Crack-Dislocation Interaction. Stress and Displacement Field calculations using Goursat's Complexes Potentials.

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Abstract: Applying complex analytical methods, the influence of a dislocation in the crack behavior is presented for two-dimensional problems. Assuming the interaction to occur under elastic conditions, the Goursat's Complex Potential Method is used to solve the basic equations of stresses and displacements. The use of complex potentials in this type of problem requires a previous study of the interaction between dislocation and crack in the semi-infinite medium in which the tentative potential needs to satisfy the conditions of the free surface. Once these conditions are met, the next step consists of the application of the conformal mapping $\xi = \sqrt{\xi}$ *, so that the mathematical approach of the interaction between dislocations in the semi-infinite medium can be used in the study of the interaction of two symmetrics dislocations placed and one semi-infinite crack in the infinite medium under external loading. The method of complex potential has shown to be well adapted for the solution of such problems.*

Keywords: *complex potentials, stresses, displacements, semi-infinite medium, infinite medium.*

1 – Bi-dimensional approach to the crack-dislocation interaction problems.

To study the behavior of a crack under the combined action of crack induced external and internal loadings allows one to observe the resulting stress and displacement fields and specially the changes in the stress intensity factors which raises up from the presence of dislocations. The analysis of crack propagation problems in which the crack-dislocation interaction is present can be reduced to the solution of plane states. The two-dimensional treatment of such problems leads to a simplification in the formalism of plane states and therefore assumes an analytic solution. The use of mathematics of complex numbers in the crack-dislocation interaction consists on changing analytical solutions developed by several authors into simpler and easy to be calculated equations. Nobutaka and Takamura (1985) have proposed a model for calculating stresses and displacements applying Westergard's "Z" function. However, one of the conditions which allow the use of this function is that the problem must show symmetry. Hence, solution of non symmetric problems, such as a semi-infinite plane, cannot easily be obtained. In what follows, the stress and the displacement fields in an infinite and a semi-infinite media, with or without the presence of a dislocation at the crack tip, is proposed.

2 – The Method of Complex Potentials applied to the solution of Elasticity problems.

The use of this Method is intended to mathematically and numerically model the influence of dislocations in the shielding (or anti-shielding) effect on a crack for two-dimensional models. Since this is problem of two-dimensional symmetry, the Method of Complex Potentials was used as a tool for developing the fundamental stress and displacement equations. These equations will be necessary for setting up the stress and the displacement fields rose from the crack-dislocation interaction (Michot, 2001 and Bissa, 2002). There are a few states of particular interest. The state of plane deformation is frequently assumed in the analysis of stress systems. When one of the principal strains is zero, one says the body is in a state of plane deformation. When equilibrium equations are written in terms of stresses:

$$\begin{cases} \sigma_{xx,x} + \sigma_{yx,y} = 0 \\ \sigma_{xy,x} + \sigma_{yy,y} = 0 \\ \sigma_{zz} = \upsilon(\sigma_{xx} + \sigma_{yy}) \end{cases} \tag{1}$$

Multiplying the second equation by j and adding it to the first one, it follows ($j^2 = -1$):

$$(\sigma_{xx,x} + j\sigma_{yx,x}) + (\sigma_{xy,y} + j\sigma_{yy,y}) = 0 \tag{2}$$

Let $u(x, y)$ be a function describing the displacement expressed in terms of the complex variables $z = x + jy$ and $\bar{z} = x - jy$. These expressions can be mathematically reworked following the definition for complex numbers:

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial z}+\frac{\partial}{\partial \bar{z}} \quad \text{e} \quad \frac{\partial}{\partial y}=j(\frac{\partial}{\partial z}-\frac{\partial}{\partial \bar{z}}) \tag{3}$$

$$2\frac{\partial}{\partial z}=\frac{\partial}{\partial x}-j\frac{\partial}{\partial y} \quad \text{e} \quad 2\frac{\partial}{\partial \bar{z}}=\frac{\partial}{\partial x}+j\frac{\partial}{\partial y} \tag{4}$$

Combining equations (2) and (3) one obtains:

$$\frac{\partial}{\partial z}(\sigma_{xx}-\sigma_{yy}+2j\sigma_{xy})+\frac{\partial}{\partial \bar{z}}(\sigma_{xx}+\sigma_{yy})=0 \tag{5}$$

Considering the first term:

$$-\frac{\partial F}{\partial \bar{z}}=(\sigma_{xx}-\sigma_{yy}+2j\sigma_{xy})$$

and now the second:

$$\frac{\partial F}{\partial z}=(\sigma_{xx}+\sigma_{yy})$$

One verifies that the new function F satisfies the Cauchy-Riemman relations. Hence, assuming there exists a complex potential given by $F(z, \bar{z})$, it follows:

$$dF=-(\sigma_{xx}-\sigma_{yy}+2j\sigma_{xy})\,\partial \bar{z}+(\sigma_{xx}+\sigma_{yy})\,\partial z \tag{6}$$

Relating this potential to the displacement and introducing the complex displacement, using Eq. (4), one has:

$$\left(\frac{\partial u}{\partial z}\right)_{\bar{z}}=\frac{1}{2}\underbrace{(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}}_{\theta})+\frac{1}{2}j(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}) \tag{7}$$

By using Hooke's Law described above, with $u_z = 0$, one can write:

$$\begin{cases} \sigma_{xx}=\lambda\theta+2\mu\dfrac{\partial u_x}{\partial x} \\ \sigma_{yy}=\lambda\theta+2\mu\dfrac{\partial u_y}{\partial y} \\ \sigma_{xy}=\mu\left(\dfrac{\partial u_x}{\partial y}+\dfrac{\partial u_y}{\partial x}\right) \end{cases} \tag{8}$$

One has earlier identified that:

$$(\sigma_{xx}+\sigma_{yy})=\lambda\theta+2\mu\frac{\partial u_x}{\partial u_y}+\lambda\theta+\frac{\partial u_y}{\partial u_y}=2\lambda\theta+2\mu\theta=2\theta(\lambda+\mu)=\frac{\partial F}{\partial z} \tag{9}$$

Analogously, it follows:

$$(\sigma_{xx}-\sigma_{yy}+2j\sigma_{xy})=2\mu\left(2\frac{\partial u_x}{\partial \bar{z}}+2j\frac{\partial u_y}{\partial \bar{z}}\right)\approx 4\mu\frac{\partial u}{\partial \bar{z}}=-\frac{\partial F}{\partial z} \tag{10}$$

Integrating Eq. (10) with respect to $\bar{z}$, one finds that $F(z, \bar{z})$ relates to:

$4\mu\, u=-F(z,\bar{z})+\varphi(z)$, where $\varphi(z)$ is an integration function.

Differentiating with respect to z, it follows that:

$$4\mu \frac{\partial u}{\partial z} = -\frac{\partial F}{\partial z} + \varphi^{'}(z)$$

Rewriting the above equation:

$$\frac{\partial F}{\partial z} = \varphi^{'}(z) - 4\mu \frac{\partial u}{\partial z} = \overline{\varphi^{'}(z)} - 4\mu \overline{\frac{\partial u}{\partial z}}$$

$$2\frac{\partial F}{\partial z} = \varphi^{'}(z) + \overline{\varphi^{'}(z)} - 4\mu\theta$$

Hence:

$$\frac{\partial F}{\partial z} = \frac{1}{2}\frac{(\lambda+\mu)}{(\lambda+2\mu)}\left(\varphi^{'}(z) + \overline{\varphi^{'}(z)}\right) \tag{11}$$

Integrating Eq. (11) with respect to z and knowing that λ, μ are constants:

$$F(z,\bar{z}) = 2\left(\varphi(z) + z\,\overline{\varphi^{'}(z)} + \overline{\psi^{'}(z)}\right), \tag{12}$$

where $\psi^{'}(z)$ is a function of integration.

When function (11) is integrated with respect to z, one introduces a new function $\overline{\psi(z)}$ of $\bar{z}$ in the resultant expression and:

$$\frac{\partial\overline{\psi(z)}}{\partial z} = \frac{\partial\psi(z)}{\partial\bar{z}} = 0$$

Similarly, $\overline{\varphi^{'}(z)}$ does not depend on z:

$$\frac{\overline{\partial\varphi'(z)}}{\partial z} = \frac{\partial\ \varphi^{'}(z)}{\partial\bar{z}} = 0 \Rightarrow \int \overline{\varphi^{'}(z)}\,dz = \varphi^{'}(z)z$$

One can conclude that:

$$\begin{cases} \left(\sigma_{xx} + \sigma_{yy}\right) = 2\left(\varphi^{'}(z) + \overline{\varphi^{'}(z)}\right) \\ \left(\sigma_{xx} - \sigma_{yy} + 2j\sigma_{xy}\right) = 2\left(\bar{z}\,\varphi''(z) + \psi'(z)\right) \\ 2\mu\ u = k\ \varphi(z) - z\,\overline{\varphi^{'}(z)} - \overline{\psi(z)} \end{cases} \tag{13}$$

where $k = 3 - 4\nu$, φ and ψ are functions of z, thus defining Goursat's Potentials.

3 – Case 1 – Displacement field calculation – Semi-infinite medium without a crack and presenting two symmetric dislocations.

Let a dislocation defined by $b = b_1$ be close to a free surface, as shown in Fig. (1). This dislocation will displace the free surface following the shape shown in the sketch.

Considering the infinite medium of Fig. (2), stress component σ_{xx} must be zero in y-axis due to the symmetry of the problem. However, shear stresses in y-axis must not be zero, *i.e.*, $\sigma_{xy} > 0$.

Figure (1) shows a sketch of a semi-infinite medium containing a dislocation displaced from the system origin. To solve the stress and the displacement fields in this case, using the complex potentials, due to the absence of an image dislocation to the left of the y-axis, the free surface conditions ($\sigma_{xx} = \sigma_{xy} = 0$) will only be valid if the dislocation potential be added to the augmented potential.

$$\varphi'_2 = 2\frac{\alpha\ b_2 a}{\pi}\frac{1}{(z+\bar{a})^2} \quad , \quad \varphi'_1 = 2\frac{\alpha\ b_1 a}{j\pi}\frac{1}{(z+\bar{a})^2}$$

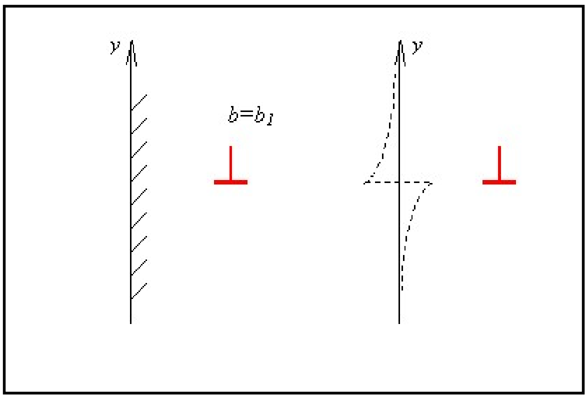


Figure 1. Schematic displacement due to the presence of a dislocation in a semi-infinite medium.

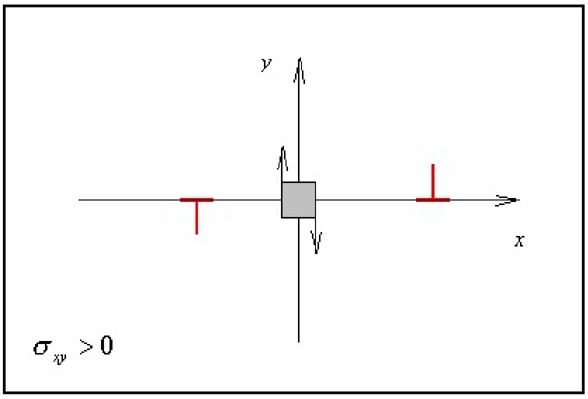


Figure 2. Relations of normal and tangential stresses in an infinite medium.

Principal stresses are related according to equation system (13). The following relation is satisfied: $2\mu\ u = k\ \varphi(z) - z\ \overline{\varphi'(z)} - \overline{\psi(z)}$, where $k = 3 - 4\nu$. Hence, displacement fields can be numerically determined. Using Maple V, Release 3 for Windows, a code was developed considering the conditions:

$a_1 = 1, a_2 = 5 \rightarrow a = 1 + j5$ / $b_1 = 1$, $b_2 = 0 \rightarrow b = 1$ / $x = 0$, the calculated displacements are shown in Fig. (3).

Figure (3) shows that the normal displacement u_x due to the presence of the dislocation $b = b_1$, has a total peak amplitude equals to the Burgers's vector as its input value $b = b_1 = 1$. Hence, one can conclude that the total lattice displacement has occurred for $b_1 = 1$, that is, a unit of length. One observes that the total tangential displacement u_y is negative, that is, displacements which are due to the presence of the dislocation $b = b_1 = 1$ are oriented towards direction $-y$. After superposing them, the green curve represents the sum of the tangential and the normal displacements. One notes that the tangential displacement shifts the curve to the left. In the numerical simulation, by considering the added potential as null, the tangential component disappears and the curve shape is the normal displacement itself. The shape of the displacement curves is independent of the distance from the crack to the y-axis in the semi-infinite medium. Numerical simulation has proved the necessary free surface conditions were matched, that is, stresses σ_{xy} and σ_{xx} assume a null value on the boundary of the semi-infinite medium, the y-axis. It was also verified through the numerical simulation that the normal displacement is independent of the dislocation position along the x-axis.

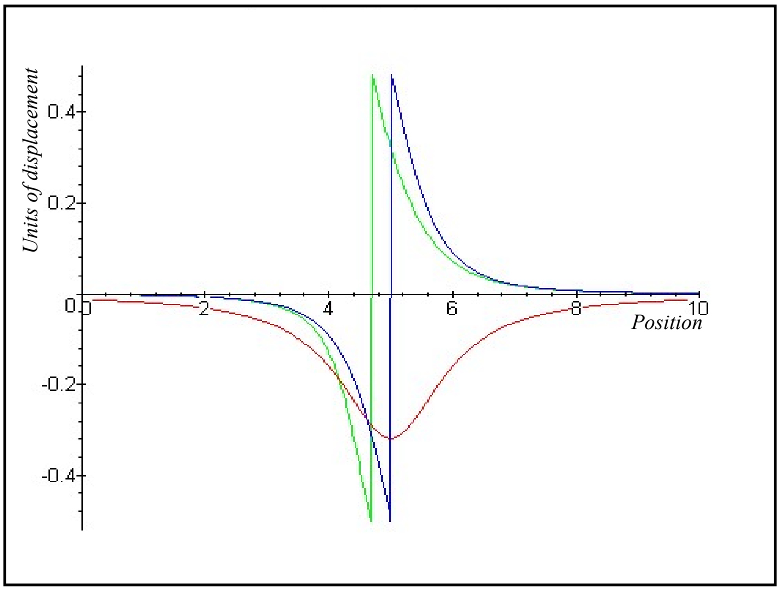


Figure 3. Superposition of displacement curves on the free surface of a semi-infinite medium. u_x (blue curve), u_y (red curve) and u_x and u_y (green curve), (Bissa, 2002).

4 – Case 2 – Calculation of the Displacement Field – Semi-infinite medium under the presence of a semi-infinite crack and dislocations.

4.1 – Conforming mapping.

The study of the crack-dislocation interaction allows one to evaluate whether the displacement field imposed by internal loading (dislocations) will result in a shielding effect at the crack tip. If so, the stress intensity factor induced by the dislocation must be lesser than zero ($K_d < 0$).
The displacement field is known for the problem of a dislocation within a semi-infinite medium. However, if one introduces a crack and a dislocation in an infinite medium some considerations must be taken.

The mapping described in Fig. (4) consists on a change of variables, *i.e.*:

$$\varphi'(z) \quad \Rightarrow \quad \varphi'(Z = \sqrt{z}) \tag{14}$$

Applying the chain rule to (14), the following relations are also valid:

$$\frac{d\varphi(z)}{dz} = \frac{d\varphi(z)}{dZ}\frac{dZ}{dz}$$

Hence, if $Z = \sqrt{z}$, one concludes that:

$$\frac{d\varphi(z)}{dz} = \frac{d\varphi(z)}{dZ}\frac{dZ}{dz}$$

$$\varphi'(z) = \varphi'(Z)\frac{1}{2\sqrt{z}}$$

$$z = C_{const}e^{j\pi} \quad \to \quad Z = \sqrt{C_{const}}e^{j\frac{\pi}{2}}$$

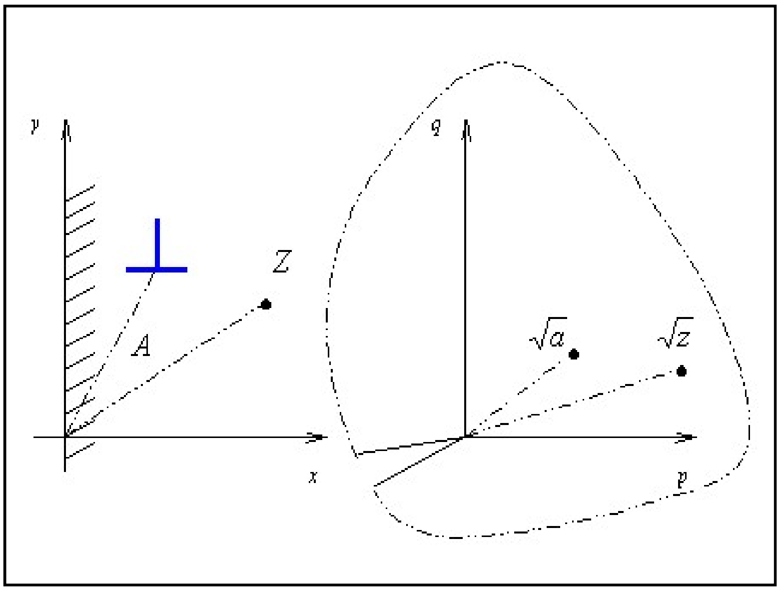


Figure 4. Conformal mapping.

That is, the proposed conformal mapping induces the opening of the crack and locates it on the negative side of *x*-axis. Fig. (4) shows the stress component σ_{xx} can be written as:

$$\sigma_{xx} = \mathrm{Re}\left\{ 2\varphi'(Z) - \overline{Z}\varphi''(Z) - \psi(Z) \right\} \tag{15}$$

$\sigma_{xx} = \mathrm{Re}\left\{ 2\varphi'(z) - \overline{Z}\varphi''(z) - \psi(z) \right\}$, according to previous verification.

Thus, complex potential equations may be applied to the case of a semi-infinite medium containing one or two dislocations.

4.2 – Complex potencials – Thomson's Method.

Thomson (1982) used *Complex Potentials of Goursat* to determine the stress and the displacement fields arising from the interaction of a crack and a dislocation. Complex potentials assume the following form:

$$\varphi'(z)_{Total} = \varphi'(z)_{Dislocation} + \varphi'(z)_{Additional\ Potential} \tag{16}$$

They are presented below:

$$\varphi' = \frac{b}{j}\left\{\frac{1}{z-a}\left(\sqrt{\frac{a}{z}}+1\right)\frac{1}{z-\overline{a}}\left(\sqrt{\frac{\overline{a}}{z}}-1\right)\right\} + \frac{b}{-j2}\left\{\left(\frac{a-\overline{a}}{\left(z-\overline{a}\right)^2}\right)\left(\sqrt{\frac{\overline{a}}{z}}+\sqrt{\frac{z}{\overline{a}}}-2\right)\right\}$$

$$\varphi' = \frac{jb}{\sqrt{z}}\left(\frac{1}{\sqrt{z}+\sqrt{a}} - \frac{1}{\sqrt{z}-\sqrt{a}}\right) + \frac{j\overline{b}}{\sqrt{z}}\left(\frac{\left(a-\overline{a}\right)}{2\sqrt{\overline{a}}}\right)\left(\frac{1}{\left(\sqrt{z}-\sqrt{\overline{a}}\right)^2}\right) \tag{17}$$

Equation (17) can be expressed as:

$$\varphi' = jb\left(Ln\left(\sqrt{z}+\sqrt{a}\right) - Ln\left(\sqrt{z}-\sqrt{a}\right)\right) + \frac{j\overline{b}}{2\sqrt{z}}\left(\frac{\left(a-\overline{a}\right)}{\sqrt{\overline{a}}}\right)\left(\frac{1}{\left(\sqrt{z}-\sqrt{\overline{a}}\right)^2}\right) \tag{18}$$

Defining the variable ω as $\omega' = \varphi' + \psi' + z\varphi''$, one has:

$$\omega' = -\frac{j\bar{b}}{\sqrt{z}}\left(\frac{1}{\sqrt{z}+\sqrt{\bar{a}}} - \frac{1}{\sqrt{z}-\sqrt{a}}\right) + \frac{j\bar{b}}{\sqrt{z}}\left(\frac{(a-\bar{a})}{2\sqrt{a}}\right)\left(\frac{1}{\left(\sqrt{a}+\sqrt{z}\right)^2}\right) \tag{19}$$

Equation (19) can be expressed as:

$$\omega' = -jb\left(Ln\left(\sqrt{z}+\sqrt{\bar{a}}\right) + Ln\left(\sqrt{z}-\sqrt{a}\right)\right) - \frac{j\bar{b}}{2\sqrt{z}}\left(\frac{1}{\sqrt{\bar{a}}}\right)\left(\frac{1}{\left(\sqrt{z}-\sqrt{\bar{a}}\right)}\right) \tag{20}$$

The limit of $\mathrm{Re}\{z\} > 0$ as z tends to 0 are $\sqrt{z}\omega'$ e $\sqrt{z}\varphi'$, leading to:

$$\sqrt{z}\varphi' \sim jb\left(\frac{1}{\sqrt{\bar{a}}} + \frac{1}{\sqrt{a}}\right) + \frac{j\bar{b}}{2\sqrt{\bar{a}}}\left(\frac{(a-\bar{a})}{\bar{a}}\right)$$

$$\sqrt{z}\omega' \sim jb\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{\bar{a}}}\right) - \frac{jb}{2\sqrt{\bar{a}}}\left(\frac{(\bar{a}-a)}{\bar{a}}\right)$$

Hence:

$$\lim_{z\to 0}(\sigma_{22} - j\sigma_{12})\sqrt{z} = \lim_{z\to 0}(\overline{\omega'} + \varphi')\sqrt{z}$$

$$\frac{(K_I - jK_{II})}{\sqrt{2\pi}} \sim 2jb\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{\bar{a}}}\right) + \frac{jb}{(\bar{a})^{3/2}}(a-\bar{a}) = \overline{K}$$

$$K_b = \frac{(K_I + jK_{II})}{\sqrt{2\pi}} = -2jb\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{\bar{a}}}\right) - \frac{jb}{(a)^{3/2}}(\bar{a}-a) \tag{21}$$

$$K_d = \frac{K_b\mu}{2\sqrt{2}\,\pi\,(1-\upsilon)} \tag{22}$$

5 – Numerical simulation – Calculation of the displacement field – Case of an infinite medium under the presence of a semi-infinite crack and dislocations.

To a better understanding the crack-dislocation interaction a program was conceived under the Maple V Rel. 3 environment, in which the displacement field due to that interaction.

The tangential and the normal displacements, u_x and u_y, respectively, may be expressed as:

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \frac{K_I}{2E}\sqrt{\frac{r}{2\pi}}\begin{pmatrix} (2k-1)\cos\frac{\theta}{2} - \mathrm{sen}\,\frac{3\theta}{2} \\ (2k+1)\,\mathrm{sen}\,\frac{\theta}{2} - \mathrm{sen}\,\frac{3\theta}{2} \end{pmatrix} \tag{23}$$

$$\begin{pmatrix} u_x \\ u_y \end{pmatrix} = \frac{K_I(1+\upsilon)}{2E}\sqrt{\frac{r}{2\pi}}\begin{pmatrix} 0 \\ (2k+1)+1 \end{pmatrix}$$

$$u_y = \frac{K_I(1+\upsilon)(2k+2)}{2E}\sqrt{\frac{r}{2\pi}} \tag{24}$$

Since $E = 2\Gamma(1+\upsilon)$, displacements can be determined by:

$$u_y = \frac{K_I 4(1-\upsilon^2)}{E}\sqrt{\frac{r}{2\pi}} = \frac{K_I 2(1-\upsilon^2)}{\Gamma}\sqrt{\frac{r}{2\pi}} \tag{25}$$

Using the potentials "φ'", "ω'" e "ψ'", we can determine the displacements, substituting the first order of the potentials in Eq. 26 below.

$$\begin{aligned} 2\mu\ u_x &= \mathrm{Re}\left\{2\mu\ u\right\} \\ 2\mu\ u_y &= \mathrm{Im}\left\{2\mu\ u\right\} \end{aligned} \tag{26}$$

Displacements are still related to a variable called ρ such that the following equation must hold:

$$u = Function\,[\rho + u_x, u_y] \tag{27}$$

Stress intensity factors for the complex potentials are found by calculating:

$$K_b = 2jb\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{\bar{a}}}\right) + j\frac{\bar{b}(a-\bar{a})}{\sqrt[3]{a}} \tag{28}$$

$$K_{Id} = \mathrm{Re}\{\frac{K_b\mu}{4\sqrt{2\pi}\,(1-\upsilon)}\} \tag{29}$$

$$K_{IId} = -\,\mathrm{Im}\{\frac{K_b\mu}{4\sqrt{2\pi}\,(1-\upsilon)}\} \tag{30}$$

Again, Maple was used and a program was developed in which displacement fields were determined for an infinite medium containing a semi-finite crack. Complex potential equations following Thomson's Method were used to calculate the displacement fields as well as the stress intensity factors. The program allows the evaluation of one or two symmetric disclocations interaction with a crack at its tip and the value of the stress intensity factor, K_d, due to the presence of the dislocations.

$$\text{Position of dislocations: } \begin{cases} a_1 = 10 \quad \rightarrow \quad a_2 = 30 \quad \rightarrow \quad a = 10 + j30 \\ a_1 = 10 \quad \rightarrow \quad a_2 = -30 \quad \rightarrow \quad a = 10\text{ - }j30 \end{cases}$$

One finds the result:

$\sigma_{xx} = 0$, $\sigma_{xy} = 0$, $\rho = 10{,}8$, $K_{Id} = -\,0{,}128103\mu$ and $K_{IId} = -\,0{,}128103\mu$.

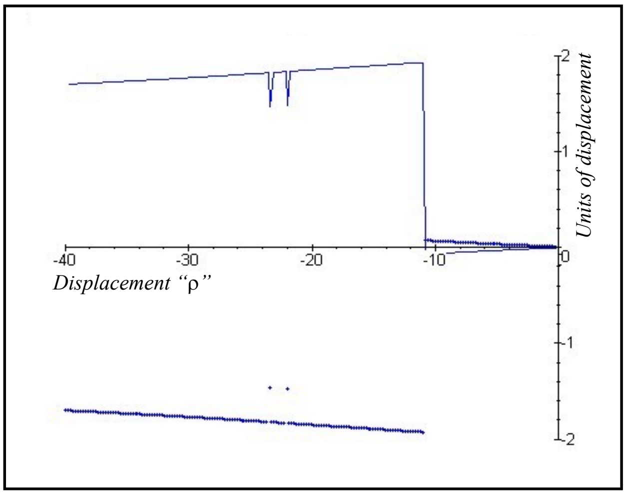
One can see that the free surface condition of the problem along the y-axis is plainly satisfied. Stress intensity factor induced by the presence of dislocations is negative in Modes I and II. Hence, the internal loading induced by those dislocations shields the crack tip. Figures 5-a and b show results for this case.

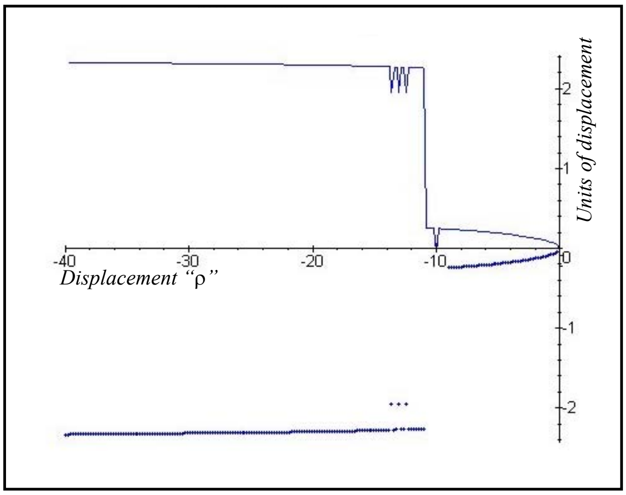
The sign of the normal displacement component u_y at the upper part of the crack helps to evaluate whether the dislocation imposes negative displacements to the crack tip (corresponding to a shielding effect). One verifies that the normal displacement assume positive values for points located far from the crack, These values become smaller as one approaches the crack tip, where the coordinate system is placed. Negative values can mathematically be found for normal displacements. However, they represent an overlaping of the upper and the lower lips of the crack, which has no physical meaning.

Starting with $\rho_{(\mathrm{limit})} = \frac{a_0}{2(1-\cos\theta)}$, where θ is the angle defined by the arc tangent of the dislocation position with respect to the crack tip and a_0 defines the initial position, one observes that, at the ρ_{limit} value, the displacement curve shows a step for which the amplitude equals to twice the Burgers' vector. The discontinuity of the displacement curve is due to the presence of a dislocation at the crack tip. If two dislocations are to be considered, two steps would show up in the curve.

This step is related to the definition of the logarithm obtained from the definition of complex potentials for an infinite medium containing a semi-finite dislocation. Function $\ln\left(\frac{1}{\sqrt{z}-\sqrt{a}}\right)$ is defined in the closed interval $[-\pi, +\pi]$. Thus, when the resulting vector defined by $\sqrt{z}$ and $-\sqrt{a}$ reverses its position wiiith respect to positive and negative values of y-axis, the function presents a step. In the calculations, it was verified that the normal displacement diminishes as one walks away from the crack, thus indicating the validity of Saint Venant's Principle.

Figure (5.a) shows the shape assumed by the upper and the lower lips of the crack under the action of the internal loading imposed by two dislocations symmetrically positioned relative to the crack tip and an external loading of the same intensity of the internal loading but with opposite sign. In Figura (5.b), one notes the shape assumed by the crack when it is loaded with an external force that causes equal intensity but opposite sign of the loading induced by the presence of the two dislocations. One verifies that the external loading greater than the one imposed by both dislocations results in negative displacements in the interval going from ρ_{limit} and the crack tip, that is, an opening occurs in the lower lip of the crack.





(a) (b)

Figure 5a. Crack shape under an external loading, K_e, equals to the internal loading imposed by the dislocation, K_d – both sides; 5b – K_e is 2.4 times greater than the loading imposed by the dislocation K_d – both sides, (Bissa, 2002).

Based on these results, one can conclude that the presence of dislocations at the tip of a crack induces displacements which opposite in sign to the ones caused by an external opening loading. Hence, the presence of dislocations also induces a shielding effect. The negative stress intensity factor induced by the dislocations demonstrates that there exists a closing tendency of the crack, that is, the plastic displacements due to the external loading are opposite to those induced by the dislocations. The crack induced stress intensity factor in *Mode I* is $K_{id} = -0{,}128103\mu\, b^{1/2}$, which is equivalent to 0,15 MPa, or approximately 15% of the fracture toughness of Si. Therefore, results obtained for $K < 0$ and $u < 0$ can be regarded as good and one concludes that the presence of dislocations are responsible for shielding the crack tip.

6 – Conclusion.

The Goursat's Complex Potential Method represent an elegant and simple solution for Elasticity problems. From these equilibrium equations one can determine the stress and the displacement fields using analytical functions. The complex mathematics is used due to the elasticity model symmetry. The crack-dislocation interaction approach to achieve the *stress field* is quite well developed. The same cannot be said about the *displacement field*, since there is a considerable complexity in the numerical simulation due to the logarithmic nature used in the model. From the results, it was verified that the presence a dislocation at the crack tip leads to an internal loading inducing stress and displacement fields acting as crack shielding agents and leading to displacements which are contrary to the crack opening tendency. The additional potentials were fundamental to calculate the stress and the displacement fields, insuring the satisfaction of some of the boundary conditions. A relation between the semi-infinite and the infinite media can be established by a conformal mapping of coodinates. The stress intensity factor a the crack-tip under external loading is positive in *Mode I*, $K_e > 0$, while the presence of dislocations induces a negative value, $K_d < 0$. This indicates there exists a competition of opposite displacements, those induced by dislocations, which cause elastic relaxation, and the plastic displacements, which are imposed by the external loading.

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