

PRESSURE DROP IN TURBULENT CHANNEL FLOW WITH POROUS AND SOLID FINS

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Abstract. *In this work, numerical solutions are presented for turbulent flow in a channel containing fins made with porous material. A macroscopic two-equation turbulence model is employed in both the porous region and the clear fluid. The equations of momentum, mass continuity and turbulence transport equations are written for an elementary representative volume yielding a set of equation valid for the entire computational domain. These equations are discretized for the method of control volume and the system of equations algebraic resultant, and resolved for the method SIMPLE. Results are presented for the velocity and turbulence kinetic energy fields, as a function of Reynolds and pressure drop.*

Keyword: Porous media, Numerical Solution, Turbulence

1. Introduction

Turbulent fluid flow in channels containing porous and solid obstacles is found in a number of engineering equipment such as shell-and-tube heat exchangers. Several other applications are found in the chemical and petroleum industries, leading to a great interest by many research groups in order to mathematically model and realistic describe this type of flow (Hwang (1997), Yang and Hwang (2003), de Lemos and Tofaneli (2003)).

For permeable structures, modeling of turbulent flow in porous media has been investigated by Pedras and de Lemos (2001a-b, 2003) and de Lemos and Pedras (2001), where the authors introduced the "Double Decomposition" concept for developing macroscopic equations for the mean and turbulent fields. Later, the model was extended to account for a macroscopic interface between a porous medium and a clear flow (Silva and de Lemos (2003)). Such methodology can be applied to flow over porous baffles.

Based on the foregoing, this work presents numerical solutions of turbulent flow in a channel containing solid and porous baffles, as illustrated in the geometry of Figure 1. In an accompanying paper (de Lemos and Tofaneli (2003)), a detailed study on the flow over porous fins was presented. Here, attention is focused on the cases of solid obstructions. Results are compared with experiments by Hwang (1997) and with the numerical work of Yang and Hwang (2003).

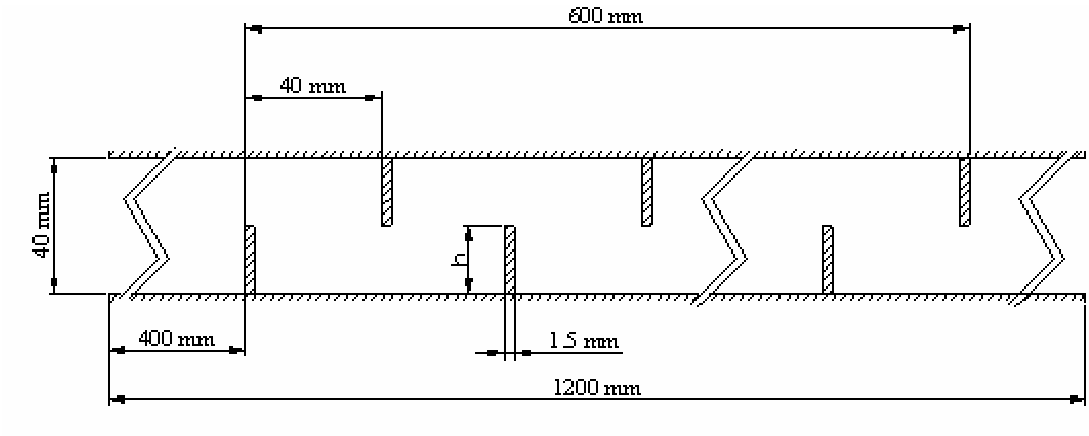


Figure 1. Channel with porous or solid fins.

2. Mathematical model

The mathematical model here employed has its origin in the works of Pedras and de Lemos (2001a). The implementation of the jump condition at the interface was considered in Silva and de Lemos (2003) based on the theory proposed in Ochoa-Tapia and Whitaker (1995a-b) Therefore, these equations will be here just reproduced and details about their derivations can be obtained in the mentioned works. These equations are:

Macroscopic continuity equation:

$$\nabla \cdot \overline{\mathbf{u}}_D = 0 \tag{1}$$

where, $\overline{\mathbf{u}}_D$ is the average surface velocity ('seepage' or Darcy velocity). The equation (1) represents the macroscopic continuity equation for an incompressible fluid.

Macroscopic momentum equation:

$$\left[\nabla\cdot(\rho\frac{\overline{\mathbf{u}}_D\overline{\mathbf{u}}_D}{\phi})\right]=-\nabla(\phi\langle\overline{p}\rangle^i)+\mu\nabla^2\overline{\mathbf{u}}_D+\nabla\cdot(-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i)-\left[\frac{\mu\phi}{K}\overline{\mathbf{u}}_D+\frac{c_F\phi\rho|\overline{\mathbf{u}}_D|\overline{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2}$$

where the last two terms in equation (2), represent the Darcy-Forchheimer contribution. The symbol K is the porous medium permeability, c_F is the form drag coefficient (Forchheimer coefficient), $\langle\overline{p}\rangle^i$ is the intrinsic average pressure of the fluid, ρ is the fluid density, μ represents the fluid viscosity and ϕ is the porosity of the porous medium. The macroscopic Reynolds stress $-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is given as,

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i=\mu_{t_\phi}2\langle\overline{\mathbf{D}}\rangle^v-\frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I} \tag{3}$$

where

$$\langle\overline{\mathbf{D}}\rangle^v=\frac{1}{2}\left[\nabla\left(\phi\langle\overline{\mathbf{u}}\rangle^i\right)+\left[\nabla\left(\phi\langle\overline{\mathbf{u}}\rangle^i\right)\right]^T\right] \tag{4}$$

is the macroscopic deformation tensor, $\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$ is the intrinsic turbulent kinetic energy, k, e μ_{t_ϕ}, is the turbulent viscosity which is modeled in Pedras and de Lemos (2001b) similarly to the case of clear flow, in the form,

$$\mu_{t_\phi}=\rho c_\mu\frac{\langle k\rangle^{i^2}}{\langle\varepsilon\rangle^i}$$

Transport equations for $\langle k\rangle^i$ and its dissipations rate $\langle\varepsilon\rangle^i=\mu\langle\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^T}\rangle^i/\rho$ are proposed in Pedras and de Lemos (2001a) as:

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle k\rangle^i\right)+\nabla\cdot\left(\overline{\mathbf{u}}_D\langle k\rangle^i\right)\right]=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_k}\right)\nabla\left(\phi\langle k\rangle^i\right)\right]+P^i+G^i-\rho\phi\langle\varepsilon\rangle^i \tag{5}$$

$$\rho\left[\frac{\partial}{\partial t}\left(\phi\langle\varepsilon\rangle^i\right)+\nabla\cdot\left(\overline{\mathbf{u}}_D\langle\varepsilon\rangle^i\right)\right]=\nabla\cdot\left[\left(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon}\right)\nabla\left(\phi\langle\varepsilon\rangle^i\right)\right]+\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}\left[c_1P^i+c_2G^i-c_2\rho\phi\langle\varepsilon\rangle^i\right] \tag{6}$$

where c_1, c_2, c_3 and c_k are model constants, $P^i=-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D$ and $G^i=C_k\rho\frac{\phi\langle k\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}}$ are generation rates of $\langle k\rangle^i$ due to gradients of $\overline{\mathbf{u}}_D$ and due to the action of the porous matrix.

At the interface, the conditions of continuity of velocity, pressure, turbulent kinetic energy k and its dissipation rate ε, in addition to their respective diffusive fluxes, are given by,

$$\overline{\mathbf{u}}_D\big|_{0<\phi<1}=\overline{\mathbf{u}}_D\big|_{\phi=1} \tag{7}$$

$$\langle\overline{p}\rangle^i\big|_{0<\phi<1}=\langle\overline{p}\rangle^i\big|_{\phi=1} \tag{8}$$

$$\langle k\rangle^v\big|_{0<\phi<1}=\langle k\rangle^v\big|_{\phi=1} \tag{9}$$

$$(\mu+\frac{\mu_{t_\phi}}{\sigma_k})\frac{\partial\langle k\rangle^v}{\partial y}\bigg|_{0<\phi<1}=(\mu+\frac{\mu_t}{\sigma_k})\frac{\partial\langle k\rangle^v}{\partial y}\bigg|_{\phi=1} \tag{10}$$

$$\langle\varepsilon\rangle^v\big|_{0<\phi<1}=\langle\varepsilon\rangle^v\big|_{\phi=1} \tag{11}$$

$$(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon})\frac{\partial\langle\varepsilon\rangle^v}{\partial y}\bigg|_{0<\phi<1}=(\mu+\frac{\mu_t}{\sigma_\varepsilon})\frac{\partial\langle\varepsilon\rangle^v}{\partial y}\bigg|_{\phi=1} \tag{12}$$

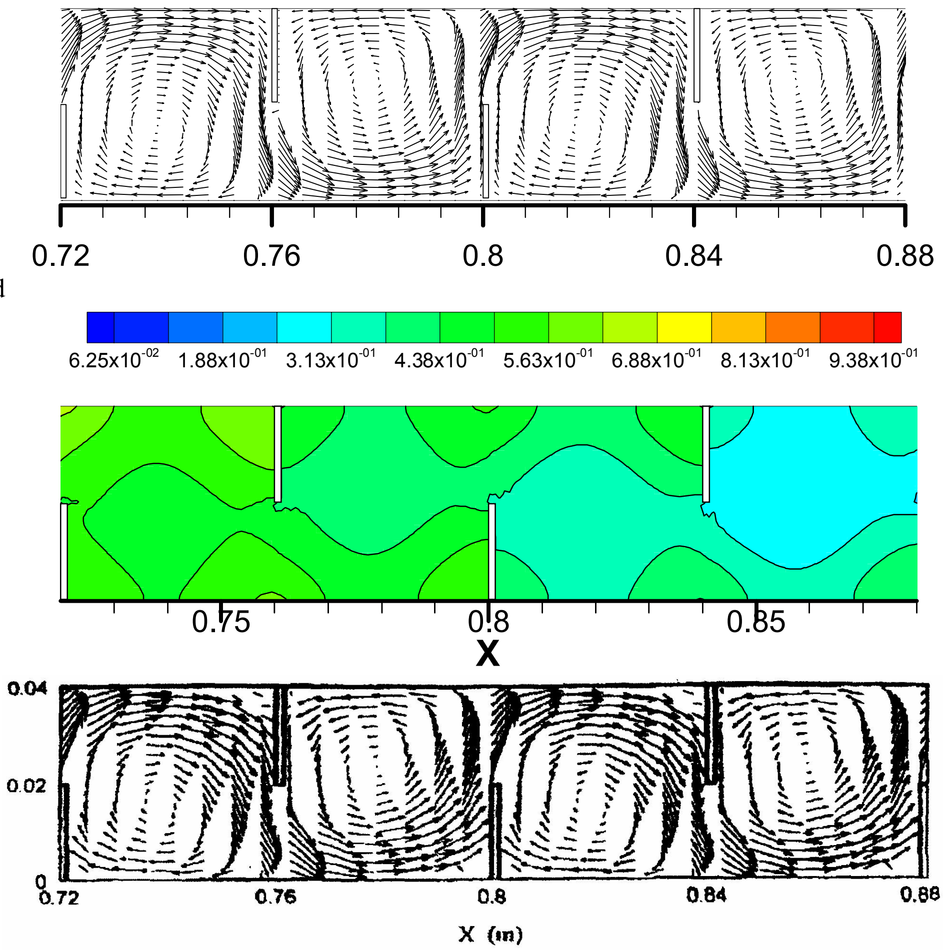
The jump condition at the interface is given by,

$$\left(\mu_{ef}+\mu_{t_\phi}\right)\frac{\partial\bar{u}_{D_p}}{\partial y}\bigg|_{0<\phi<1}-\left(\mu+\mu_t\right)\frac{\partial\bar{u}_{D_p}}{\partial y}\bigg|_{\phi=1}=\left(\mu+\mu_t\right)\frac{\beta}{\sqrt{K}}\bar{u}_{D_i}\Big|_{\text{interface}} \tag{13}$$

The non-slip condition for velocity is applied to all walls.

3. Numerical method

The numerical method utilized to solve the flow equations is the Finite Volume method applied to a boundary-fitted coordinate system. Equations (1)-(2) subjected to boundary and interface conditions (7)-(13), were discretized in a two-dimensional control volume involving both clear and porous media. The numerical method used in the resolution of the equations above was the *SIMPLE* algorithm of Patankar (1980). The interface is positioned to coincide with the border between two control volumes, generating, in such a way, only volumes of the types 'totally porous' or 'totally clear'. The flow equations are then resolved in the porous and clear domains, being respected the interface conditions mentioned earlier. Details of the numerical implementation can be seen in Pedras and de Lemos (2001b) and Silva and de Lemos (2003).



a) Velocity field

b) Pressure contours

c) Velocity field

Figure 2: Velocity vectors and pressure contours for solid-type baffles for *Re*=30000 and *h*/*H*-0.5: a-b) Present results, c) Yang and Hwang (2003).

4. Results and discussion

Figure 2 shows a comparison between the present results and the ones obtained by Yang and Hwang (2003), where 16 solid fins were placed along the 1400mm of the channel of Figure 1. The drawings shows the calculate velocity and pressure fields for Reynolds number equal to 30000. A qualitative comparison with results by Yang and Hwang (2003) indicates that velocity values match at corresponding locations inside the channel. As far as pressure values, no such distribution was reported in Yang and Hwang (2003). It is interesting also to observe that away from the entrance and within the section shown in Figure 2, the flow becomes spatially periodic along the main direction. Also, the flow is forced against the walls, which may improve heat exchange rates between the heated surfaces and the fluid. In the overall, one could say that the present work shows a good agreement with the work of Yang and Hwang (2003).

Statistical turbulent fields for k and ε are shown in Figure 3, also for Re=30000. As can be observed in Figure 3a, the largest levels of turbulence kinetic energy are close of the walls, opposed to the fin tips. It this wall region, the fluid is forced to impinge upon the walls, enhancing local mixing rates and converting mean mechanical kinetic energy into turbulence. The turbulence kinetic energy dissipation rate behaves quite similarly.

Table 1 finally shows the friction factor for the channel filled with fins of the solid and porous. Results are compared with data by Hwang (1997) and similar computations by Yang and Hwang (2003). For the experimental results obtained by Hwang (1997), the following correlation was used:

$$f = d_1 \mathrm{Re}^{d_2} \tag{14}$$

where d_1 and d_2 are coefficients given by: d_1 =31.64 and d_2 =0, *i.e.*, the friction factor is independent of the Reynolds number. As can be observed, the present result seems to be in good agreement with the correlation by Hwang (1997). Also interesting to emphasize are the conformity of the present numerical methodology with the literature for both cases involving either solid or porous baffles.

5. Concluding remarks

This work presented results for the numerical solution of turbulent flow in a channel containing solid and porous and baffles. Discretization of the governing equations used the finite volume method of and the set of algebraic equations was solved by the SIMPLE method. The presented numerical results were compared with found experimental data and numerical simulation in literature and indicated that a qualitatively good agreement was obtained.

6. Acknowledgments

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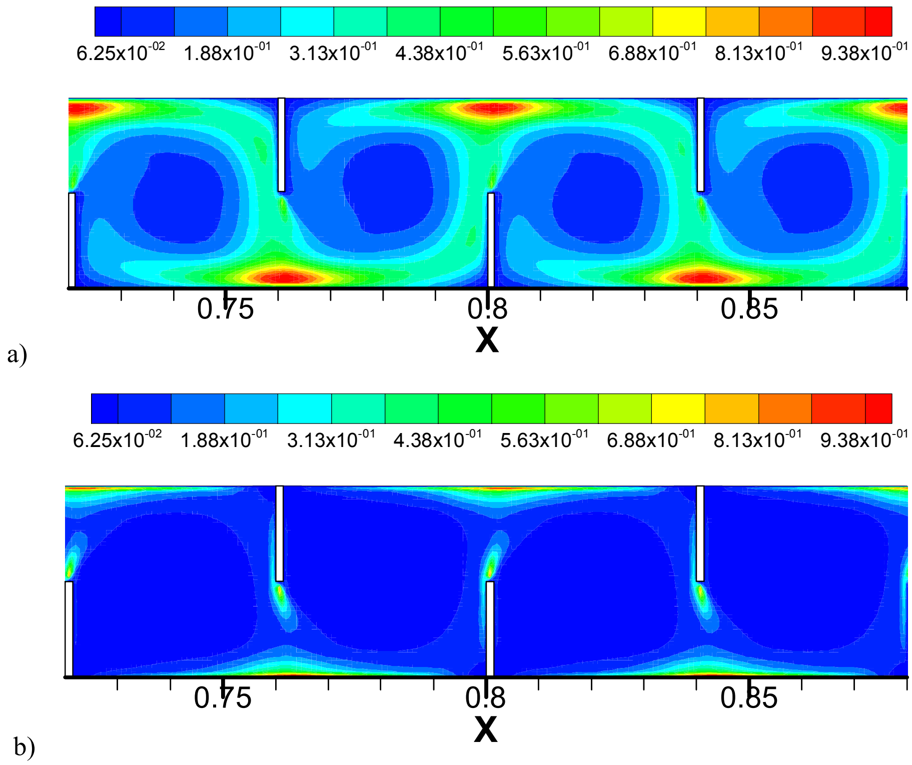

Figure 3: Statistical fields for solid fin case with Re=30000: a) k, b) ε

Table 1: Friction factors for channel of Figure 1 for porosity $\phi = 0.42$ and particle diameter $D_p = 0.72\times10^{-3}$m.

Re_H	h/H=0.25		h/H=0.5		h/H=0.75	
	Hwang (1997)	Present Results	Hwang (1997)	Present Results	Hwang (1997)	Present Results
30.000	3,68	5,98	22,75	24,18	49,88	59,39
50.000	3,44	4,71	22,75	24,97	49,88	58,17
			Solid Baffles			
			Hwang (1997)	Yang and Hwang (2003)	Present Results	
30.000			31.64	29.4	35.01	

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