

FE-MODEL UPDATING OF BEAM STRUCTURE PRESENTING NON PROPORTIONAL DAMPING

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Abstract. *This work discusses the effects considering damping in a finite element model updating of flexible structures. The finite element results are compared with experimental data in order to evaluate the correlation of the models and, subsequently, it is updated aiming at defining a very reliable FE-model. The formulation of the updating equations is based in the force balance equation and the set of equations is solved in an iterative way. The complex character of the measured receptances is incorporated in the formulation of the problem, taking the mechanism of non proportional damping. The updating parameters of the model can be its geometrical/physical parameters and also the coefficients of proportionality. The results show a great improvement of the correlation of the FRF(s) of the models, also improving the confidence of the analyst in the finite element model.*

Keywords: *FEM, Modal Analysis, Damping, Correlation, Model Updating.*

1. Introduction

With the advent of the modern computation, the finite element modelling has been widely used in several areas of engineering manly due to its great versatility and facility for simulation of different operation conditions and loading of a system, in a fast way and with low cost. The mathematical model formulated by finite element, usually, it involves an estimating of several parameters of the physical model and, in some situations, these parameters are roughly known. These parameters, typically associated with the stiffness and mass characteristics, demand an updating when one needs better representation of the dynamic behavior of the model, and also the damping characteristics that is often not taken into the FEM, Schaak (2000).

The model created from the finite element method is defined by a set of differential equations of movement represented in a matricial form. In the case of structural dynamics, usually they are the mass and stiffness matrices. These matrices are calculated in terms of physical and geometric parameters of the structure. If these parameters are not known with enough accuracy, they result in an analytical model that it presents a dynamic behavior discrepancy, if compared with the real physical model (Lammens, 1995; Pereira, 1996; Mottershead et all, 2000). Another aspect to be considered in the definition of the dynamics behavior of the analytical model is the effect of the damping, that is always present in the experimental data.

The characterization of damping forces in structural vibration has been a active research area in structural dynamics. However, the understanding of the mechanisms of damping is still very limited and the finite element models usually do not include damping (Ewins and Inman, 2001; Friswell Mottershead, 1998). In this context, the purpose of this work is to incorporate a model of viscous non proportional damping in the finite element modelling aiming at representing the effect of the damping present in the structure. Correlation techniques will be used to analyze the reliable of the model and depending on the discrepancy of the models, the parameters of the finite element model will be updated based on the experimental data. It is also shown how the updating of damping parameters can improve the correlation of the models, manly the FRF(s) correlation.

2. Theoretical Fundaments

In the nature there exist different types of damping mechanisms like viscous, hysteretic, Coulomb, etc. In mechanical structures the effect of damping could come from a simple mechanism of damping or even from a combination of different mechanisms that are responsible for the dissipation of energy of the system, usually, in the form of the heat, Richardson and Potter (1975). Different mathematical models can be used to represent that loss of energy, and the viscous model has been widely used, in this case, mainly because its simplicity and pratice for modal analysis purpose. This approach could represent the effect of proportional or non proportional damping properties of a structure. The effect non proportional is found mainly in structures constituted by regions with different types of materials and in structures that present joints and connections, Clough and Penzien (1993).

The proportional damping describes the damping effect as being distributed uniformly along the whole structure and the non proportional ones refers to the damping as being distributed distinctly in each region of the structure. In the first case, the damping could be defined as proportional the mass matrix by a coefficient α, or the stiffness matrix by a coefficient β, or in most general case, proportional to the mass and stiffness matrix as described by the eq. (1).

$$[C] = \alpha[M] + \beta[K] \tag{1}$$

From the ortogonality properties of the mass and stiffness matrices, equation (1) permits one defines the modal damping ratio in terms of the natural frequency and proportionality coefficients, eq. (2), Clough and Penzien (1993). In this case, the damping ratio is inversely proportional to the frequency by the coefficient α and directly proportional to the frequency by the coefficient β:

$$\zeta_r = \frac{\alpha}{2\omega_r} + \frac{\beta\omega_r}{2} \tag{2}$$

For the case of non proportional damping, the damping matrix of the model is defined from the damping of each different regions of the model. In this case, the distribution of the damping is not proportional in the whole model, but it is assumed to be proportional for each distinct region of the model. It can be defined a damping matrix proportional to the mass and stiffness matrices of each distinct region of the model and the global damping matrix is defined by a combination of damping of each region. Figure (1) shows schematically the mass, stiffness and damping matrices for a structure that presents different coefficients of proportionality for *a* and *b* regions.

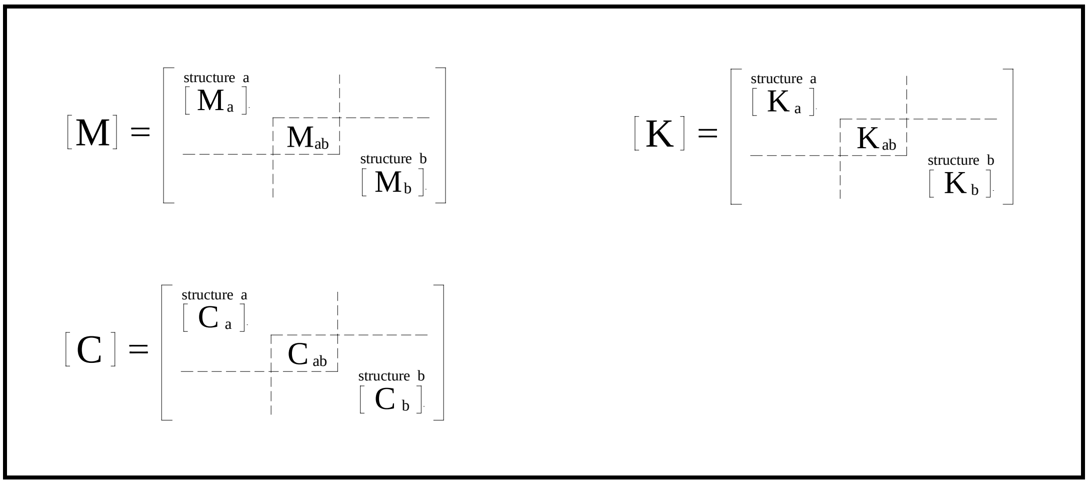

Figure 1. Matrices of the system with combined properties

An interesting aspect to be enphasize here, is that in the proposed updating methodology, the parameters are updated without affecting the properties of connectivity of the matrices. So, it permits the engineer to have a physical interpretation of the variation of the updated parameters and consequently, an evaluation of the physical consistence of the model. That is quite relevant in the updating of structures that present joints and connections or structures constituted of more than a type of material. Structures of this nature, present a different distribution of damping at each regions and the distribution of the forces of damping will be not uniform.

The damping of each region of the model is calculated from the expression (3) and subsequently they are combined to define the whole structure:

$$[C_k] = \alpha_k[M_k] + \beta_k[K_k] \tag{3}$$

Where: $k=1,2,...,nmat$;
nmat - different materials and/or regions.

For the case of non proportional damping, the matrix of the whole structure cannot be diagonalized and it will present a coupling in the coefficients C_{ij} $(i \neq j)$. In some situations, an approximate solution can be obtained by ignoring the off-diagonal coupling coefficients of modal damping matrix, Clough and Penzien (1993).

2.1. Model Correlation

The representativity of FEM can be evaluated through correlation techniques. In this work, the correlation of the analytical and experimental models will be evaluated through the relative difference of the natural frequencies, comparison of mode shapes and the comparison of FRF(s).

The correlation of the natural frequencies can directly be expressed by relative differences:

$$\varepsilon_{\omega} = \frac{\omega_r^A - \omega_r^E}{\omega_r^E} \times 100\% \tag{4}$$

Where: ω_r^A - r-th analytical natural frequency;

ω_r^A - r-th experimental natural frequency;

ε_{ω} - frequency residue.

The mode shapes correlation can be expressed by using the well known Modal Assurance Criterion (MAC):

$$MAC = \frac{\left| \{\psi^E\}^T \{\psi^A\} \right|^2}{\left(\{\psi^A\}^T \{\psi^A\}\right) \left(\{\psi^E\}^T \{\psi^E\}\right)} \tag{5}$$

Where $\{\psi^E\}$ and $\{\psi^A\}$ are experimental and analytical mode shapes respectivily. MAC iqual 1 indicates a perfect correlation between the two modes, while MAC iqual 0 indicates that the two modes do not show any correlation.

The correlation of the models, through FRF(s) it can be evaluated through the visualization of the superposition of the analytical FRF and measured one, in the equivalent points of the model, and also, its correlation can be quantified through Frequency Response Assurance Criterion (FRAC), which can be defined for the *j-th* degree of freedom as:

$$FRAC_j = \frac{\left| \{H^E(\omega_k)\}_j^T \{H^A(\omega_k)\}_j \right|^2}{\left(\{H^E(\omega_k)\}_j^T \{H^E(\omega_k)\}_j\right) \left(\{H^A(\omega_k)\}_j^T \{H^A(\omega_k)\}_j\right)} \tag{6}$$

Where $\{H(\omega_k)\}$ is a row vector describing the receptance for all frequencies ω_k . The value of FRAC iqual 1 indicate a perfect correlation between the analytical and experimental FRF(s), while value iqual 0 indicates that the two FRF(s) do not show any correlation. A large difference of magnitude of the FRF(s) caused by the damping, can result in low FRAC-values (< 0.1) even they visually presents acceptable correlation, as discussed in Lamnens (1995).

2.2. Updating Process

The proposed technique has been discussed in previous papers (Pereira and Borges, 2001; Doi and Pereira, 2002) and the detailed formulation of the problem will not be presented. Only the principle and the main characteristic of the procedure will be discussed. The procedure operates at an element level or macro region of the model and the changes, during the updating process, could directly be related to the variation of the physical parameters of each element or macro region of the model. As the procedure makes the updating at local level, the symmetry, positive definiteness and connectivity characteristics of the FEM remain uncorrupted and the updated model can easily be compared with the original one. This feature facilitates the physical understanding of the model changes. The procedure describes the discrepancy between analytical and experimental models as a force residue defined in term of an error function based on the input variables (force).

$$\varepsilon(\{p\}) = \{F^X\} - \{F^A\} \tag{7}$$

Rewriting equation (7) in terms of the dynamic stiffness matrix $[Z^A]$, it gives:

$$\varepsilon(\{p\}) = \{F^X\} - [Z^A(\{p\})]\{X^A\} \tag{8}$$

where: $[Z_k^A] = [K] + i\omega_k[C] - \omega_k^2[M] = [H(\omega)]^{-1}$ - dynamic stiffness matrix;

$[H(\omega)]^{-1}$ - receptance matrix;

p - parameters of the analytical model.

By using Taylor's expansion, the residue expression could be defined, after some re-arrangement, in terms of the first order sensitivity of the dynamic stiffness matrix and the difference of the analytical and experimental receptance.

$$\{\varepsilon(\{\Delta p_i\})\} = \sum_{i=1}^{np} [H^A] \frac{\partial [Z^A]}{\partial p_i} \Delta p_i \{H^X\}_j + \{\Delta H\}_j \tag{9}$$

The variables Δp's are parameters corrections that minimise the discrepancy between the two models. An approximated solution of the problem is obtained by using the least square approach to solve the residue function for the selected updating parameters, p_i. In a compact form it becomes:

$$[S_k]\{\Delta p\} = \{\Delta H_k\}_j \tag{10}$$

Equation (10) is valid at the updating frequency point's ω_k, and it defines a set of m-equations in the updating parameters. The selection of these updating frequencies does not attend a systematic criterion. It should be define a minimum number of frequencies able to provide a reference receptance matrix that contains, in a well-balanced way, all relevant information contained into the full measured receptance matrix, Pereira (1996). The updating set equation is solved using the SVD (Singular Value Decomposition) method and the changes in the parameters Δp_i are used to obtain, in an iterative way, an estimate of the updated mass, stiffness or damping matrices. For the case of damped model, the updating is accomplished in two stages:

- In the first stage the model is updated in terms of the mass and stiffness matrices, without considering the damping matrix, i.e., the variation of the dynamic stiffness matrix is caused by the mass and stiffness matrices correction in the parameter's p_i:

$$[\Delta K] = [K(p_i + \Delta p_i)] - [K(p_i)] \tag{11}$$

$$[\Delta M] = [M(p_i + \Delta p_i)] - [M(p_i)] \tag{12}$$

- In the second stage, the coefficients of proportionality α and β are updated and the dynamic stiffness matrix is corrected take into account the damping matrix change:

$$[\Delta C] = \left((\alpha_i + \Delta\alpha_i)[M(p_{UPD})] - \alpha_i [M(p_{UPD})] \right) + \left((\beta_i + \Delta\beta_i)[K(p_{UPD})] - \beta_i [K(p_{UPD})] \right) \tag{13}$$

It should be emphasised, since updating is provided at element or region level the physical and geometric parameters, are maintained unaffected during the updating of the proportionality coefficients α and β, and the matrix of damping is updated in terms of the variation of the coefficients α and β. These coefficients can be also updated at elements level or regions of the structure, what makes possible the updating of the proportionality coefficient for the different region of the model simultaneously, i. e., non proportional damping.

3. Case Studied: Beam with Adhesive - Non Proportional Damping

A beam structure has been used to evaluate damping effect in a FRF-based finite element model updating approach. An analytical modelling by finite element and a modal test of the structure were carried out and the results were compared aiming at defining the correlation of the models. It is also discussed how updating of damping parameters could improve the correlation between the analytical and mesaured FRF(s) of the model.

The structure consists of a steel beam of rectangular section of 31 mm x 4.5 mm and 1 m long. The analytical modelling of the structure uses beam elements. The model contains 21 nodes and 20 elements, analysed in a free-free condition. Table 1 shows the material and geometrical properties of the structure.

Table 1. Material properties (steel)

Properties	Values
Modulus of elasticity	210 e 9 [N/m^2]
Specific mass	7860 [kg/m^3]
Poisson's coefficient	0.30

The experimental data were obtained from an experimental modal test. The structure was excited by impulsive force (hammer) applied in a point located in the position 0.3 m from the extremity in a vertical direction *y*, node 4. Responses were captured in the same direction of excitation, using axial accelerometers distributed along of the beam. The measuring were made in a frequency range of 0 to 600 Hz, at a reduced number of points when compared to the finite element model. Figure 3 shows a scheme of the structure.

The damping properties of the model were modified by using an adhesive tape manufactured by the 3M. This type of material is used in fuselages of airplanes, blade of turbines and other vibrant surfaces aming at reducing noise and vibration level of the system. The form and amount of adhesive in the structure will permit to simulate a condition of non proportional damping in the model. Figure (2) shows the application of the adhesive in beam.

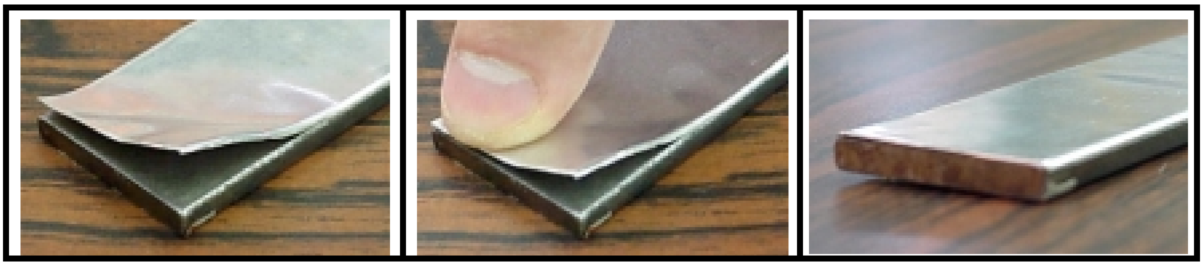

Figure 2. application of the adhesive

The adhesive tape was glued in the two faces of the beam but only in some regions of the model (elements 6, 7 and 8) as it can be seen in the fig. (3). In this case it causes a effect of non proportional damping in the structure.

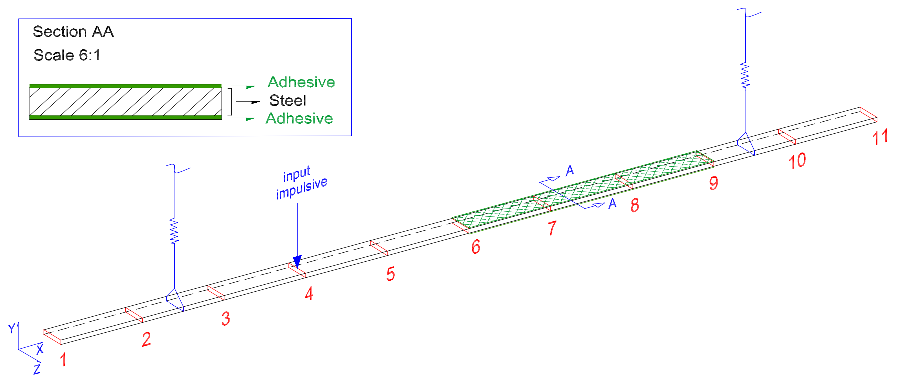

Figure 3. Structure test with non proportional damping

3.1. Updating of the Geometric Parameters

The initial comparation of the models shows a discrepancy between the analytical and experimental results, which demands an updating of the FEM.

The updating were carried in two stage: in the first stage the updating consists of updating the parameters of the FEM without considering the effects of damping. In this case, the selected updating parameters were the geometric parameters of the beam since the material properties were assumed correct. These parameters were updated independenty for each region of the model, region one contains elements (1, 2, 3, 4, 5, 9 and 10) and region two contains the rest.

Figure (4) shows superposition of FRF(s) of the experimental, analytical and updated models. The updated parameters are inertias in *z* and the cross section areas of the two regions. Figure (5) shows the evolution of the 4 updated parameters.

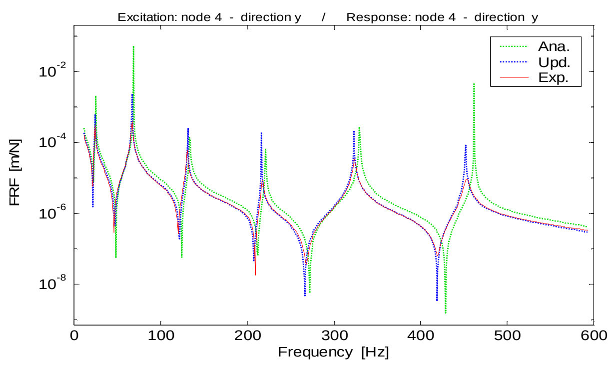

Figura 4. Experimental, analytical and updated FRF(s)

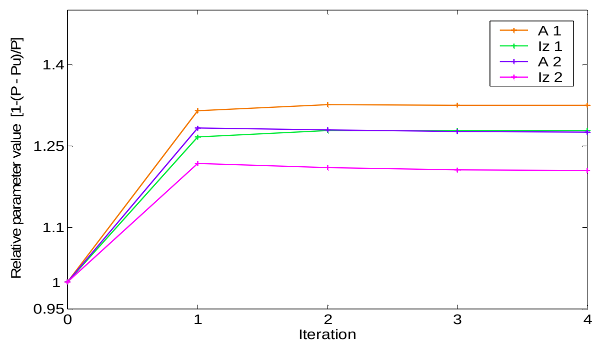

Figura 5. Evolution of the updated parameters

The table (2), shows the values of the updated parameters of two regions of the model, cross section area and inertia in relation to the axis z. It is important to observe that the region 2 should have cross section area and inertia larger than the region 1, to maintain the physical consistence of FEM.

Table 2. Values of the updated geometric parameters

Parameters	Estimated values	Updated values
A_1	144.9 e-6	199.1 e-6
I_{Z1}	255.5 e-12	340.2 e-12
A_2	151.2 e-6	206.0 e-6
I_{Z2}	290.3 e-12	370.8 e -12

Through the table (3), it can be seen that the initial correlation of the mode shapes is very good, MAC-values close 1. However there exist a differences in the natural frequencies of the model, that although small (around 2%), they are considerable in the case of a simple structure as a beam. After the updating of the parameters, those differences of frequencies decreased significantly and the MAC-values stayed very good as it can be seen in tab. (3).

Table 3. Modal parameters and correlation of the models before and after the updating

Experimental model		Analytical model			Updated model		
Mode	ω_{EXP} [Hz]	ω_{ANA} [Hz]	$\frac{\omega_{ANA}-\omega_{EXP}}{\omega_{EXP}}$ (%)	MAC (%)	ω_{UPD} [Hz]	$\frac{\omega_{AJU}-\omega_{EXP}}{\omega_{EXP}}$ (%)	MAC (%)
1	24.3	25.1	3.72	99.8	24.5	0.82	99.8
2	67.2	68.7	2.23	99.9	67.2	0.00	99.9
3	130.3	133.9	2.76	99.8	131.2	0.69	99.8
4	217.1	220.9	1.75	99.6	216.4	-0.32	99.6
5	323.8	329.9	1.88	99.9	323.3	-0.15	99.9
6	453.2	461.7	1.87	99.9	452.1	-0.24	99.9

The correlation of the models, after the first stage of the updating process, is very good comparing the frequencies and mode shapes. However, it shows not a good results for the FRF(s). Figure (6) shows that the correlation of the FRF(s) is very low, FRAC-valores are around 0.1. These lowers values are caused mainly by the discrepancies of amplitudes of FRF(s) in the resonances, as it can be verified in the fig. (4).

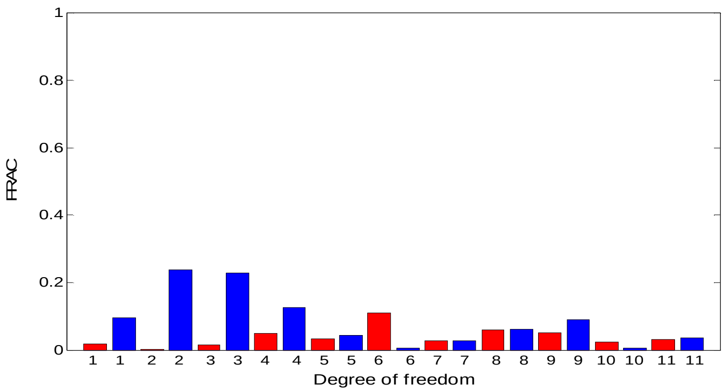

Figure 6. FRAC-values before and after the updating of the geometric parameters

3.2. Updating of Damping

In the second stage, the updating focus only in the proportionality coefficients of damping, without considering the updated parameters of the first stage. It is worth to point out that introduction of the damping in the mathematical model causes a small reduction in the resonance frequencies that it does not compromise the updating of the first stage. Since the damping is non proportional, the damping matrix is not diagonal and the damping ratio cannot also be calculated as in the model of proportional damping. The values of the parameters modal of the analytical model were obtained by modal analysis from the analytical FRF(s). In this case, the form of obtaining those parameters is quite convenient, since the analytical FRF(s) contain the damping effect. The coefficients α and β estimated initially increased their values after the updating, tab. (4).

Table 4. Values of the coefficients of damping

Coefficients	Estimated	Updated
α_1	3.00	5.03
β_1	1.00 e-7	1.21 e-6
α_2	3.00	9.63
β_2	1.00 e-6	6.71 e-6

The initial estimate of the coefficients α and β, resulted in a low damping in the structure, shown a discrepancy in the amplitudes of FRF(s) as it can be observed in the plot of the analytical and experimental FRF(s), fig. (7). It can be also observed that the updating of the coefficients α and β improves the representativity of the dynamic behavior of the structure. Figure (8), it shows the evolution of the coefficients α and β during the updating process for region one without added damping and region two with added damping.

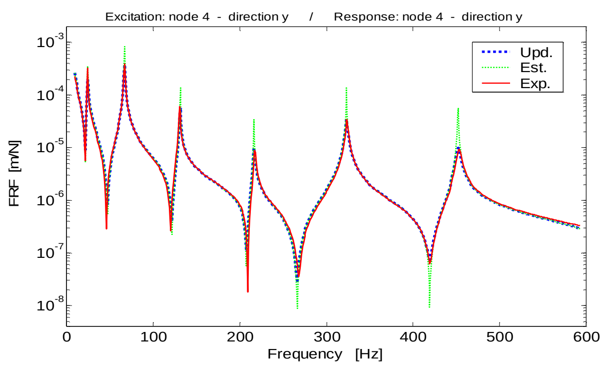

Figure 7. Experimental, analytical and updated FRF(s)

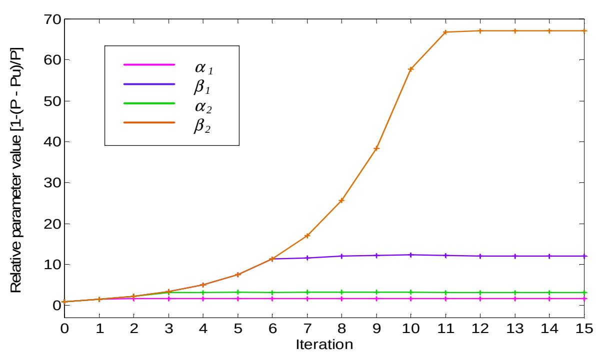

Figure 8. Evolution of the updated parameters

Table (5) shows discrepancy damping ratio before the updating if compared with the damping ratio of the experimental model. That discrepancy, decreases strongly after the updating of the coefficients of proportionality.

Table 5. Values of the damping ratio

Experimental model		Analytical model		Updated model	
Modo	ζ_{EXP} (%)	ζ_{ANA} (%)	$\frac{\zeta_{EXP} - \zeta_{ANA}}{\zeta_{EXP}}$ (%)	ζ_{UPD} (%)	$\frac{\zeta_{EXP} - \zeta_{UPD}}{\zeta_{EXP}}$ (%)
1	1.30	0.98	-24.62	1.98	52.31
2	0.82	0.36	-56.10	0.84	2.44
3	0.56	0.19	-66.07	0.51	-8.93
4	0.48	0.12	-75.00	0.42	-12.50
5	0.41	0.08	-80.49	0.43	4.88
6	0.56	0.07	-87.50	0.53	-5.36

The representativity of the model can be better observed in the fig. (9). It shows that the correlation of the FRF(s) of the models are good, an accentuated increase of the FRAC-values is observed after the introduction and updating of the non proportional damping in FEM. It indicates a great improvement of the FEM that presents a more reliable dynamic behavior.

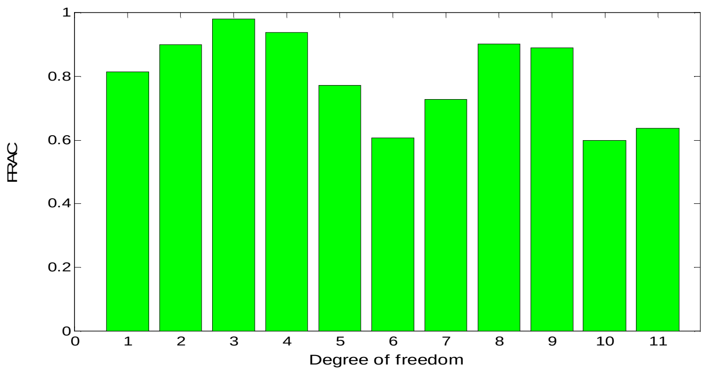

Figura 9. FRAC-values after the introduction and updating of the damping

4. Conclusions

In this paper it has been discussed the iteration from results of finite element modelling with experimental data, looking for a more reliable FEM, which represents the tested structure more realistically. The introduction of non proportional damping in the finite element model updating shows that the represetativity of the FE model could be improved.

The updating of the damping parameters improved the FRAC-values, showing a better correlation of the models not only in terms of the difference frequencies and mode shapes, i.e., MAC-values but also in terms of their FRF(s) that presented a good correlation analysing the FRAC-values.

5. Acknowledgement

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6. References

Clough, R. W.; Penzien, J., 1993, “Dynamics of structures”, Ed. McGraw-Hill, USA, 738p.

Doi, R. M. and Pereira, J. A., 2002, “Analysis of the Effects of Damping in the Models Validation”, In: 26th International Seminar on Modal Analysis-ISMA, Leuven, Belgium, CD-ROM.

Ewins, D. J. and Inman, D. J., 2001, “Structural Dynamics 2000: current status and future directions”, Ed. Research Studies Press Ltd., Great Brittain, 482 p.

Friswell, M. J. and Mottershead, J. E., 1998, "Editorial", Mechanical Systems and Signal Processing, vol.12, London, pp.1-6.

Lammens, S., 1995, “Frequency Response Based Validaton of Dynamic Structural Finit Element Model”, Doctor’s Thesis, Katholieke Universiteit Leuven, Belgium.

Maia, N. M. M. and Silva, J. M. M., 1997, "Theoretical and Experimental Modal Analysis," Ed. Research Studies Press Ltd., Tauton, 468 p.

Mottershead, J. E., Mares, C., Friswell, M. and James, S., 2000, “Selection and Updating of Parameters for an Aluminium Space-frame Model”, Mechanical System and Signal Processing, USA, vol.14, No.6, pp.17-18.

Pereira, J.A. and Borges, A.S., 2001,"Using of Model Updating Considering Damped FRF(s)", in: 16th Brazilian Congress of Mechanical Engineering, Uberlândia, Brazil, CD-ROM.

Pereira, J. A., 1996, "Structural Damage Detection Methodology using a Model Updating Procedure based on Frequency Response Functions - FRF(s)", Tese de Doutorado, Universidade Estadual de Campinas, Faculdade de Engenharia Mecânica, Campinas, Brasil.

Richardson, M. and Potter, R., 1975, “Viscous vs. Structural Damping in Modal Analysis”, Proceeding of the 46th Shock and Vibration Symposium.

Schaak, H., 2000, "Industrial Model Updating of Civil Four-Engine Aircraft in NASTRAN Environment – An Overview", in: 25th International Seminar on Modal Analysis-ISMA, Leuven, Belgium, CD-ROM.

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