DYNAMIC ANALYSIS AND PARAMETRIC IDENTIFICATION IN THE USE OF TECHNIQUES OF CONTROL ADAPTIVE IN THE CANTILEVER-BEAM

JOÃO CARLOS BARBOSA DA SILVA
Department of Mechanical Engineering, Center of Technology, UFPB – Campus I
CEP 58059-900, João Pessoa, Paraíba, Brazil.
jkbarbosa@yahoo.com.br

JOSÉ FELÍCIO DA SILVA
Department of Mechanical Engineering, Center of Technology, UFPB – Campus I
CEP 58059-900, João Pessoa, Paraíba, Brazil.
felício@funape.ufpb.br

VALDEMIR MARIANO
Department of Mechanical Engineering, Center of Technology, UFPB – Campus I
CEP 58059-900, João Pessoa, Paraíba, Brazil.
cynmar@uol.com.br

HIRAN DE MELO
Federal University of Campina Grande
CEP – 58000 – 000, Campina Grande, Paraíba, Brazil.
hiran@dee.ufcg.edu.br

Abstract. Identification problem consists essentially of the search of a model that determines the relationship that exists between the input and output signal, according to some approach. The parametric identification has a fundamental paper in the theory of modern control, and one of the used methods is least square. The method least-square recursive is important, because it facilitates to estimate the parameters of a certain model, as the data of the process are available. In the present work, it is used of modeling techniques, seeking the establishment of a group of equations that can monitor the previous analysis of the behavior dynamic and vibratory of the system cantilever-beam appropriately. After, it tries to verify the answer vectors in the domain of the frequency starting from a procedure using techniques of identification of parameters - least-square, least-square recursive.

Keywords. *Identification, dynamic, least-square, control*

1. Introduction

Parametric identification plays a fundamental role in the theory of modern control, and one of the mostly used methods is the least square. Model reference adaptive controller, for example, uses recursive least square estimator for formation of the control law (Aström and Wittenmark, 1995). In control the problem is concentrated in the selection of the input signal. The choice of the input signal requires some knowledge of the process. In adaptive control the parameters of a process change continuously, it is necessary to have estimation methods that update the parameters recursively.

The recursive methods are important, because they allow the parameters estimation of a certain model when the data of the process are available (Aguirre, 2000). The basic idea of this research work is to calculate and estimate the coefficients of stiffness of the one cantilever-beam with three degrees of freedom with the use of the methods of parametric estimation.

2. Characteristics of the system

The proposed system consists of a aluminum cantilever-beam, with a length of $L = 0.8$m, transverse section area, A=9.60×10^{-5}m, moment of inertia of transverse section area, I= 2.88×10^{-10}m^4, density, ρ=2690 Kg/m, module of elasticity was admitted, E =7.03×10^{10}N/m^2 mass, $m = m_1 = m_2 = m_3 = 0.069Kg$. For the present study, the system presented in the figure (1) was used.

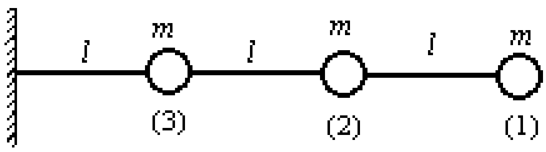


Figure 1 – Representation of the cantilever beam, with concentrated masses.

The flexibility influence coefficient a_{ij} is defined as the displacement in i due a unity force applied in j. With forces f_1, f_2 and f_3 acting upon points 1, 2 and 3. We can apply the principle of superposition in order to determine the displacements resulting from all forces by simply summing up the individual contribuitions (Thomson, 1978).

$$\begin{aligned} x_1 &= a_{11}f_1 + a_{12}f_2 + a_{13}f_3 \\ x_2 &= a_{21}f_1 + a_{22}f_2 + a_{23}f_3 \\ x_3 &= a_{31}f_1 + a_{32}f_2 + a_{33}f_3 \end{aligned} \tag{1}$$

In matrix form,

$$\{x\} = [a]\{f\} \tag{2}$$

where

$$[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{3}$$

is the flexibility matrix. Premultiplying (2) by the inverse of the flexibility matrix we obtain,

$$[a]^{-1}\{x\} = \{f\} = [k]\{x\} \tag{4}$$

therefore, the stiffness matrix that is given by the inverse of the flexibility matrix,

$$[a]^{-1} = [k] \tag{5}$$

By adopting the moment of area method, the deviation in the several points is the same as the moment of the curve M/EI in relation to the point in subject. The flexibility matrix for the proposed system shown in figure (1) is given by,

$$a = \frac{l^3}{3EI}\begin{bmatrix} 27 & 14 & 4 \\ 14 & 8 & 2.5 \\ 4 & 2.5 & 1 \end{bmatrix} \tag{6}$$

substituting the values in (6) and solving comes,

$$a = \begin{bmatrix} 0.2276 & 0.1180 & 0.0337 \\ 0.1180 & 0.0674 & 0.0211 \\ 0.0337 & 0.0211 & 0.0084 \end{bmatrix} \tag{7}$$

therefore, the global stiffness matrix can be written in the form

$$K = \begin{bmatrix} 63.8784 & -146.0077 & 109.5058 \\ -146.0077 & 401.5212 & -419.7721 \\ 109.5058 & -419.7721 & 730.0385 \end{bmatrix} \tag{8}$$

3 – Mathematical model

In the development of the mathematical model it is necessary to identify the components of the system and establish its individual characteristics. Such characteristics are ruled by physical laws, (Kirchhoff, Fourier laws, etc., according to the nature of the system) and they are described in terms of system parameters. The fundamental law that rules the mechanical systems is Newton's second law. It can be applied to any mechanical systems (Ogata, 1998).

If the force $f(t)$ is function of the time with small variations, then the dynamics of a uniform beam will be appropriately described by a mass-spring model of a simple degree of freedom (Craig, 1981). The analysis of several structures are based on the models of multiple degrees of freedom.

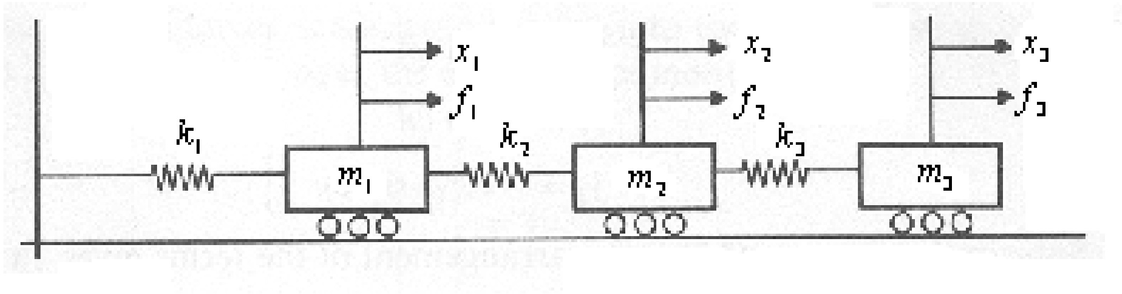


Figure 2 – System, mass –spring, equivalent of the beam with three degrees of freedom.

Let us consider the system in Fig. (1), as a mass-spring system equivalent with three degrees of freedom, as shown in figure (2). The differential equations of motion obtained from the free-body diagram, for the system in study, applying Newton's second law to mass m_i ($i = 1,2,3$), can be written in the form,

$$\begin{aligned} &m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = f_1 \\ &m_2\ddot{x}_2 - k_2x_1 + (k_2 + k_3)x_2 - k_3x_3 = f_2 \\ &m_3\ddot{x}_3 - k_3x_2 + k_3x_3 = f_3 \end{aligned} \tag{9}$$

the equation (9) can be represented by equation (10), Meirovitch (1975),

$$[M]\{\ddot{X}\} + [K]\{X(t)\} = \{F(t)\} \tag{10}$$

where $[M]$,$[K]$ and $\{\mathrm{F(t)}\}$ are respectively the mass matrix, stiffness matrix and the vector excitation forces.

The elements of the mass and stiffness matrices are given by group of expressions (11) and (12).

$$\begin{aligned} &M_{11} = m_1;\ M_{12} = 0;\ M_{13} = 0; \\ &M_{21} = 0;\ M_{22} = m_2;\ M_{23} = 0 \\ &M_{11} = 0;\ M_{32} = 0;\ M_{33} = m_3 \end{aligned} \tag{11}$$

$$\begin{aligned} &K_{11} = K_1 + K_2;\ K_{12} = -K_2;\quad K_{13} = 0; \\ &K_{21} = -K_2;\quad K_{22} = K_2 + K_3;\ K_{23} = -K_3; \\ &K_{31} = 0;\quad K_{32} = -K_3;\quad K_{33} = K_3; \end{aligned} \tag{12}$$

A dynamic system constituted of a finite number of concentrated elements can be described by differential ordinary equations in which time is the independent variable. A differential equation order n can be represented by a differential equation of first order (Ogata, 1998). As each differential equation of the proposed system is of 2nd order, there are six initial conditions,

$$x_1 = x_1,\quad x_2 = x_2,\quad x_3 = x_3,\quad x_4 = \dot{x}_1,\quad x_5 = \dot{x}_2,\quad x_6 = \dot{x}_3 \tag{13}$$

Deriving and applying the differential equations of the mathematical model we obtain, after algebraic manipulations,

$$\begin{cases} \dot{x}_1 = x_4,\quad \dot{x}_2 = x_5,\quad \dot{x}_3 = x_6 \\ \dot{x}_4 = \dfrac{-(k_1 + k_2)}{m_1}x_1 + \dfrac{k_2}{m_1}x_2 + \dfrac{f_1}{m_1} \\ \dot{x}_5 = \dfrac{k_2}{m_2}x_1 - \dfrac{(k_2 + k_3)}{m_2}x_2 + \dfrac{k_3}{m_2}x_3 + \dfrac{f_2}{m_2} \\ \dot{x}_6 = \dfrac{k_3}{m_3}x_2 - \dfrac{k_3}{m_3}x_3 + \dfrac{f_3}{m_3} \end{cases} \tag{14}$$

The first three equations don't depend on the dynamics of the system, while the last three do (Levy and Wilkinson, 1978). In matrix form we have,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{(k_1+k_2)}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{(k_2+k_3)}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_3}{m_3} & -\frac{k_3}{m_3} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 \\ 0 & 0 & \frac{1}{m_3} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} \tag{15}$$

that is,

$$\dot{X} = Ax + Bu \tag{16}$$

where the vectors and matrix can be easily identified considering x_1 as output, i.e., $Y = x_1$, the output equation is

$$[Y] = [1\;0\;0\;0\;0\;0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + [0\;0\;0] \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix} \tag{17}$$

that is,

$$Y = Cx + Du \tag{18}$$

where the vectors and matrices can be easily identified considering x_1 and $\dot{x}_1$, as output or

$$Y = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1\;0\;0\;0\;0\;0 \\ 0\;0\;0\;1\;0\;0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0\;0\;0 \\ 0\;0\;0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \tag{19}$$

where the vectors and matrices can be easily identified.

Equations (15) and (17) represent the state equations of the system in study. In the mathematical model, i.e., in the differential equations, the system parameters appear as coefficients. If the coefficients are constant, we say that the system is time invariant; if not, the system is considered time variant.

4 – Excitation with Synthetic Signal

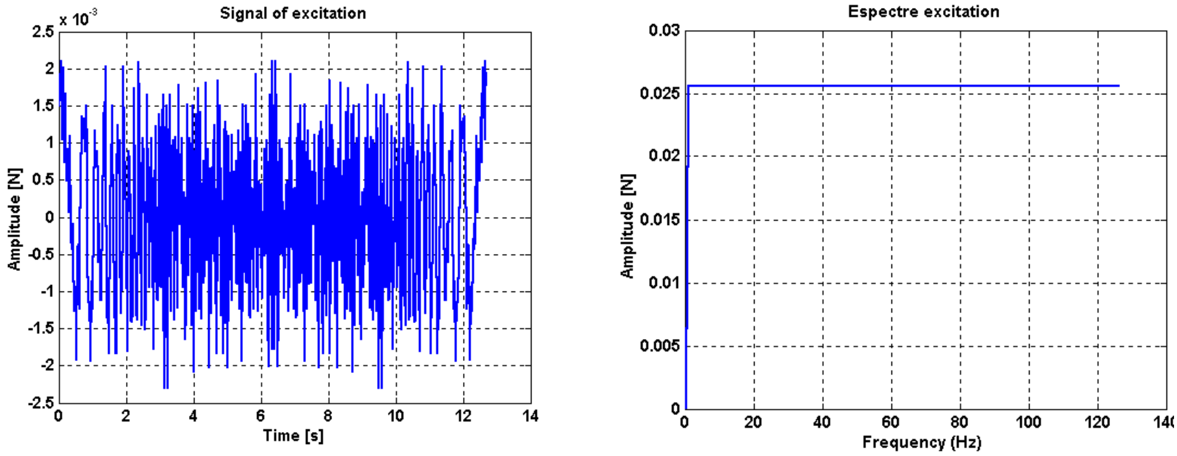
A signal modeled from synthesis of periodic signals that possesses the same statistical properties of a white noise, i.e., null medium value, constant variance, density spectral potency planes, given by the model in the form of the equation (20), is considered for simulation effect.

$$F(t) = \sum_{n=1}^{\infty} \beta_n e^{i\omega t} \tag{20}$$

where β_n is a temporary series adjusted with angle of phase that compose the signal between 0 and π (Schroeder, 1970). As the form of the equation (20) is the real signal of the solution of the system in the equation (10) it can be simplified. The signal can have the form below.

$$F(t) = \mathrm{Re}\left[\sum_{n=1}^{\infty} \beta_n e^{i\omega t} \right] \tag{21}$$

The excitation signal generated in such way is constituted of a large band signal in frequency of the type white noise, with the advantages of being periodic. The Fig. (3) shows the signal generated, in the time and frequency domain.



(a) (b)

Figure 3 – Excitation signal in time (a) and in frequency (b)

5. Estimators

5.1. Least Square

The Least-square method, in the estimation parameters of the mechanics system, is the result of the formulation in the matrix form (Mariano and Oliveira, 1997),

$$[A]\{\phi\}=\{b\} \tag{22}$$

where,

$[A]$: Rectangular matrix whose elements are composed of the input and output measures;

$\{\phi\}$: Vector or matrix of the parameters to esteem;

$\{b\}$: Vector or matrix of the input;

It is supposed that errors are present in the measures because of unadjusted data. Under this assumption, the equation (22) can then be written as,

$$[A]\{\phi\}=\{b\}+\{\varepsilon\} \tag{23}$$

where,

$\{\varepsilon\}$: Measured error.

The least square method principles, establish that the parameters can be obtained by the minimization of an error function (Aguirre, 2000), i.e., the sum of squares due to error in each mensuration, in relation to the calculated values from the used parameters in the equation (22).

The sum of squares due to error, can be written as:

$$\mathrm{E}=\varepsilon^{\mathrm{T}}\varepsilon=(b-A\phi)^{\mathrm{T}}(b-A\phi) \tag{24}$$

Making the product in (24) and considering the equality $b^{\mathrm{T}}A\phi=A^{\mathrm{T}}\phi^{\mathrm{T}}b$ we obtain:

$$\mathrm{E}=b^{\mathrm{T}}b-2\phi^{\mathrm{T}}A^{\mathrm{T}}b+\phi^{\mathrm{T}}A^{\mathrm{T}}A\phi \tag{25}$$

where $(\,)^{\mathbf{T}}$ denotes the transpose of a matrix.

The solution that minimizes the quadratic medium error is:

$$\frac{\partial E}{\partial \phi} = 0 \therefore A^{\mathrm{T}} A \hat{\phi} = A^{\mathrm{T}} b \tag{26}$$

As $A^{\mathrm{T}} A$ is a square matrix, (26) can be pre-multiplied by $(A^{\mathrm{T}} A)^{-1}$ resulting in,

$$\hat{\phi}_{MQ} = (A^{\mathrm{T}} A)^{-1} A^{\mathrm{T}} b \tag{27}$$

Thus, $\hat{\phi}_{MQ}$ constitutes the estimator for least squares of the unknown parameters of ϕ. The only restriction in relation to the estimator is that the matrix product $A^{\mathrm{T}} A$ is nonsingular

5.2 Recursive estimator of the least square

Recursive techniques are desirable for two reasons. First, it is possible, by using recursive techniques, to estimate parameters of a certain model, when the data of the process are ready for use. In second place, they are also useful in the resolution of numeric problems whose solution of an alone time would be difficult at once(Aguirre, 2000).

In adaptive controllers the observations are obtained sequentially in real time. It is then desirable to make the computations recursively to save computation time. Computation of the least-square estimate can be arranged in such a way that the results obtained at time $(t-1)$ can be used to get the estimates at time t.

The equations presented in (28) constitute the estimator for recursive least squares.

$$\begin{cases} K_t = \dfrac{P_{t-1}\psi(t-1)}{\psi_t^T P_{t-1}\psi_t + 1}; \\ \hat{\theta}_t = \hat{\theta}_{t-1} + K_t\left[y(t) - \psi_t^T \hat{\theta}_{t-1}\right]; \\ P_t = P_{t-1} - K_t \psi_t^T P_{t-1} \end{cases} \tag{28}$$

6. Formulation for identification of parameters of the system

The use of matrices in the formulation of problems involving analysis and vibrations control is very applied, mainly in systems of several degrees of freedom. Oliveira (1989) proposed a method of parameters identification in the frequency domain. The system of linear equations to the identification process in the frequency domain can be written in the form,

$$\begin{bmatrix} x_n^C & y_n^C & z_n^C \\ \multicolumn{3}{c}{\ldots\ldots\ldots\ldots} \\ x_n^S & y_n^S & z_n^S \end{bmatrix}_{2N\times 3} \times \begin{bmatrix} K_{11} & K_{21} & K_{31} \\ K_{12} & K_{22} & K_{32} \\ K_{13} & K_{23} & K_{33} \end{bmatrix}_{3\times 3} = \begin{bmatrix} F_{xn}^C + M_{11}x_n^C(n\omega_0)^2 & F_{yn}^C + M_{22}y_n^C(n\omega_0)^2 & F_{zn}^C + M_{33}z_n^C(n\omega_0)^2 \\ \multicolumn{3}{c}{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots} \\ F_{xn}^S + M_{11}x_n^S(n\omega_0)^2 & F_{yn}^S + M_{22}y_n^S(n\omega_0)^2 & F_{zn}^S + M_{33}z_n^S(n\omega_0)^2 \end{bmatrix}_{2N\times 3} \tag{29}$$

The equation (29) in compact notation, can be represented by the system of lineal equations in the form,

$$[A]\{\phi\} = \{b\} \tag{30}$$

where

$[A]$ = Rectangular matrix $2N \times 3$ containing the real and imaginary components of the displacements;
$\{\phi\}$ = Square matrix 3×3 containing the unknown stiffness coefficients ;
$\{b\}$ = Rectangular matrix of order $2N \times 3$ containing the Euler-Fourier coefficients of the inertia and applied forces.

The parameter identification process (inverse problem) in the frequency domain (Silva and Oliveira, 1998), is obtained by system of lineal equations represented by equation (10).

It can be verified by equation (29), if the solution of the state vector in terms of the displacement response in the frequency domain is consistent.

7. Results and discussions

The determination of the process parameters is a basic element for the adaptive control (Aström and Wittenmark, 1995). Here will be presented the simulation results through the least square methods and recursive least square for the parameters estimation, that is particularly simple if the model has the linear property in the parameters.

For a simulation effect, the excitation force f_1 used in this paper is obtained from the synthesis of low-peak factor periodic signal presented in the equation (20) and the forces $f_2 = 0$ and $f_3 = 0$.

Figure (4) shows the displacement signal in the frequency domain, where the excitation force acting in the mass m_1 is considered. With effect, the three vibration modes are excited.

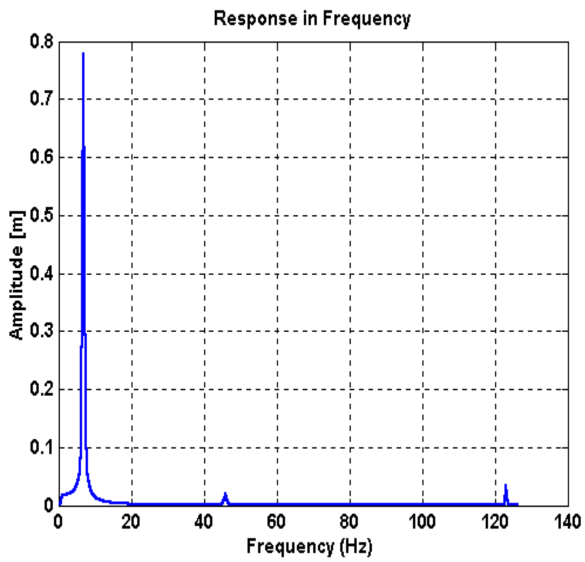


Figure 4. Espectrum response.

Figure (5) shows the time response of the proposed system, where the excitation force has actuating in the mass m_1

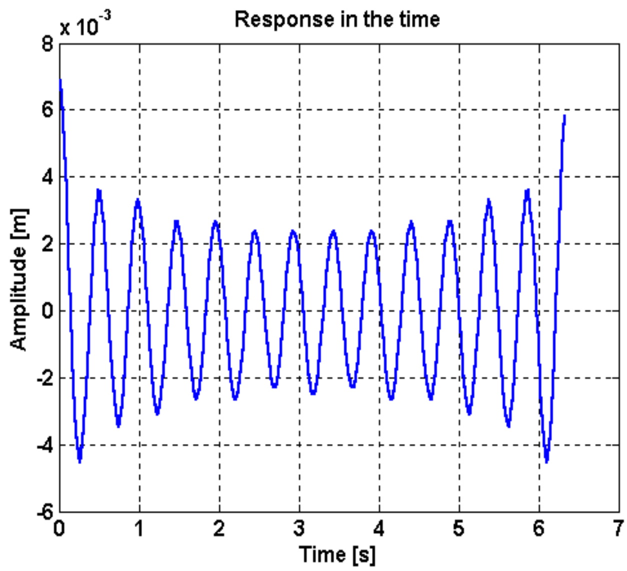


Figure 5. Response in time

It is observed by the simulation results, that the excitation in displacement happens in all the modes, as it can be visualized by the maximum peacks of the spectrum of the response around the resonance frequencies and compared with the eigenvalue data in the table (1) in rad/sec but that can be converted in Hertz, a unit generally used in the studied structure type in the present work.

Table 1. Eigenvalue of the system in (rad/s)

λ_1	λ_2	λ_3
$-0.0000 \pm 1.1144i$	$-0.0000 \pm 7.2969i$	$-0.0000 \pm 19.6053i$

To verify if the solution in terms of the displacement signals is correct, the direct method of resolution in frequency is used (Lalane, 1984) where, starting from the inverse problem, It identifies the stiffness coefficients, (Silva, 1999), through the system of lineal equations obtained in (29) and the least square estimators and recursive least square (Aguirre, 2000).

Table 2. Stiffness coefficients: Theoretical, estimated and error in (%)

Identification of Parameters - Least square stiffness (N/m)									
	K_{11}	K_{12}	K_{13}	K_{21}	K_{22}	K_{23}	K_{31}	K_{32}	K_{33}
Theoretical	63.8784	-146.0077	109.5058	-146.0077	401.5212	-419.7721	109.5058	-419.7721	730.0385
Estimated	63.8784	-146.0077	109.5058	-146.0077	401.5212	-419.7721	109.5058	-419.7721	730.0385
Error (%)	0.1335×10^{-10}	0.1088×10^{-10}	-0.0105×10^{-10}	0.1785×10^{-10}	0.1175×10^{-10}	-0.0134×10^{-10}	0.3087×10^{-10}	0.143×10^{-10}	-0.0156×10^{-10}

As a result of this procedure, the table (2) presents the identification results, using the least square method, of parameters of stiffness of the system and its respective error in percentage.

Table 3. Stiffness coefficients: Theoretical, estimated and error in (%)

Identification of Parameters - Recursive Least square stiffness (N/m)									
	K_{11}	K_{12}	K_{13}	K_{21}	K_{22}	K_{23}	K_{31}	K_{32}	K_{33}
Theoretical	63.8784	-146.0077	109.5058	-146.0077	401.5212	-419.7721	109.5058	-419.7721	730.0385
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Error (%)	0.1335×10^{-10}	0.1088×10^{-10}	-0.0105×10^{-10}	0.1785×10^{-10}	0.1175×10^{-10}	-0.0134×10^{-10}	0.3087×10^{-10}	0.1430×10^{-10}	-0.0156×10^{-10}

Table (3) presents identification results using the recursive least square method. Showing that the verification of the vectors in displacement in the domain of the frequency is consistent, characterizing the consistency of the used mathematical model.

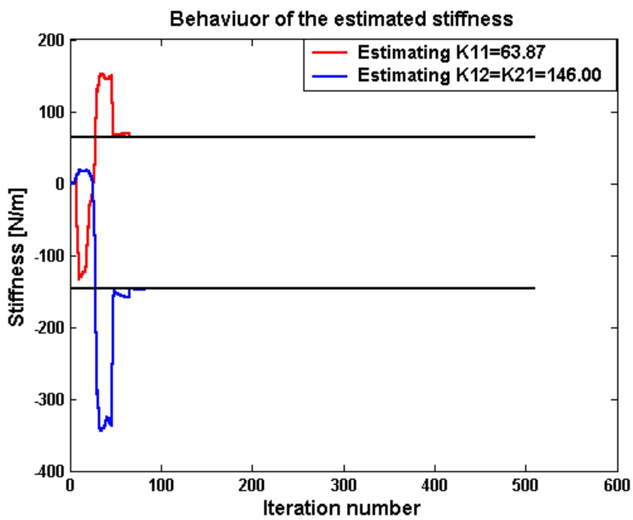

Figure 6. Behavior of stiffness estimation K_{11}, K_{12}, K_{21}

Figure (6) shows the behavior of stiffness estimation of K_{11}, K_{12}, K_{21} through of the method of the recursive least square. The system was simulated from a synthesized input signal of the white noise type.

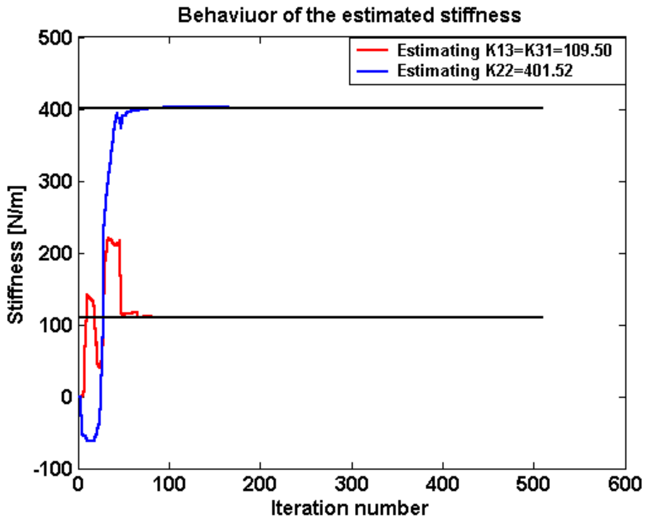

Figure 7. Behavior of stiffness estimation K_{13}, K_{31}, K_{22}

In a similar way, Fig. (7) and Fig. (8) show the behavior of the estimate of the stiffness K_{13}, K_{31}, K_{22} and K_{23}, K_{32}, K_{33}

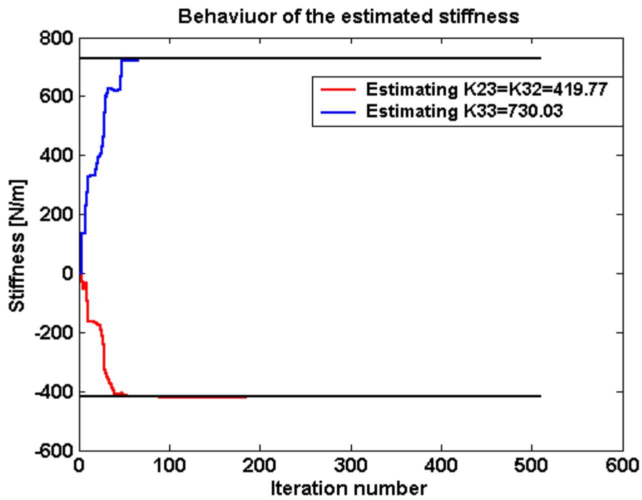

Figure 8. Behavior stiffness estimation K_{23}, K_{32}, K_{33}

It was verified in Fig. (6), (7), and (8) through the estimate curves, that the synthetic excitation signal, makes the parameters have a fast convergence. A good identification was observed. One of the factors that contributed to this was the definition of $p=1\times10^6 \times (I)$, once p is the matrix that indicates the precision of the identified parameters (Aguirre, 2000).

8. Conclusion

In the resolution of identification problems it is very important to validate the results. Especially for adaptive systems in which the identification is accomplished automatically.

The method of the least square is a basic technique for parameters estimation. The method is particularly simple if the model has the property of being lineal in the parameters. In this case the estimate for least square can be calculated analytically.

The modelling of a system (cantilever beam) with concentrated masses was shown in this work, where three degrees of freedom were considered for simulation effect.

The response was verified in displacement in the frequency time domain being used the inverse process. The coherence and correlation among the vibration modes, considering three degrees of freedom, were fully satisfied, as it be observed by the response curve in the presented frequency.

The estimate results presented in the Table (2) and (3), clearly shows that is possible, through least square and recursive least square, to obtain reliable results of simulation and eventually they can be used in adaptive control techniques.

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