

OPTIMUM SPACING IN AN ARRAY OF VERTICAL PLATES WITH TWO-DIMENSIONAL PROTRUDING HEAT SOURCES COOLED BY NATURAL CONVECTION

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Abstract. *A numerical and experimental analysis was performed to find an optimal spacing between plates that maximizes the heat dissipation, per unit volume, for a prescribed temperaure in a stack of parallel vertical plates cooled by natural convection. Each plate contains protruding two-dimensional heat sources that are sparsely distributed. It was varied both the distance between plates and power dissipated per plate. In the numerical analysis the SIMPLEC algorithm, based on the finite volume method, was used for solving the pressure velocity coupling. Numerical and experimental temperature profiles and optimum spacing values were compared and good agreements was observed.*

***Keywords**. Natural Convection, Protruding heat sources, Vertical Channels, Optimal spacing*

1. Introduction

Arrays of vertical printed circuit boards are frequently encountered in natural convection cooling of electronic equipment. Many studies have been carried out on natural convection heat transfer from vertical parallel plates with emphasis on the optimum thermal design of the cooling system. Bodoya and Osterle (1962) analytically derived a criterion for an optimum plate to plate spacing for which the heat dissipation is maximum. Levy *et al.* (1975) determined experimentally an optimum plate to plate spacing, which minimizes the plate temperature for a given rate of convective heat transfer. Using composite relationships Bar-Cohen and Rohsenow (1984) developed correlations for natural convective heat transfer in heated channels covering the two asymptotes, one for a fully developed channel regime and the other for a single plate boundary layer regime. An optimum spacing correlation was obtained by maximizing the total heat dissipation through the channel. Both uniform heat flux and uniform surface temperature boundary conditions were considered. For these cases, symmetric and asymmetric heating conditions were analyzed. Anand et al. (1992) determined the optimal spacing numerically by calculating the flow and temperature fields based on a finite-difference formulation of the governing equations. Bejan (1995), based on scale analysis, inferred the optimum spacing the maximize the heat transfer rate in an stack of isothermal smooth plates. It was obtained asymptotic expressions for heat transfer rate in the channel for the limits of fully developed flow and the boundary layer flow limit. The two expressions were plotted as a function of the plate to plate spacing , and the optimum spacing was estimated as the distance between plates correspondent to the position were the two asymptotes intersect. The method of solution used was proposed initially by Bejan (1984). Fujii et al. (1996) carried out numerical and experimental studies on natural convection heat transfer to air from an array of vertical parallel plates with protruding, discrete and densely distributed heat sources, and a procedure for estimating the optimum spacing of the parallel plates was discussed. Morrone et al. (1997) analyzed numerically the problem of optimizing the plate separation of an open, vertical channel, symmetrically heated, cooled by natural convection. The elliptic Navier-Stokes equations and the finite-difference discretization technique were applied. Geisler & Bar-Cohen (1997) derived composite relations for analytical modeling of heat transfer from arrays of vertically oriented populated Printed Circuit Boards, PCBs. They considered both isothermal and isoflux, single-sided, conditions. A pressure loss correlation which accounts for the effect of the chip packages on the fluid dynamics for laminar fully developed flow in a narrow channel was employed. The relations presented can be used to optimize the spacing between PCBs in order to maximize the total heat transfer from a given base area or volume.

The above studies deal with channels formed by smooth or densely distributed protruding plates. In the present work an optimum spacing between plates was estimated, numerically and experimentally, in an stack of vertical channels with protruding and not densely distributed two-dimensional heat sources. Numerical solutions were obtained to the full elliptic steady state Navier-Stokes equations using the SIMPLEC algorithm. A scale analysis was also presented in order to show the existence of an optimum spacing between plates. It was varied the distance between plates and total power dissipated per plate. Numerical and experimental temperature profiles and optimum plate to plate spacing results were compared and good agreements was observed.

2. Numerical Analysis

Figure (1) shows the physical model and coordinate system. An infinite number of plates is placed in a vertical parallel arrangement with equal spacing d. Each plate has the same height *l* and thickness b. On one surface of the plates it is mounted seven two-dimensional protruding heat sources separated by the distance s_p. The heat sources have the same dimensions. The total heat generation in each plate is set to be the same. The solution domain is chosen to be the region bounded by the broken line in Figure (1).

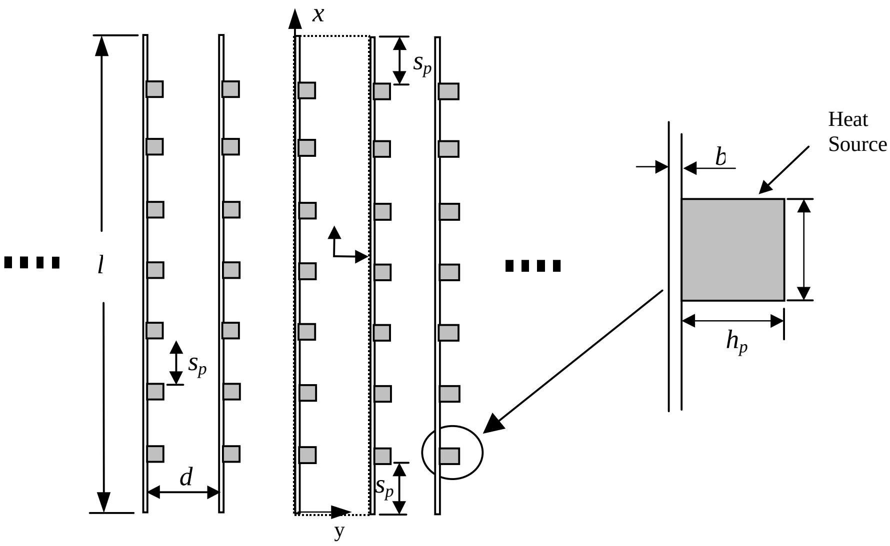


Figure 1 - Physical model and coordinate system.

Airflow is assumed to be at steady state, laminar and two-dimensional. The air thermo-physical properties are constant, except for the density in the buoyancy term of the momentum equation, which follows the Boussinesq approximation. The heat conduction in the plates and in the heat sources is taken into account. It is admitted uniform heat generation within the heat sources. The radiative heat transfer among the plates and the ambient is not accounted. The harmonic mean formulation suggested by Patankar (1980) is used to handle abrupt variations in thermophysical properties, such as the thermal conductivity across the interface of two different media.

The governing equations are expressed in dimensionless form as follow:

Continuity equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

Momentum equation in *X* direction

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{Pr}{Ra}\right)^{1/2}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\theta \tag{2}$$

Momentum equation in *Y* direction

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{Pr}{Ra}\right)^{1/2}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$

Energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{Pr}{Ra}\right)^{1/2}\frac{k_i}{k_{air}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+f\ x\ S^* \tag{4}$$

where:

- f=1 for the protruding heat sources and f=0 for the rest of the domain;
- k_i is thermal conductivity of the correspondent region. i=1,2 and 3, for air, plate and heating sources, respectively;
- The source term S* is being given by

$$S^*=\frac{2L}{npt\times XPT\times YPT\times\sqrt{Pr\,Ra}} \tag{5}$$

where npt is the number of protruding heat sources.

A unique form is used to express the energy equation in the fluid and the solid regions. A very high value for the dynamic viscosity was employed in the solid regions in order to make the velocities in these regions equal to zero.

The dimensionless variables in the above equations are defined by

$$X = \frac{x}{d}, \quad Y = \frac{y}{d}, \quad L = \frac{1}{d}, \quad B = \frac{b}{d}, \quad H_p = \frac{h_p}{d}, \quad L_p = \frac{l_p}{d},$$

$$\theta = \frac{T - T_i}{q''d / k_{air}}, \quad U = \frac{u}{u_o}, \quad V = \frac{v}{u_o}, \quad P = \frac{p - p_h}{\rho u_o^2} \tag{6}$$

$$Ra = \frac{q''qd^4\beta}{k_f \nu_f \alpha_f}, \quad Pr = \frac{\nu}{\alpha}$$

where *q"* is defined based on the total surface area of the plate as

$$q'' = \frac{Q}{2A} = \frac{Q}{2LW} \tag{7}$$

and the reference velocity, u_o is defined by

$$u_o = \left(d^2 g\beta \frac{q''}{k_{air}} \right)^{1/2} \tag{8}$$

The boundary conditions are:

Channel entrance (X = 0): $\theta = V = 0$; $P = -0.5U_m^2$

Channel Walls (Y = 0 and Y = B + 1): U = V = 0

Channel exit (Y = l/d): $\frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial \theta}{\partial X} = P = 0$

Periodic boundary condition was imposed with respect to the temperature at the plate surface, i.e: θ(X,0) = θ(X,B+1). The pressure values at the channel entrance and exit were obtained from potential flow theory.

The governing equations were discretized using the control volume formulation described by Patankar (1980), where velocity control volume are staggered with respect to the pressure and temperature control volumes. Coupling of the pressure and velocity fields was treated using the SIMPLEC algorithm (Van Doormal and Rathby, 1984), with the power-law scheme. The conjugate problem of conduction and convection was dealt by using the harmonic averaging thermal conductivity at the interfaces solid-fluid, Patankar (1980). The periodic boundary condition imposed with respect to the temperature at the plate surface was handled by using the CTDMA algorithm (Cyclic TriDiagonal Matrix Algorithm), from Patankar et al (1977), to solve the discretized energy equation. No specification of the wall temperature is required in this formulation. The equations were solved in a non-uniform grid crowded near the solid walls. The number of nodes was varied from 622~56 to 622~64, depending on the distance between plates. The convergence of the iterative procedure was tested by the following criterion

$$\frac{\left| \Gamma_{i,j}^{n} - \Gamma_{i,j}^{n-1} \right|_{max}}{\left| \Gamma_{i,j}^{n} \right|_{max}} \leq 5 \times 10^{-6} \tag{9}$$

where Γ stands for *U*, *V*, θ and the maximum residual in the continuity equation.

3. Experimental Analysis

An array of five fiber glass plates was accommodated in a metallic structure that is used in telecommunications devices and that allows variation of the distance between plates. The plates are numbered from 1 to 5 for convenience. Each plate was 365mm heigth (*l)* and 340mm width (w*)*, with 1,5mm thickness and it had seven heat sources mounted on its surface. The protruding heat sources were constructed from two aluminum bars 12,25mm height, 340mm width and 6,125mm thickness, with one resistance wire inserted between them. The elements resulted were screwed into the fiber glass plates and an equal spacing of 34,5mm was adopted. The protruding heat sources were connected in a way that any desired power level could be set to any given element, independently of the others. Power was supplied to the plates by regulated D.C. sources and both sides of the channels were closed to prevent lateral air flowing. In order to reduce the radiation heat transfer influence, the heat sources were polished with diamond paste. The structure was maintained about 1m from the ground and placed in a quiet room. Temperature measurements were obtained by using calibrated thermocouples 36 AWG type J, a switch and a digital thermometer. Special care was taken to embed the thermocouples in the aluminum and in the fiber glass surfaces. A very small hole was drilled in their surfaces, which was covered with a thin layer of thermal paste, and the thermocouples were fixed with epoxy adhesive. Experiments were performed varying the distance between plates and the total heat generation rate, Q_T, set the same for all plates during the tests. The distance between plates was ranged from 2 to 4cm, what corresponds to ratio (L = *l*/d) between 9 and 18, and the total heat generation per plate from 20 to 60W, corresponding to Rayleigh number values ranging from $1.x10^4$ to $8x10^5$. A schematic view of the experimental apparatus is shown in Fig. (2). It is the same used by Avelar (2001).

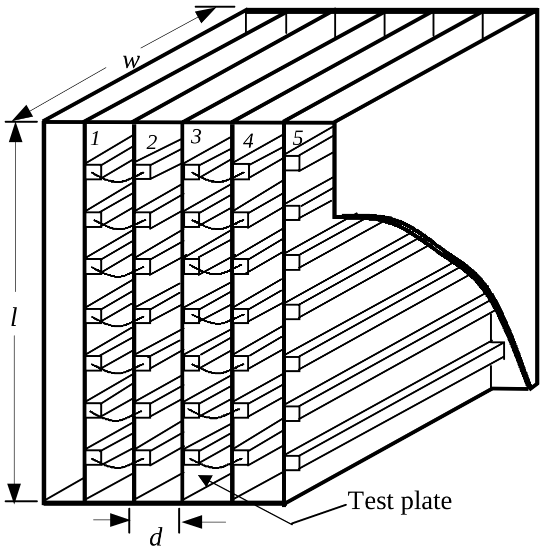
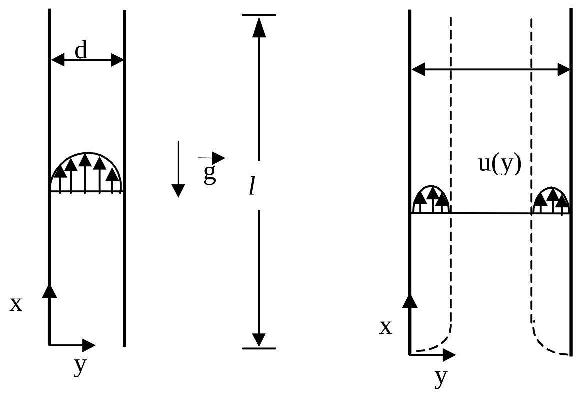


Figure 2 - Schematic view of experimental apparatus.

4. Optimum Spacing

For an allowable maximum temperature rise, the heat dissipated from a channel decreases with decreasing spacing, but the total number of plates increases. Hence, there is an optimum number of plates per unit volume (Peterson and Ortega, 1984).

The optimum spacing issue can be fully comprehended making use of a scale analysis similar to the one used by Bejan (1995) for the case of isotherm plates. Considering two asymptotic limits for natural convection flow in vertical channels illustrated in Fig (3)



(a) fully developed flow. (b) One plate isolated in an infinite medium.

Figure 3 - Asymptotic limits for natural convection flow in vertical channels.

Limit (a): l/d → ∞

Admitting that the channel is very long, it can be considered the fully developed flow hypothesis

$$v = 0 \text{ and } u = u(y) \tag{10}$$

Furthermore, from the momentum equation in *y* direction, it can be showed that the pressure in the fully developed region is a function of *x* only. Hence, the momentum equation in this direction is

$$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx} + \rho \ g \tag{11}$$

Since both ends of the channel are open to the ambient of density ρ_o, the pressure gradient can be considered the same as the hydrostatic external field

$$\frac{dP}{dx} = -\rho_o g \qquad (12)$$

Using the Boussinesq approximation for the density difference *(ρ -ρ_o)* yields,

$$\mu \frac{d^2u}{dy^2} = \rho g \beta (T_o - T) \qquad (13)$$

Considering that the order of magnitude of the buoyancy term can be determined by substituting the fluid temperature, *T*, by the bulk temperature, T_f

$$\mu \frac{d^2u}{dy^2} = -\rho_o g \beta (T_f - T_o) \qquad (14)$$

The solution of Eq (14) provides a parabolic velocity profile, similar to the Hagen-Poiseulle flow, which integrated yields the mass flow rate per channel unit depth

$$\dot{m} = \frac{\rho g \beta d^3 (T_f - T_o)}{12\nu} \qquad (15)$$

In a uniform heat flux channel, the integral energy balance for a fluid control volume in the channel, from the channel entrance up to a *x* position, yield

$$\dot{m} c_p (T_{f,x} - T_o) = 2xq'' \qquad (16)$$

Then, the term *(T_f– T_o)* in Eq (15), can be replaced by its mean value along the channel (Bar-Cohen and Rosenow, 1984), which corresponds to the value in the middle of channel, *x =l/2*,or

$$\overline{(T_f - T_o)} = \frac{lq''}{\dot{m} c_p} = \frac{T_{f,l} - T_o}{2} \qquad (17)$$

Replacing Eq (17) in Eq (15), we have that in the case of channels submitted to the uniform heat flux condition, the mass flow rate is given by

$$\dot{m} = \frac{\rho g \beta d^3 (T_{f,l} - T_o)}{24\nu} \qquad (18)$$

The total heat transfer rate in a channel, *q'*, is determined making *x = l* in Eq. (16)

$$q' = 2lq'' = \dot{m} C_p (T_{f,l} - T_o) \qquad (19)$$

In the limit l/d $\rightarrow \infty$, the difference between the fluid mean temperature and wall temperature is small, in terms of order of magnitude, when compared with the difference between this temperature and the air temperature in the channel entrance. Hence,

$$(T_{f,l} - T_0) \quad \approx \quad (T_{w,l} - T_0) = \Delta T \qquad (20)$$

where $T_{w,l}$ is the wall temperature in the position *l*. Manipulating Eqs (18), (19) and (20) it is obtained

$$q''_a = \frac{\rho g \beta d^3 C_p \Delta T^2}{48 \nu l} \qquad (21)$$

Limit (b): l/d$\rightarrow$0

For a isolated vertical plate uniformly heated in an infinite medium, from Bejan (1995) we have

$$\frac{q''}{(T_{w,x} - T_o)} \frac{x}{k} = 0.6 \left(\frac{g\, q'' \beta}{\alpha} \frac{x^4}{\nu\ \ k} \right) \quad \text{, or} \qquad (22)$$

$$q''_b = 0,6 \frac{\Delta T k}{l} Ra_l^{1/4} \tag{23}$$

Where

$$Ra_l = \frac{g\beta l^3 \Delta T}{\nu\alpha} \tag{24}$$

Now, considering a set of parallel plates with depth *w*, as indicated in Fig (4)

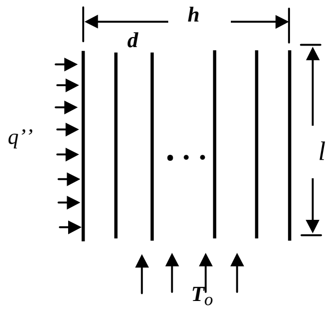


Figure 4 – Array of vertical plates cooled by natural convection.

The overall heat dissipated in a stack with (*h/d*) plates, for the limit (a), results in

$$q_a = q''_a \, 2lw\left(\frac{h}{d}\right) = \frac{\rho g \beta c_p \, d^3 \Delta T^2}{24\nu} w\left(\frac{h}{d}\right) \quad \Rightarrow \quad q_a \; \alpha \; d^2 \tag{25}$$

and for the limit (b),

$$q_b = q''_b \, 2lw\left(\frac{h}{d}\right) = 1{,}2\, k\, \Delta T\left(\frac{h}{d}\right) w\, Ra_l^{1/4} \quad \Rightarrow \quad q_b \; \alpha \; d^{-1} \tag{26}$$

The power law that drives q_a e q_b variation with the distance *d* indicates that a maximum value of the total heat dissipated in the set will take place when the distance between plates be such that

$$q_a \cong q_b \tag{27}$$

Solving Eq (27) it is obtained

$$d_{opt} \cong 3{,}06\, l\, Ra_l^{-1/4} \tag{28}$$

Which replaced in Eq. 25 or 26 yields,

$$q_{max} \cong 0{,}39\, k\Delta T\left(\frac{hw}{l}\right) Ra_l^{1/2} \tag{29}$$

Hence, the maximum value of heat dissipated per unit volume and temperature difference is

$$\frac{q_{max}}{lwh\,\Delta T} \cong \frac{q'''}{\Delta T} \cong 0{,}39\left(\frac{k}{l^2}\right) Ra_l^{1/2} \tag{30}$$

This result give a scale that allows defining a dimensionless heat dissipated per unit volume and temperature difference as

$$q^* = \frac{q''' l^2}{k\Delta T Ra_l^{1/2}} \tag{31}$$

The optimum spacing was estimated numerically and experimentally from maximum values of temperature excess in the plate for each value of power and distance between plates analyzed.

5. Results and Discussion

Figures (5) and (6) show temperature excess profiles, for the situation of uniform heating of the plates, for several values of power, and distance between plates equal to 2 cm and 4 cm. The temperature excess, *ΔT*, is defined as the difference between the heat source temperature and the air inlet temperature, T_o.

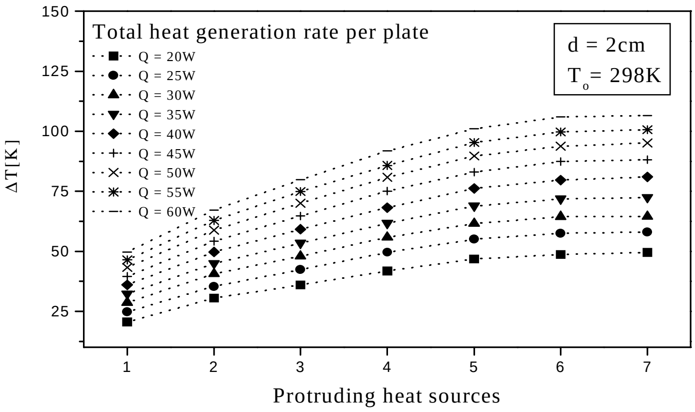


Figure 5 - Temperature excess profiles - d = 2 cm.

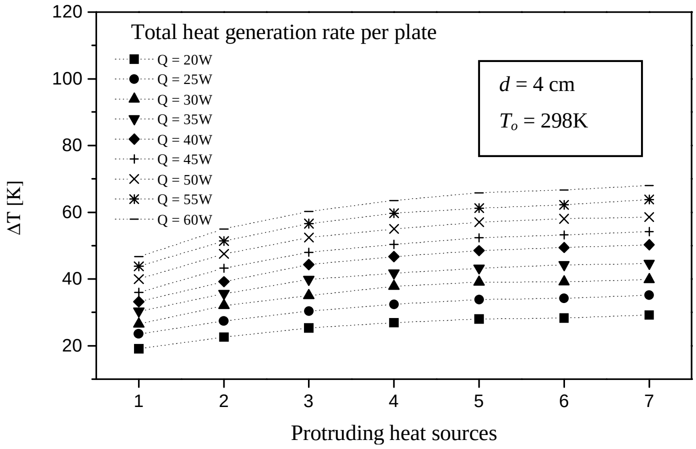


Figure 6 - Temperature excess profiles - d = 4 cm.

From Fig. (5) it can be observed that for the smallest plate spacing, 2cm, the temperature profile is almost linear from the second to the fifth element for all values of total heat generation rate. This behavior was described by Kelkar and Chouddhury (1993), who numerically investigated the periodically fully developed natural convection in a vertical channel with equally spaced surface mounted heat generation blocks. When this regime is established, the flow pattern in each module is the same, and the temperature difference between correspondent points in adjacent modules is constant. On the other hand, for large values of the plate to plate distance, the temperature profile approaches that corresponding to a vertical plate exposed to an infinite medium. The less accentuated temperature profile, displayed in Fig (6) for the distance d = 4 cm, can be explained from the Nusselt number correlation for a vertical wall subject to a surface condition of uniform heat flux (Bejan, 1995), for which the temperature excess is proportional to the vertical distance raised to the $1/5^{th}$ power.

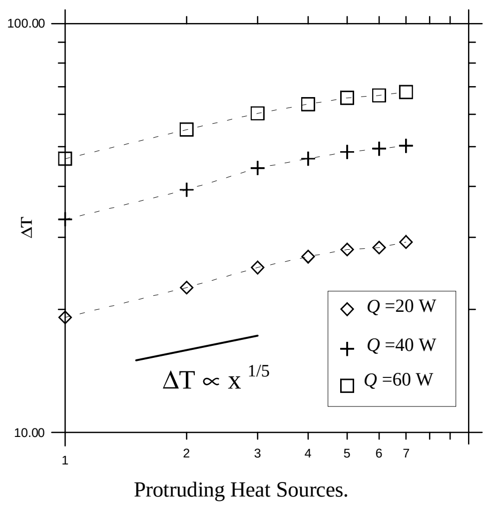


Figure 7 – Experimental values of Temperature excess – d = 4 cm.

As illustrated in Fig. (7), this power law can be verified by presenting the experimental values of the excess temperature, for a large value of the distance d, in a logarithmic scale.

In Fig. (8) and (9) it is compared numerical and experimental values of dimensionless temperature excess for the value of total heat generation rate per plate equal to 25W and plate spacing equal to 2cm and 3,5cm.

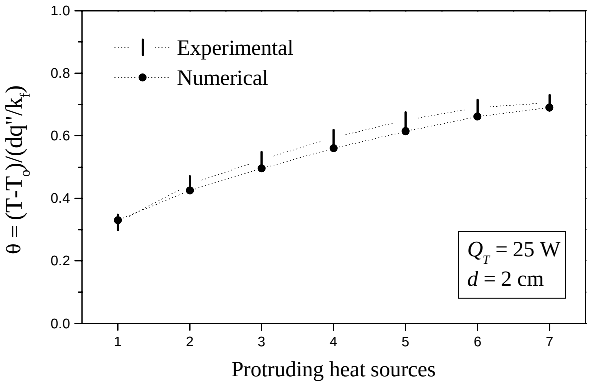


Figure 8 - Numerical and experimental values of temperature excess – Q = 25 W – d = 2cm.

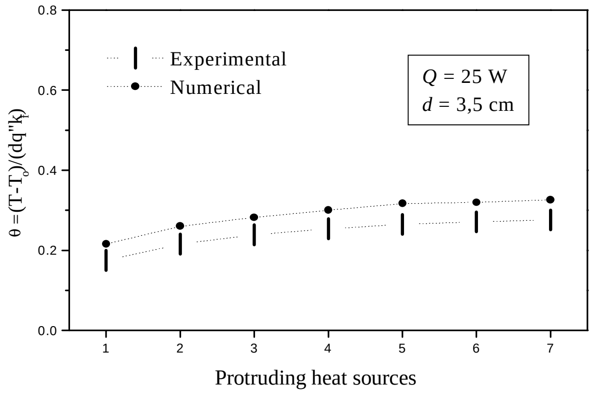


Figure 9 - Numerical and experimental values of temperature excess – Q = 25 W – d = 3,5cm.

As can be noted from Fig. (8) and Fig. (9), good agreement was observed between numerical and experimental results. For the distance 2cm differences of about 8% were observed, while for the greatest distance 3,5cm difference were around 15%.

Figures (10) and (11) present isotherms, obtained from the numerical analysis, for the distance between plates equal to 2cm, and 3,5cm and total heat generation rate of 20W and 35W. The channel is displayed in the horizontal direction for convenience.

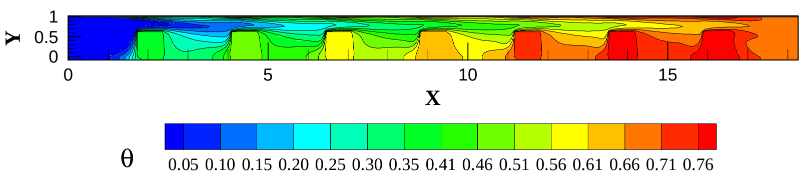


Figure 10 - Isotherms - *d* = 2cm - *Q* = 20W.

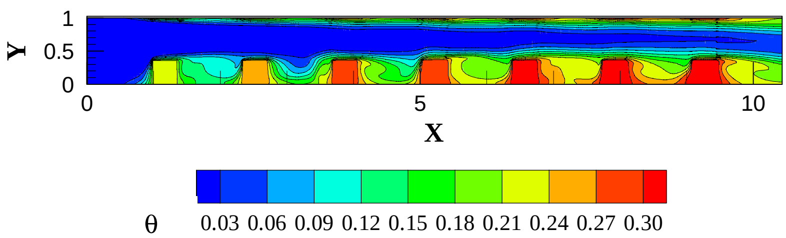
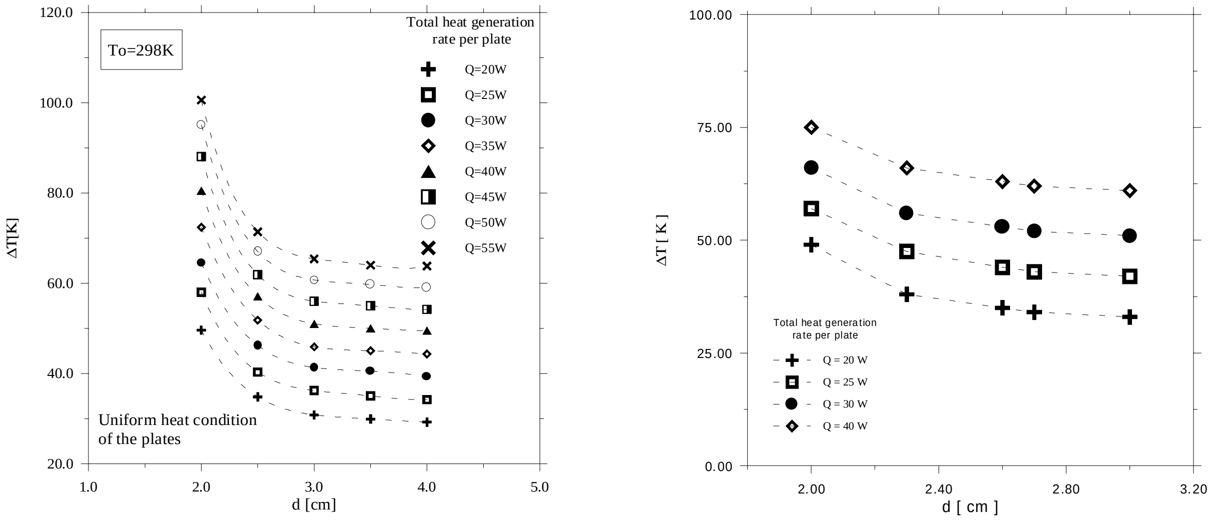


Figure 11 - Isotherms - *d* = 3,5cm - *Q* = 25W

By comparing Figs. (10), and (11), it can be noticed a longer unheated fluid length for the distance of 3,5cm, with the flow in the channel approaching the condition of a plate in an infinite medium. These figures shows also that the isothermal lines are densely distributed on the surfaces that are parallel to the main flow direction and near the protrusions bottom corners. What indicates that the heat transfer rate are higher in these regions.

Figures (12 a) and (12 b) illustrates experimental and numerical maximum values of temperature excess, verified in the heat source closer to the channel exit, for several values of heating generation rate per plate.



(a) Experimental Values. (b) Numerical Values.

Figure 12 – Maximum values of temperature excess in the plate – experimental results.

For a specified value of maximum temperature excess of 50K, the results presented in Fig (12) were graphically \interpolated and the value of the plate-to-plate distance correspondent to the specified value of temperature excess was achieved. From this value it was obtained the heat dissipated in a channel per unit volume, given by

$$q''' = \frac{Q}{ldw} \tag{32}$$

which was made dimensionless by using Eq. (31).

Numerical and experimental dimensionless values of heat flux, *q**, are presented in Fig. (13) for the specified value of temperature excess of 50 K as a function of the dimensionless value of the distance between plate, *D*. The optimum spacing corresponding to the distance between plates for which the highest *q** value can be obtained.

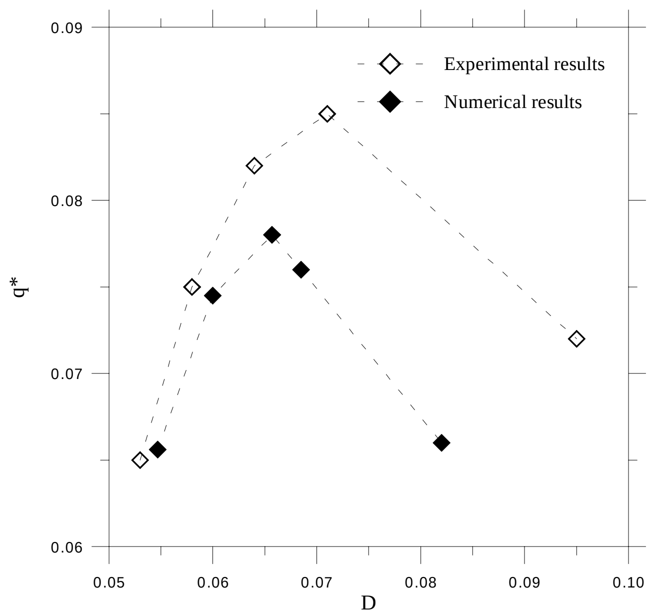


Figure 13 – Optimum spacing between plates.

From Fig. (13) it is possible to perceive the existence of an optimum distance between plates from both numerical and experimental results. The optimum spacing found experimentally and numerically are in a reasonable agreement and the differences observed are due to the fact that the curve of numerical results does not have as many points as the experimental curve. That is because of the difficulty in running the numerical program, which is very time-consuming.

It can also be noticed from Fig. (13) that the maximum value of q^* obtained from the experimental values is around 0,085, while from Eq. (30) the maximum value was found equal to 0,39. This discrepancy occurs because the scale analysis that yielded Eq. (30) was carried out for the case of uniformly heated smooth plates and in the present work the plates have protruding heat sources sparsely distributed. However, the scale analysis showed that it is adequate to identify a correct scale for the maximum value of heat dissipated in the channel per unit of volume and temperature difference.

6. Conclusions

The results obtained from the present analysis suggest the following conclusions:

As distance between plates increases, the temperature profile in the channel decreases rapidly, approaching that corresponding to a vertical plate exposed to an infinite medium. It was verified numerically and experimentally;

An optimum spacing was estimated numerically and experimentally and a reasonable agreement was observed;

For the smallest plate to plate distance, 2cm, it was verified that the flow is periodically fully developed between the second and the fifth protruding heat source, i. e., the flow patter is almost the same around each element, and the temperature variation between correspondent points in adjacent elements is constant;

Good agreement between numerical and experimental values of temperature excess was observed. For the distance between plates of 2cm, differences of about 8% were observed, while for the greatest distance, 3,5cm differences were around 15%.

7. Acknowlewdgents

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