary to know the induced non-dimensional downwash, which may be regarded as an effective angle of attack. From the boundary conditions for small perturbations, the relationship between the normalwash and the solid boundary displacement is given by

$$\left\{\frac{w}{U_\infty}\right\} = \frac{1}{U_\infty}\left(\frac{\partial\{z\}}{\partial t} + U_\infty \frac{\partial\{z\}}{\partial x}\right) \quad . \tag{3}$$

In aeroelastic analysis it is usually more convenient to employ a modal representation of the aereolastic model. Hence, the equations of motion can be rewritten as

$$[m]\{\ddot{\eta}\} + [k]\{\eta\} = \left\{\overline{F}(\eta,\dot{\eta})\right\} \quad , \tag{4}$$

and Eq. (3) can be written as

$$\left\{\frac{w}{U_\infty}\right\} = \frac{1}{U_\infty}\left([\Phi]\frac{\partial\{\eta\}}{\partial t} + U_\infty [\Phi_x]\{\eta\}\right) \quad , \tag{5}$$

since $\{z\} = [\Phi]\{\eta\}$ and $\{\overline{F}\} = [\Phi]^T\{F\}$, where $[\Phi]$ is the matrix containing the structural mode shapes.

In the right hand side of Eq. (5) a substantial derivative is applied to the modal displacement vector $\{\eta\}$. In matrix form, the substantial derivative is denoted by *D*. The aerodynamic loading vector, *F,* may be expressed by Eq. (2) with the multiplication of the pressures by an integration matrix *S*, which is constructed from the panel elements geometry. The normalwash vector can be substituted by Eq. (5), closing the right hand side of Eq. (4) as a function of the generalized coordinates of the system.

$$\{\overline{F}\} = \frac{1}{2}\rho_\infty U_\infty [\Phi]^T [S][AIC][D][\Phi]\{\eta\} \quad . \tag{6}$$

2.2. Non-Linear Model

A numerical procedure for solving the Reynolds-averaged Navier-Stokes equations (RANS) is used to obtain viscous non-linear solutions so that correction factors could be developed. The numerical method employs an implementation of Roe's flux difference splitting (FDS) method (Roe, 1981; and Vinokur, 1988), which is capable of good shock resolution and, thus, is considered adequate in representing shock strength and location effects. The Navier-Stokes solver used in the present work is a modified version of a code developed by Sankar and Kwon (1990). This modified code is used to obtain the non-linear pressure coefficient (C_p) distributions over the wing surface

2.3 The Correction Procedure

The method used here to obtain the correction factors is based on Eq. (2), which relates the theoretical surface pressure coefficients to the downwash and it is rewritten as

$$\{\Delta C_p^{th}\} = [AIC]\{\overline{w}\} \quad , \tag{7}$$

where $\{\overline{w}\}$ is the dimensionless downwash or the effective angle of attack. From the CFD computations, or experimental data, it is possible to obtain the nonlinear pressure coefficient differential between lower and upper surfaces of the wing which can be written as $\Delta C_p = C_p^l - C_p^u$. In the DLM, the lifting forces at each panel are concentrated on the ¼ element chord, at element midspan. Therefore, the CFD computed ΔC_p's are linearly interpolated to these locations on the wing surface.

Considering a steady state situation, Eq. (7) applies and the downwash vector may be expressed as an angle of attack, as shown below. When one rewrites Eq. (5) in the frequency domain, it is possible to obtain

$$\{\overline{w}(ik)\} = \left(\frac{1}{U_\infty}[\Phi]ik + [\Phi_x]\right)\{\eta\} \quad , \tag{8}$$

where $k = \omega b/U_\infty$ is the reduced frequency. In matrix form, Eq. (8) becomes

$$\{\overline{w}(ik)\} = ([D_R] + ik[D_I])[\Phi]\{\eta\} \tag{9}$$

The $[D_R]$ and $[D_I]$ matrices are the real and imaginary part of the substantial derivative matrix $[D]$ which relates the downwash to the physical displacement vector. This downwash may be regarded as an unsteady perturbation in angle of attack, for a given reduced frequency. Here, for the rigid pitch motions the choice is an amplitude of $\Delta\alpha=1.5^o$ in this angle. From the unsteady pressures acquired, a Fourier transformation is used to obtain the frequency-domain components for both lower and upper surfaces as follows:

$$\mathrm{Re}\left(C_{pi}\right)=\frac{k}{\pi}\int_{\tau_1}^{\tau_1+2\pi/k}C_p(\tau)sin(k\tau)d\tau \quad , \tag{10}$$

$$\mathrm{Im}\left(C_{pi}\right)=\frac{k}{\pi}\int_{\tau_1}^{\tau_1+2\pi/k}C_p(\tau)\cos(k\tau)d\tau \quad . \tag{11}$$

The discrete transformation is

$$\mathrm{Re}\left(C_{pi}\right)=\frac{kM\Delta\tau}{2\pi\Delta\alpha}\sum_{m=m_1}^{m_1+m_T}C_{p\,m}\,sin(km\Delta\tau) \quad , \tag{12}$$

$$\mathrm{Im}\left(C_{pi}\right)=\frac{kM\Delta\tau}{2\pi\Delta\alpha}\sum_{m=m_1}^{m_1+m_T}C_{p\,m}\cos(km\Delta\tau) \quad . \tag{13}$$

Thus, the complex $\Delta C_p^D(k)$ difference is obtained as

$$\Delta C_p^C(k)=\left(\frac{kM\Delta\tau}{2\pi\Delta\alpha}\right)\left(C_{pi}^{lower}(k)-C_{pi}^{upper}(k)\right) \quad . \tag{14}$$

Considering the non-dimensionalization of the Navier-Stokes formulation, the non-dimensional time is given as $\tau = \boldsymbol{a}_\infty \boldsymbol{t}/\boldsymbol{c}$. The pressure differences of Eq. (14) need to be scaled by the reduced frequency, k, the Mach number, M, the dimensionless time step $\Delta\tau$ and the amplitude of the oscillation $\Delta\alpha$. After the pressure difference, ΔC_p^C, is computed, it is possible to determine correction factors which satisfy the relation

$$\{CF\}=[AIC]^{-1}\left\{\Delta C_p^C\right\} \quad . \tag{15}$$

It is interesting to note that the *CF* vector is a downwash vector related to a disturbance pressure coefficient differential. On the other hand, the *AIC* matrix relates pressure coefficients to unit displacements (downwash). Then, it is necessary to scale the *CF* quantities by the complex displacement of each panel given by the Eq. (9). This is performed by the division of each coefficient of *CF*, which is associated to a known panel, by the complex displacement of the panel, for the same reduced frequency which generates the unsteady pressure coefficients. In this way, it is possible to rewrite Eq. (7) as

$$\left\{\Delta C_p^D\right\}=[AIC][WT]\{\overline{w}\} \quad . \tag{16}$$

Here, *WT* is a diagonal weighting matrix which affects a change in the downwash vector to take into account the transonic nonlinear effects. The diagonal elements are the scaled quantities of the vector of correction factors *CF* .The introduction of the weighting matrix in NASTRAN™ is done by the post-multiplication of the *AIC* matrix internally in the aeroelastic solver. Hence, the resulting right-hand side of the aeroelastic equations can be written as

$$\{F\}=\frac{1}{2}\rho_\infty U_\infty[S][AIC][WT][D]\{u\} \quad . \tag{17}$$

3. Results

The test case considered here is known as the AGARD wing 445.6 weakened (no. 3). Known as a standard aeroelastic configuration, the AGARD wing is discretized by the doublet lattice method as an isolated wing in different flow conditions. The aerodynamic model is comprised of 240 panels (Figure 1). The test conditions are the same as in Lee-Rausch and Batina (1993) and Yates (1988), which present numerical and experimental results, respectively. The model under study is described in Yates (1988). The Mach numbers and air densities for the cases considered here are presented in Table 1.

Table 1: Flow conditions for AGARD wing 445.6 aeroelastic analysis.

Mach	Reynolds	Density (kg/m^3)
0.678	$1,410\times10^6$	0,2082
0.901	$0,911\times10^6$	0,09947
0.960	$0,627\times10^6$	0,06339

In order to proceed with the unsteady aerodynamic calculations, a set of reduced frequencies is chosen for each of the values of the flutter Mach number. In Table 2 the chosen values for the reduced frequency are presented. It is important to remember that the values of reduced frequency for the linear aeroelastic analysis are different from the values which are input for the Navier-Stokes solver. This is so because the reduced frequency in NASTRAN is defined as $K_1 = \omega c_r / 2U$, whereas in the Navier-Stokes code this value is defined as $k = \omega c / a_\infty$. The values presented in Table 2 are the reduced frequencies values in which flutter occurs. These values were taken from the experimental data available (Yates, 1988).

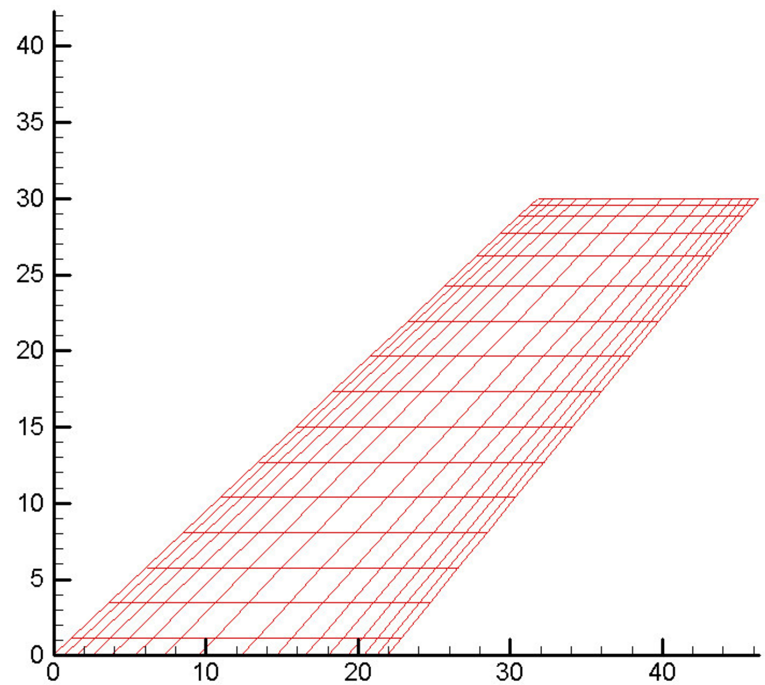

Figure 1: DLM paneling of the AGARD wing.

Table 2: Reduced frequencies for AGARD wing 445.6 aeroelastic analysis.

Mach	*k*	K_1
0,678	0,3311	0,1364
0,901	0,3070	0,0952
0,960	0,2713	0,0789

In Figs. 2 and 3, it is possible to see that the values of the pressure coefficient difference distribution along the chord length for two cases of reduced frequencies associated to the corresponding Mach number.

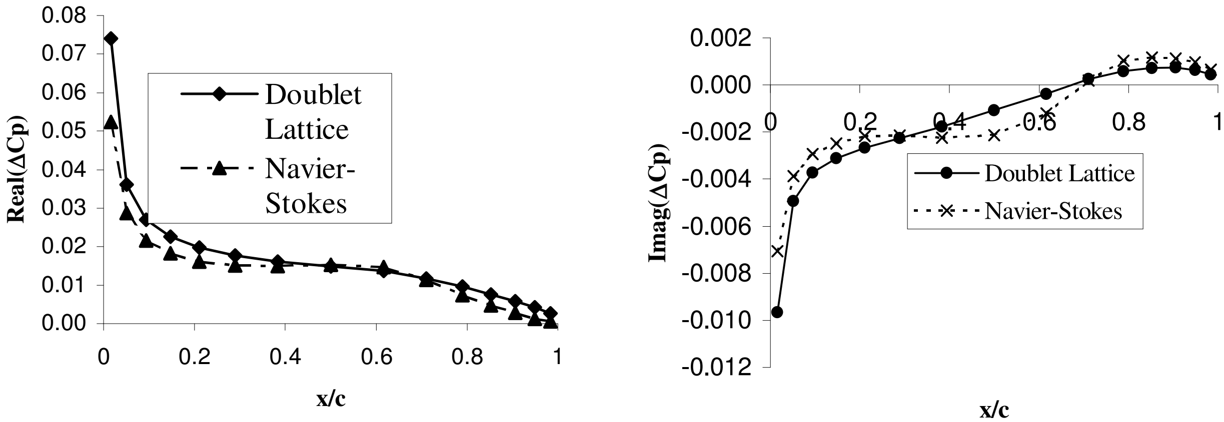

Figure 2 : Comparison between complex unsteady pressure difference distribution along chord for wing span station 23,1 %, Mach 0,96.

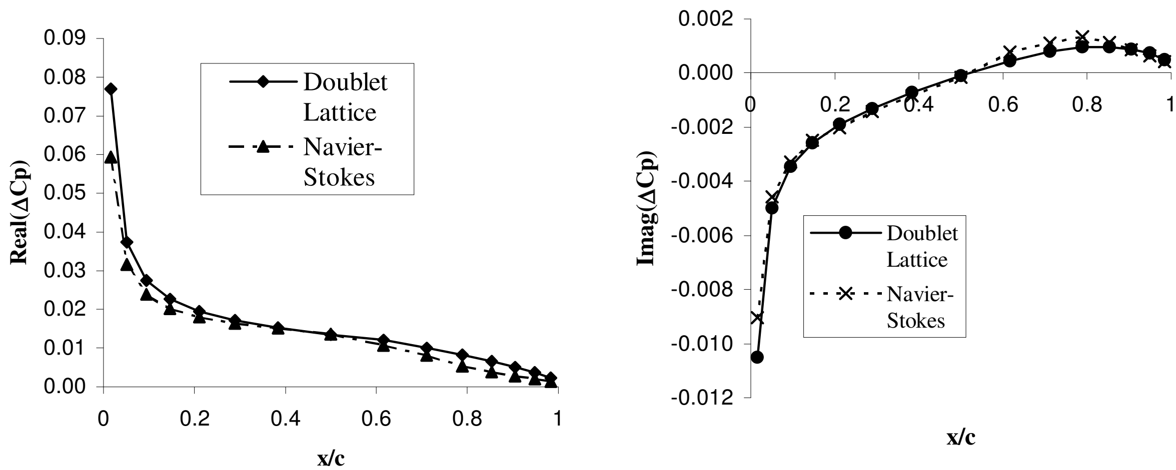

Figure 3**:** Comparison between complex unsteady pressure difference distribution along chord for wing span station 23,1 %, Mach 0,901.

One can observe in Figs. 2 and 3 that the chosen spanwise station presented here (23,10 %), where there is a formation of a shock wave, easily noted for the Mach number 0,96. The transonic effects are more pronounced for the value of the highest Mach number value principally concerning the imaginary part of the pressures.. It is interesting to note that the difference in phase in more pronounced than the difference in amplitude. The explanation for this feature is associated to the profile maximum thickness. The 65A004 airfoil has the thickness ratio (*t/c*) equals to 4 %. Hence, the shock wave formation will only occur at higher Mach numbers. As the pressure phases are more sensitive to the shock movement than the pressure amplitudes, the main differences when comparing the potential and the Navier-Stokes results are in the phase plots.

As previously described, the correction method here proposed uses the pressure coefficients differences to compute downwash correction factors to the Doublet Lattice formulation. Hence, when the correction factors are introduced in the aeroelastic analysis, this yields the results shown in Tables 3 and 4, which concern the stability of the aeroelastic system. The Tables include comparisons with some well known analysis codes (Chen *et al*, 2000) and with experimental results (Yates, 1988). In Table 3, the results related to steady correction (Silva *et al.*, 2001) are presented as a basis of comparison of the correction procedure employing steady data and unsteady data, as they are presented here. The ZTAIC method is a form of correction procedure based on a transonic AIC matrix reduced from a set of reference pressure differences from known downwash modes (Chen *et al*.,2000). The CAPTSD method is a finite difference solution in the time domain of the Transonic Small Disturbance Equation (Chen *et al*.,2000). The results shown in this tables are also presented in graphical form in Figs. 4 and 5.

Table2: Flutter speeds and frequencies for AGARD wing 445.6.

Mach Number	Experimental		Linear		Steady Correction (Silva *et al.*, 2001)		Unsteady Correction	
	V_F [m/s]	ω_F [Hz]	V_F [m/s]	ω_F [Hz]	V_F [m/s]	ω_F [Hz]	V_F [m/s]	ω_F [Hz]
0.678	231,37	17,98	239,89	17,542	213,82	21,250	218,95	21,591
0.901	296,69	16,09	299,30	15,282	275,65	17,352	290,85	16,729
0.960	309,01	13,89	329,18	14,346	315,97	15,652	323,61	14,781

Table3: Flutter speeds and frequencies for AGARD wing 445.6.

Mach Number	Unsteady Correction		CAPTSD –(non-linear) (Chen *et al.*, 2000)		ZTAIC (Chen *et al.*, 2000)	
	V_F [m/s]	ω_F [Hz]	V_F [m/s]	ω_F [Hz]	V_F [m/s]	ω_F [Hz]
0.678	218,95	21,591	234,09	19,2	231,95	19,30
0.901	290,85	16,729	290,17	15,8	294,19	16,38
0.960	323,61	14,781	N/A	N/A	N/A	N/A

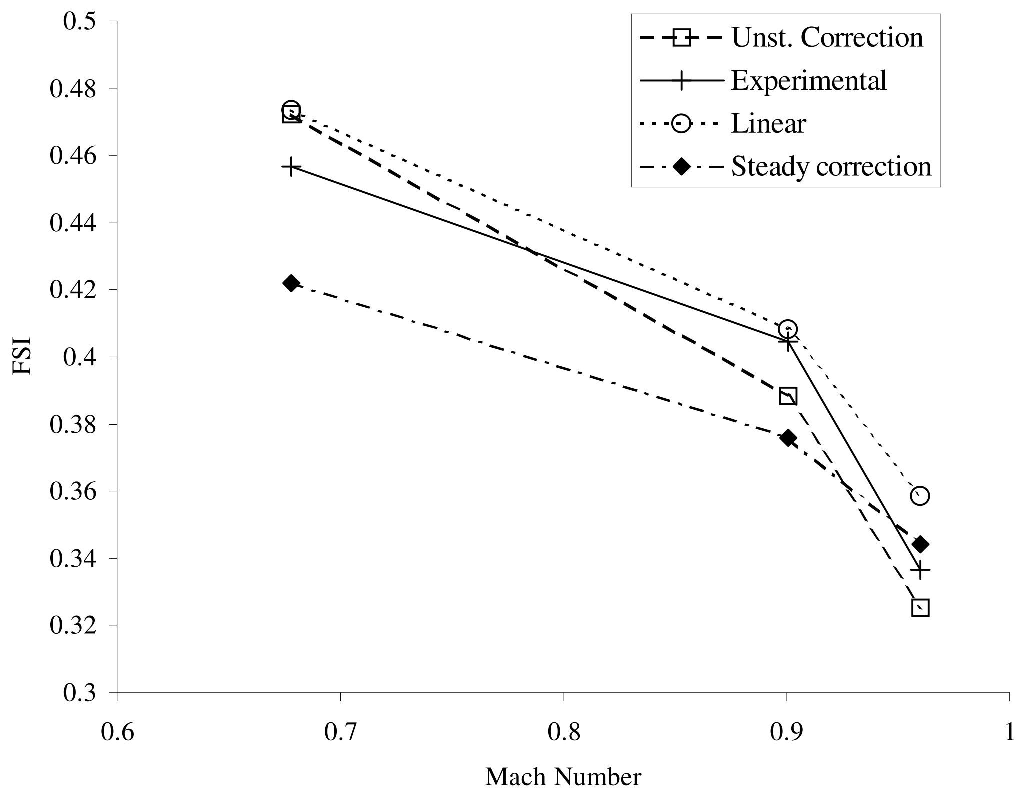

Figure 4: Flutter speed versus Mach number (wing 445.6 weakened model no.3).

One can observe in Fig. 4 that the correction here presented indicates the presence of the transonic dip which is characterized by a decrease of the slope of the flutter speed plot as a function of the Mach number, when comparing with the linear predicted one. Actually, the results in Fig 4 presents a similar behavior concerning the dip phenomenon, when comparing with the experimental data. However, at subsonic the Mach number the correction does not play effect. As the non linear an linear pressures distributions at subsonic Mach numbers presents a nearly coincident behavior for the case of the AGARD wing, the generated correction factor will not modify downwash vector as in the case of transonic flow condition. Observing Figs. 2 and 3, is possible to note the differences in pressure from the linear and non-linear calculations. Considering this, the resulting correction factors for the transonic flow conditions will properly introduce the necessary changes in the downwash vector to take into account the out of phase component concerning the non-linear behavior.

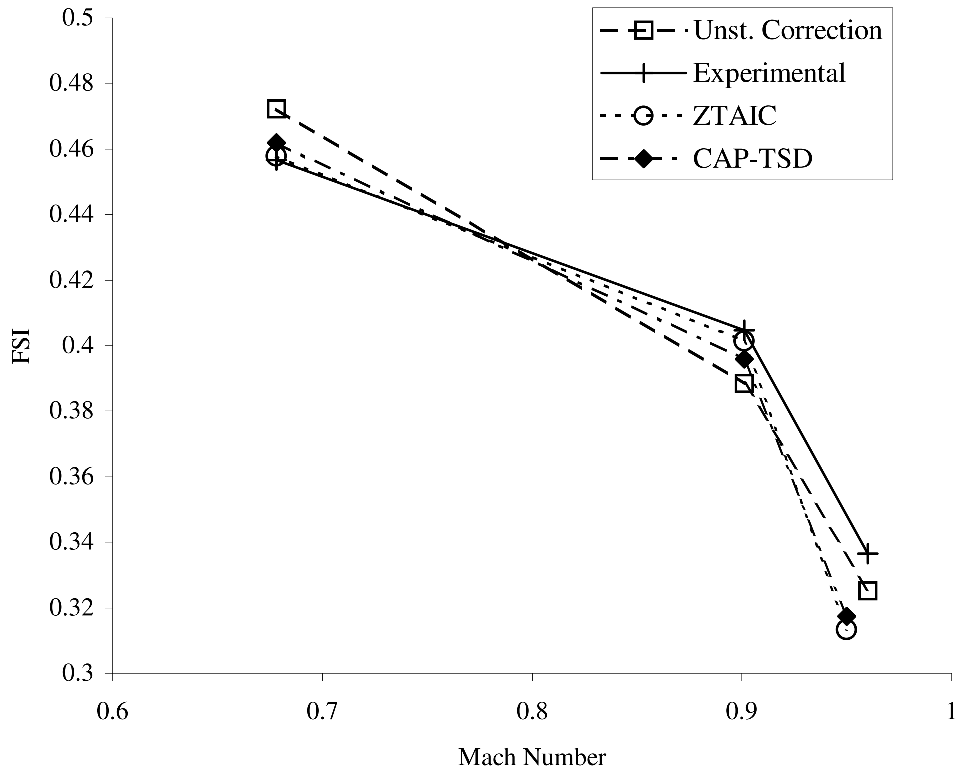

Figure 5: Flutter speed versus Mach number (wing 445.6 weakened model no.3).

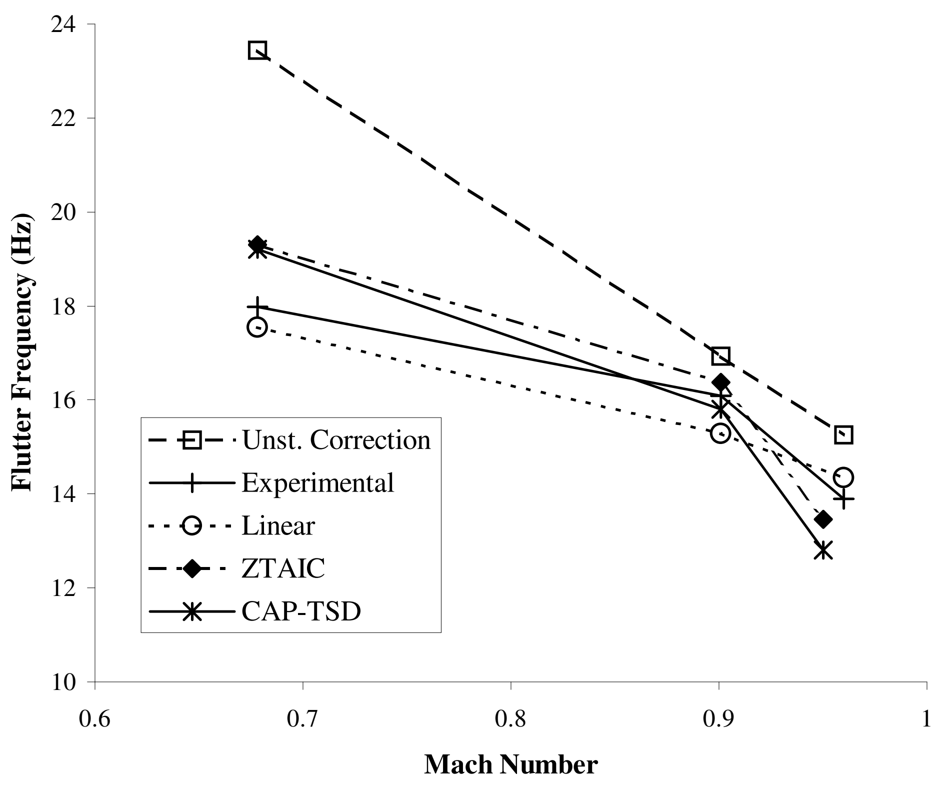

Figure 6: Flutter frequency versus Mach number. (wing 445.6 weakened model no.3).

The results presented in the work of Chen *et al.* (2000), as indicted in Fig. 5 show that the transonic dip phenomenon is well characterized in the solution with the ZTAIC method. Clearly, as already discussed, these results presents also good agreement with the experimental data. One can see also in Fig. 5 the CAPTSD calculations (Chen *et* *al*., 2000) are expected to yield better results because they are based on the nonlinear solution of the transonic small disturbance equations, and the comparison in Fig. 5 indeed shows such behavior. Moreover, the ZTAIC results are based in a more comprehensive method of AIC matrix correction because such procedure employs a set of different downwash modes. If one considers that the method presented here is based on a single pitch mode, it is clear that one should expect the ZTAIC method to yield a more conservative correlation with the experimental data, as also seen in Fig. 5. Moreover, the ZTAIC and CAPTSD methods presents an increase in the flutter dip phenomenon, which in this case is a desirable feature in transonic flutter calculations.

The flutter frequencies are also shown in Figure 6 and compared with the experimental data and the other methods. It is clear that a larger change in the flutter frequency expected since the unsteady correction procedure introduces an important variation in the imaginary part of the complex eigenvalue problem involved in the flutter solution. Hence, eigensolutions would tend to be more sensitive to the introduced complex coefficients. However, the present results concerning the flutter frequency does not agree with the experimental and the other methods.

4. Conclusions

The results presented in the paper indicate that the correction method here proposed, using unsteady downwash corrections in the AIC matrix, is capable of capturing the transonic dip phenomenon. This fact represents an improvement when comparing this procedure with the one based on steady pressure data. Previous work (Silva *et al*.,2001) has shown that aeroelastic analyses based on the steady data based correction method yield flutter speeds which are more conservative than those calculated by the linear method, which is a desirable characteristic of such procedure. The same characteristic is preserved when considering the present method. The results of Chen *et al*.(2000) also indicates the presence of transonic dip which can be identified as a decrease of the flutter speed slope as a function of the freestream Mach number, as observed in the experimental data

The flutter frequency behavior as a function of the Mach number does not present a good agreement with the experimental and the other computed results. In order to repair this discrepancies, as a next step in the continuation of the present effort would be to use a correction matrix which is constructed from a set of downwash modes. However, this is also going to increase the computational cost of the aeroelastic analysis since more unsteady CFD simulations would need to be performed in order to create the correction factors. As the the transonic dip phenomenon is predicted and the computed flutter speeds are conservative, the important aspect which remains to be analyzed, hence, is whether the additional costs of such an approach are indeed worthwhile or one should seek an altogether different form of including nonlinear aerodynamic information into the aeroelastic calculations

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