ASSESSMENT OF THE STRESS AND STRAIN LEVELS AT GEOMETRICAL DISCONTINUITIES UNDER ELASTOPLASTIC CONDITIONS: A CASE STUDY

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Abstract. *Consideration of structural failures caused by initiation and growth of cracks at stress concentration points are extremely important for safe design of structural component which will avoid accidents that may cause life and financial losses. At notch roots if the stress level exceed the yield stress the strain will become the dominant factor to control the cracking initiation process and an elastoplastic analysis will be necessary in order to determine the actual stress and strain values. In the present work the finite element analysis and the models proposed by Neuber(1961) and Glinka(1985) were used to assess the validity of the local strain approach in the estimation of the stress and strain levels on a plate with a central hole subjected to uniform axial loading. Specimens made of Al alloy 7050 T7451 and MAN-TEN steel were used to study possible effects of stress-strain curve behaviour on the stress distribution. The theoretical stress concentration factor of the hole was determined using photoelastic and numerical methods. The applicability of the models proposed by Neuber and Glinka was assessed comparing the results obtained with both methods and the results from the finite element analysis. The results show that Neuber and Glinka models may underestimate the stress values for plane stress while may overestimate the strain levels depending on the model adopted and state of stress.*

Keywords. *Stress concentration, notches, elastoplastic analysis, Neuber's rule, Glinka's rule.*

1. Introduction

Intensily loaded structural components may yield locally at stress concentration points such as holes, notches, geometric changes, keyways etc. Under cyclic loading, local plasticity may induce redistribution of the stress gradients and also fatigue crack initiation (Fillippini, 2000, Visvanatha et al, 2000). To identify the possibility of occurrence of such phenomena is fundamentally important to conduct an analysis of the cyclic strain at the root of those stress concentrators. Due the need to reduce computing effort such analysis is commonly done using the local strain approach where the stress and strain at the root of the stress concentrator are estimated as a function of the theoretical stress concentration factor and of a constitutive equation to account for the material behaviour. Among others, the Neuber rule (Neuber, 1961) is the method mostly used with this objective. Topper et al (1969), Seeger et al (1980), Glinka (1985) e Hoffman et al (1985) also proposed alternative and/or complementary methods with the same aim. The great inconvenient of those approaches is the impossibility to evaluate the stress redistribution due to yielding and the consequent geometrical changes around the root of the notch. A way to solve this problem is to use finite element method considering geometrical non-linearities and the material elastoplastic behaviour. Most of the literature approaches the stress concentration problem as a plane stress problem (Peterson, 1974) not taking in account any variation of the stress with thickness, adding another degree of difficulty to the problem. Research made by Cunha (1981) using three-dimensional photoelasticity showed that as thicker are the specimens bigger are the variations in stress state along thickness.

The present work had the objective of verifying the local strain approach for the stress concentration problem. A stress analysis was carried on a plate with a central hole subjected to a uniform loading using the finite element method – FEM an also considering the models proposed by Neuber and Glinka. The possible effects due to the behaviour of the material stress/strain curve were studied using 7050 T7451 Al alloy and Man-Ten steel constitutive curves. The theoretical stress concentration factor was determined using photoelastic and numerical methods. Comparison between the FEM results and the one obtained using Neuber and Glinka models allowed to verify the applicability of these models for the case studied.

2. Strain Estimation Techniques

2.1 Neuber's Rule

The rule of Neuber, despite being proposed for a specific geometry and loading, is the mostly used method to describe stresses and strains on geometrical discontinuities. This method is based in the premiss that the theoretical stress concentration factor K_t used to relate nominal with actual local stresses and strains stays constant up to yielding starts. After yielding the nominal and actual local stresses and strains are no longer linearly related by K_t but by the stress concentration factor K_σ and the strain concentration factor K_ε . The different local response is due to residual stresses developed as result of local yielding in the root of the notch and can be expressed by Eq. (1) (Neuber, 1961).

$$K_t^2 = K_\sigma K_\varepsilon = \frac{\Delta\sigma}{\Delta S} \cdot \frac{\Delta\varepsilon}{\Delta e} \tag{1}$$

where $\Delta\sigma$, ΔS, $\Delta\varepsilon$, Δe represent the range of local stress, nominal stress, local strain and nominal strain, respectively.

Assuming the stress/strain relation can be described by the Ramberg-Osgood equation, Eq. (2), the relation between the elastic nominal stress range, ΔS, and the stress range at the notch root, $\Delta\sigma$, is given by Eq. (3).

$$\frac{\Delta e}{2} = \frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K}\right)^{\frac{1}{n}} \tag{2}$$

$$\frac{(K_t \Delta S)^2}{4 \cdot E} = \frac{\Delta\sigma}{2}\left[\frac{\Delta\sigma}{2 \cdot E} + \left(\frac{\Delta\sigma}{2 \cdot K}\right)^{\frac{1}{n}}\right] \tag{3}$$

where n and K are the material hardening exponent and strength coefficient, respectively.

Seeger (1969) and Amstutz (1978) stated that the model proposed by Neuber gives conservative estimations for the root notch strain and that the accuracy of the results do not depends on K_t and the material. Eq. (3) is valid till the yielding onset at the notch root. To consider the situation of general yielding at the notch root section, a generalization of Neuber's model was proposed by Seeger e Heuler (Seeger et al, 1980).

2.2 Equivalent Strain Energy Density (*ESED*)

Several methods related to the solution of the elastic stresses and strains problem around geometrical discontinuities have been proposed in the literature (Howland, 1930, Neuber, 1946, Peterson , 1974). Schijve (1980) showed that the elastic stress distribution at the root of different kinds of geometrical discontinuities are similar one to another and could be satisfactorily characterized by two parameters: the notch root and the theoretical stress concentration factor K_t . The availability of solutions for elastic stresses and strains at the notch root made possible to calculate the energy density distribution at such points. It was also shown that in cases of local and non-generalized yielding, the distribution of energy density in the plastic zone is approximately equal to what is observed in linear-elastic regime, which allows to infer the material linear elastic behaviour controls the plastic zone strain (Hutchinson, 1968, Walker, 1974).

The energy density in the plastic zone, W_σ, is equal to the energy density calculated based on the elastic solution, W_s.

$$W_s = W_\sigma \quad or \quad \int_0^{e_{ij}} S_{ij}\, de_{ij} = \int_0^{e_{ij}} \sigma_{ij}\, d\varepsilon_{ij} \tag{4}$$

For plane stress states and full elastic behaviour, the stress at the notch root can be calculated based on the nominal stress and corrected by the theoretical stress concentration factor. Thus, assuming that energy density of elastic strain at the notch root, W_S is equal to the product of the energy density from the nominal stress, W_{sn} and the square of K_t , Eq. (3) can be expressed as:

$$\frac{(K_t \cdot \Delta S)^2}{E} = \frac{\Delta\sigma^2}{E} \quad or \quad K_t^2 \cdot W_{sn} = W_\sigma = W_s \tag{5}$$

Results of research made on this energy approach performed by Glinka (1985) to estimate the non-elastic stress and strain levels at the notch root allowed to conclude that, in the presence of local yield at the root of the geometrical discontinuity, the density of energy can be calculated using Eq. (5), which may be associated to the Ramberg-Osgood constitutive relation (Bannantine, 1998, Dowling, 1999) and be expressed as Eq. (6).

$$\frac{(K_t \cdot \Delta S)^2}{8E} = \frac{\Delta\sigma^2}{8E} + \frac{\Delta\sigma}{2(n+1)}\left(\frac{\Delta\sigma}{2K}\right)^{\frac{1}{n}} \tag{6}$$

This last equation is valid for local yielding at the root of the geometrical discontinuity, making possible to calculate the local stress and strain, being known the nominal stress and theoretical stress concentration factor. It has been showed the energy density method can be used for plane stress and plane strain states for stress levels near the general yielding on the discontinuity section (Glinka, 1985).

3. Materials and Methods

3.1 Specimen Characteristics

Plates with circular holes are elements which appear frequently in structural applications, especially in the naval and aeronautic industry. The stress distribution is highly affected by the presence of the discontinuity in the neighbourhood of the discontinuity itself. At increasing distance from the hole border the notch effect upon the stress distribution will reduce gradually becoming negligible at points located at distances which are great when compared with the notch radius (Branco, 1989). The geometrical characteristics of the specimen analysed in the present work are presented in Fig. (1). The materials considered in the analysis were the Aluminium alloy 7050 T7451 and the MAN-TEN steel, whose cyclic properties are shown in Fig (2). The variation of load ΔS were applied uniformly in the direction shown in Fig. (1)

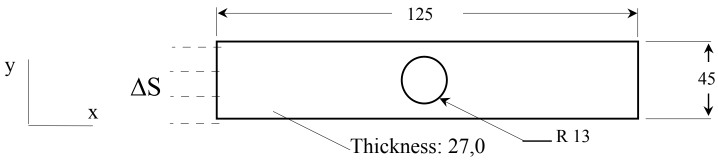

Figure 1. Geometry of the notch studied (dimensions in mm).

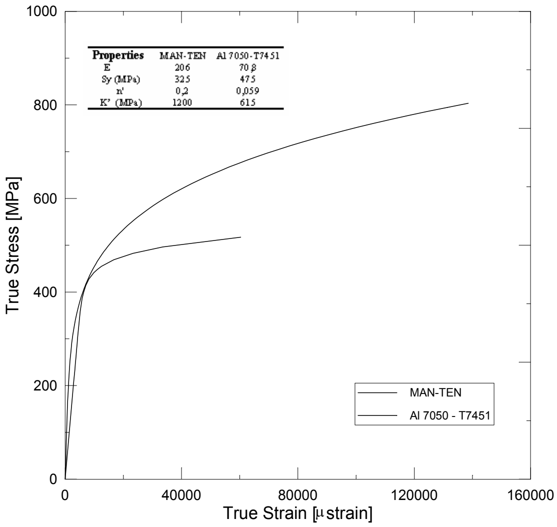

Figure 2. σ-ε curve and cyclic mechanical properties 7050 T7451Al alloy and Man-Ten steel.

3.2 Photoelastic Analysis

Plane photoelasticity techniques were used for preliminary stress analysis in the linear elastic regime in order to determine the magnitude of the theoretical stress concentration factor K_t as well as the pattern of the stress distribution. The photoelastic model of the plate with a central hole was made of a polymer plate with thickness of *6.35 mm.* To avoid effects due to machining and other preparation operations the specimens suffered a previous thermal treatment to eliminate any existing residual stresses (Frocht, 1966). The optical system used is shown on Fig. (3) and comprise a polariscope model P-150, made by Riken Keiki Fine Instruments Co., lenses and loading apparatus. The system was prepared for analysis in dark field using circular polarized light with incident white light. Several levels of loading were uniaxially applied. The material fringe value was determined by a calibration process (Durelli et al, 1965, Dally et al, 1978) and the isochromatic fringe values at the border of the hole were determined using the Tardy compensation method (Tardy, 1929, Chakrabarti, et al, 1969).

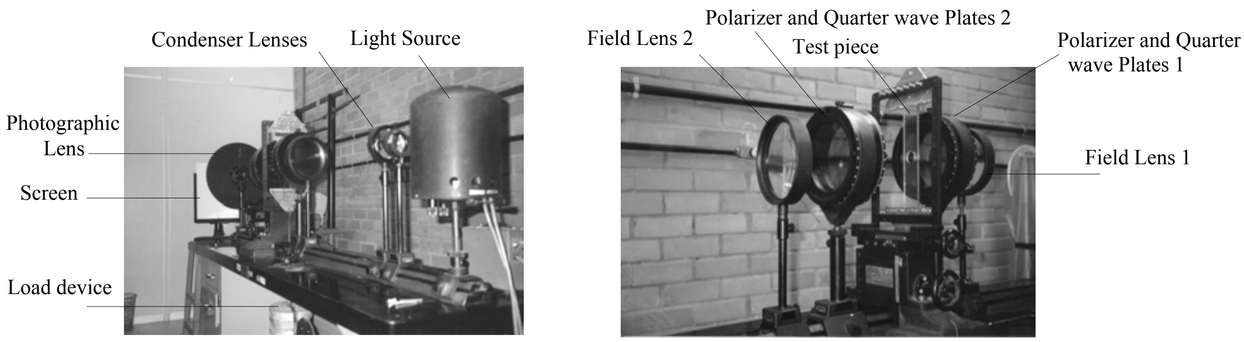

Figure 3. Polariscope set up for the photoelastic analysis.

3.3 Numerical Analysis - Finite Element Method

The finite element method simulation carried out used three dimensional and plane elements showed in Fig. (4). For the three dimensional modelling tetrahedral elements with 10 nodes and 3 degree of freedom by node were used. The symmetry of the geometry under analysis allowed to consider one-eight of the specimen, restricting the nodal dislocations on the symmetry planes. The mesh chosen had 21316 nodes and 13495 elements and it was refined on the regions of interest. This model was used to determine the stress distribution on two specific points of the discontinuity geometry, on the surface and on the mid thickness plane. For the two dimensional modelling, plane triangular elements with 6 nodes and 3 degree of freedom by node were used in a net with 15865 nodes and 7770 elements. In the plane simulation a quarter of the geometry was used, restricting the nodal dislocations to the nodes on the symmetry lines. Analysis considering plane stress state, simulating the surface of the component, and plane strain state, approaching the condition in the middle of the thickness were carried out in more detail. The model was loaded elastically in order to verify the magnitude of the theoretical stress concentration factor K_t and compare it with the values determined experimentally and also values presented in the literature. For the elastoplastic analysis, a constitutive curve for each material, as the ones shown in Fig. (2), was used. The model was loaded from zero to the yield stress in steps of 50 MPa and observed the levels of stress and strain in the root of the notch as well as the correspondent stress and strain concentration factors.

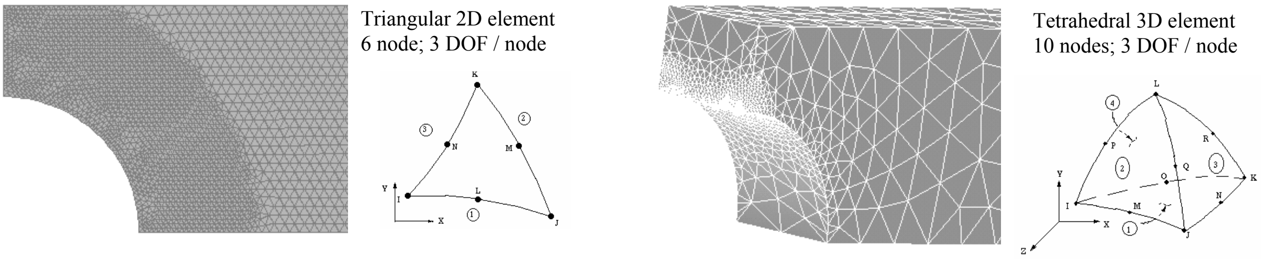

Figure 4. Geometry, Mesh and elements used in the plane and three dimensional models.

3.4 Numerical Analysis - Elastoplastic Models

The levels of stress and strain on the notch root were estimated, for the two materials, using the models of Neuber and Glinka, expressed by Eq (2) and (5), respectively. The solutions were obtained using iterative numerical methods. The equation of Ramberg-Osgood was used as the material constitutive relation between stress and strain in both regimes, elastic and plastic, in order to obtain the total strain. Considering the theoretical stress concentration factor for the linear-elastic regime obtained by finite element and photoelastic methods and varying the nominal applied stress from zero to the threshold of yielding, it was possible to evaluate the elastoplastic behaviour of stress and strain as well as to evaluate the parameters involved for situations near the generalized yielding on the discontinuity section.

4. Results and Discussion

After thermal treatment for stress relieving, the calibration process indicated a fringe value of 6581 N/fr.m for the photoelastic material used. In Fig. (5) the isochromatic patterns associated to axial loads of 15, 30 e 45kg shows the critical points at the notch root region *A*, the influence of component size on the fringe pattern at region *B* and the presence of compression areas at region *C*.

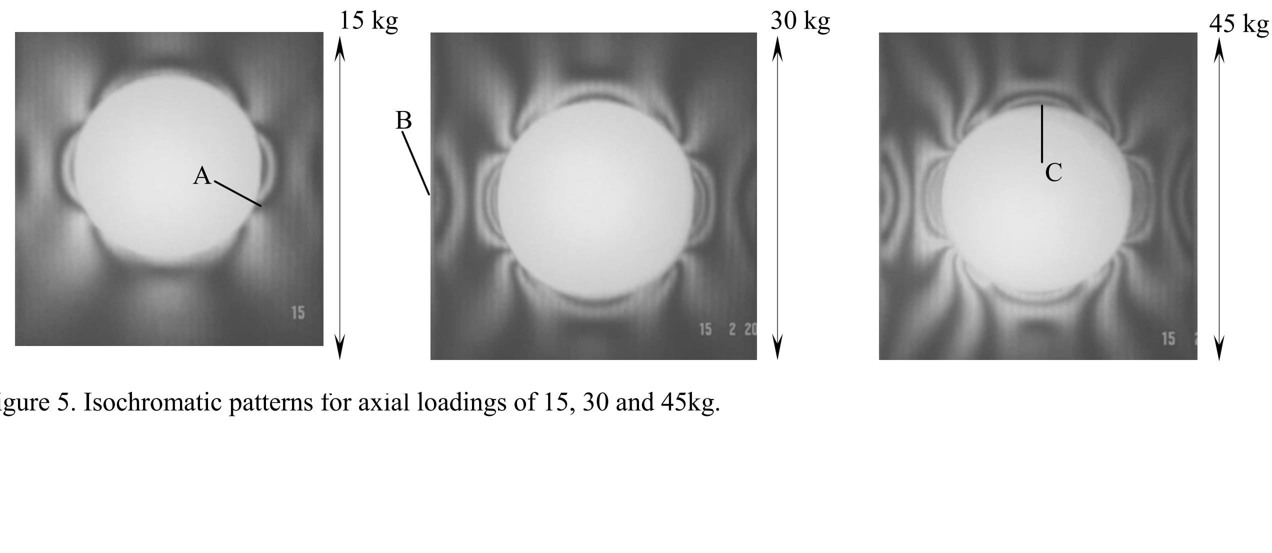

Figure 5. Isochromatic patterns for axial loadings of 15, 30 and 45kg.

To determine the value of the isochromatic fringe at the border, on the hole surface, the image of the isochromatic pattern was projected on a screen where, using the Tardy compensation method, the isochromatic fringe at nine points approaching the border was read. By an exponential fit isochromatic fringe value at the border was determined extrapolating the curve fit, as shown in Fig. (6).

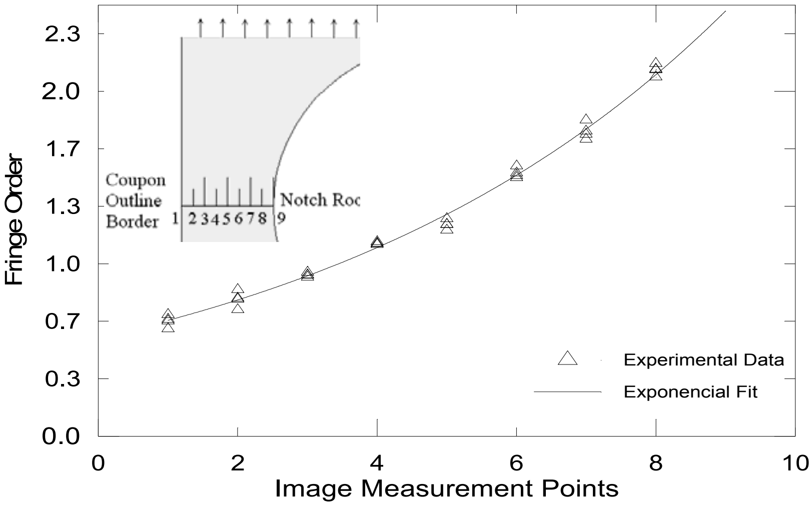

Figure 6. Determination of the isochromatic fringe value at the specimen border.

On the notch root a isochromatic fringe value of magnitude 2.46 was obtained. As the nominal stress is known from the section area and the applied loads, a theoretical stress concentration factor K_t at the notch root of 2.0 was obtained.

Fig. (7) show the stress distribution in the three dimensional model for one-eight of the geometry and nominal stress at the section discontinuity of 100 MPa. The plane stress state observed at the notch surface change to a three axial stress state inside the component. As the stresses observed in one direction present values well below to the values observed in the other directions, these results allow to assume the hypothesis that the material inside the component and in the neighbourhood of the notch develop a plane strain state. As consequence, the local stress varies with thickness, reaching in the middle plane values greater than obtained with the use of the theoretical stress concentration factor. Results recently obtained by Soares et al (2002) indicate the two dimensional approach is valid for components whose thickness do not exceed 1.5 times the hole radius, Thus, it is possible to use two dimensional finite element models associated to plane stress state in the simulation of the behaviour at the free surface and at the middle section of the component.

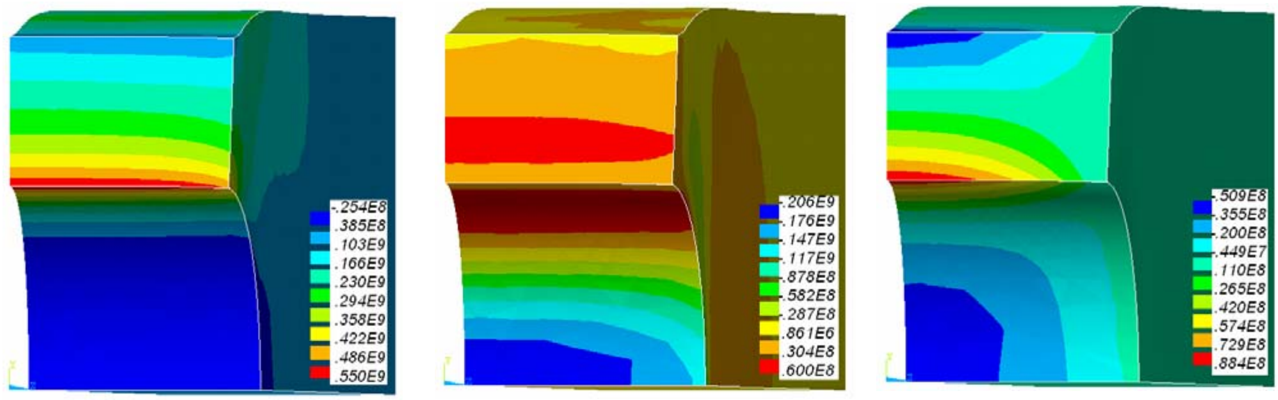

Figure 7. Stress distribution in the finite element three dimensional model for 100 MPa of nominal stress in the geometrical discontinuity section: (a) σ_{xx}, (b) σ_{yy} e (c) σ_{zz}.

Analysis of the two dimensional finite element model considering loading in the elastic regime was carried out to verify the theoretical stress concentration factor and the stress distribution in the geometrical discontinuity section. The stress concentration factor obtained was 2.12, which is not far from the value of 2.2 indicated by Dowling (1999).

Varying the stress range applied at the discontinuity section in fixed steps of 50 MPa up to the yield stress of each material, the actual stresses and strains were determined using the models of Neuber and Glinka and also using the simulations by finite element method for plane stress and plane strain states. The results are presented in Tab. (1) and (2) for Al alloy 7050 T7451 and Man-Ten steel, respectively.

Table 1. Stresses and strains at the notch root – Al alloy 7050 T7451

Nominal	Neuber		Glinka		Plane Stress		Plane Strain	
$\Delta\sigma/2$ (MPa)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)	$\Delta\sigma/2$ (MPa)
25	53,0	0,0003	53,0	0,0003	51,5	0,0002	51,4	0,0002
50	105,5	0,0005	105,1	0,0005	102,9	0,0005	102,9	0,0005
75	155,3	0,0008	153,3	0,0008	154,4	0,0007	154,4	0,0007
100	199,4	0,0011	193,9	0,0011	190,9	0,0010	198,5	0,0009
125	236,3	0,0014	226,8	0,0013	227,7	0,0013	234,4	0,0012
150	266,9	0,0018	253,7	0,0017	258,8	0,0017	268,1	0,0015
175	292,6	0,0023	276,3	0,0020	286,3	0,0021	297,1	0,0018
200	314,9	0,0028	295,7	0,0023	312,2	0,0027	322,9	0,0022
225	334,4	0,0033	312,9	0,0027	338,1	0,0034	349,6	0,0027
250	351,9	0,0039	328,4	0,0031	362,7	0,0042	375,0	0,0033
275	367,7	0,0045	342,5	0,0036	385,8	0,0052	398,4	0,0040
300	382,3	0,0051	355,4	0,0040	410,2	0,0066	422,3	0,0048
325	395,8	0,0058	367,5	0,0045	434,9	0,0082	444,9	0,0058

Table 2. Stresses and strains at the notch root – Man-Ten steel.

Nominal	Neuber		Glinka		Plane Stress		Plane Strain	
$\Delta\sigma/2$ (MPa)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)	$\Delta\sigma/2$ (MPa)	$\Delta\varepsilon/2$ (mm/mm)	$\Delta\sigma/2$ (MPa)
25	53	0,0007	53	0,0007	51,5	0,0007	51,5	0,0007
50	106	0,0015	106	0,0015	102,9	0,0014	102,9	0,0013
75	159	0,0022	159	0,0022	154,3	0,0022	154,3	0,0020
100	212	0,0030	212	0,0030	205,6	0,0029	205,6	0,0026
125	264,98	0,0037	264,96	0,0037	256,6	0,0036	256,6	0,0033
150	317,52	0,0045	317,11	0,0045	307,4	0,0043	307,7	0,0039
175	365,75	0,0053	362,5	0,0052	354,6	0,0051	358,1	0,0046
200	400,28	0,0063	392,38	0,0060	389,0	0,0059	404,1	0,0053
225	421,65	0,0076	410,78	0,0069	411,7	0,0069	438,1	0,0060
250	436,03	0,0091	423,49	0,0078	427.8	0,0080	459.8	0,0068
275	446,73	0,0107	433,15	0,0087	439.9	0,0094	477.0	0,0078
300	455,29	0,0125	440,99	0,0098	451.0	0,0112	490.1	0,0089
325	462,44	0,0145	447,6	0,0109	459.7	0,0133	501.6	0,0103
350	468,61	0,0166	453,35	0,0121	468.4	0,0159	510.8	0,0118
375	474,06	0,0188	458,46	0,0134	476.3	0,0193	519.2	0,0137
400	478,95	0,0212	463,06	0,0147	485.5	0,0238	526.8	0,0158
425	483,4	0,0237	467,26	0,0161	494.3	0,0304	534.4	0,0183
450	487,49	0,0264	471,13	0,0176	504.4	0,0412	541.7	0,0214
470	490,54	0,0286	474,02	0,0188	516.9	0,0575	548.7	0,0248

Fig. (8) show the stress distributions in the linear dominium, the threshold of yielding and near to the generalized yielding in the discontinuity section for the two materials studied. It can be seen that for high levels of nominal stress, especially near the yield stress, both materials presents a high degree of local plastic deformation which is dependent on the material and nominal stress level.

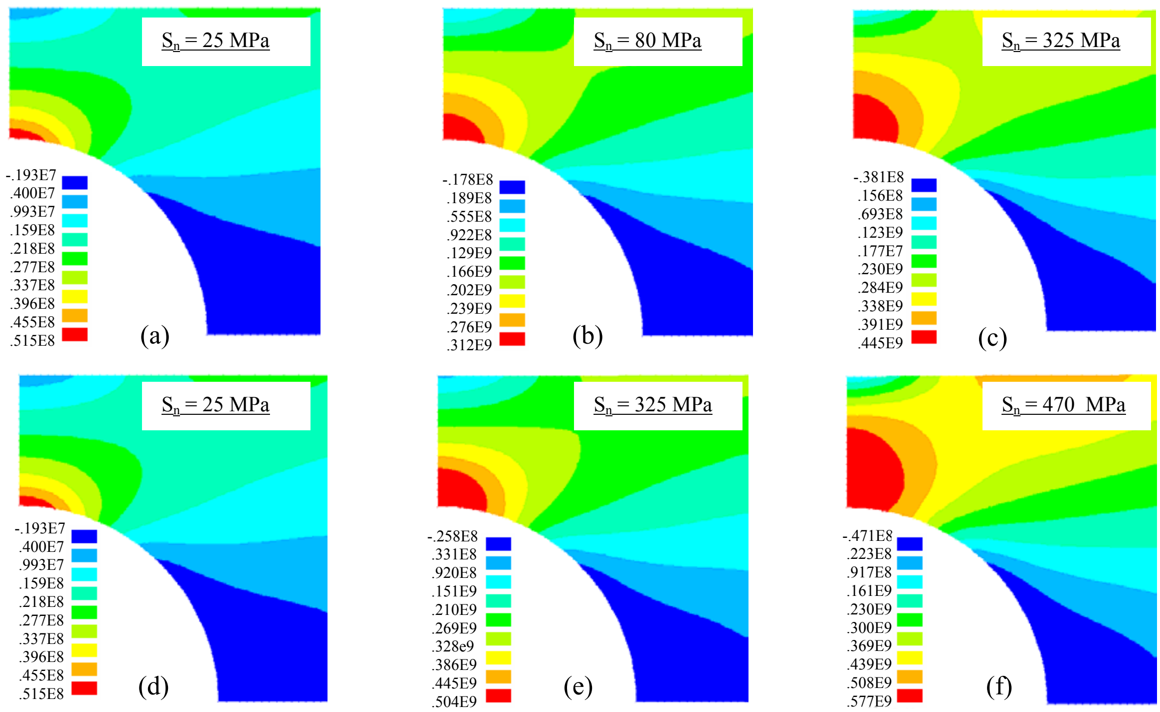

Figure 8. Stress didtribution obtained by finite element simulation. Linear-elastic regime: (a) Man-Ten; (d) Al 7050; beginning of local yielding: (b) Man-Ten; (e) Al 7050; threshold of generalized yielding: (c) Man-Ten (f) Al 7050.

The graphs of Fig. (9), built with the results presented in tables (1) and (2), show the influence of nominal stress on the notch section upon the stress concentration factor.

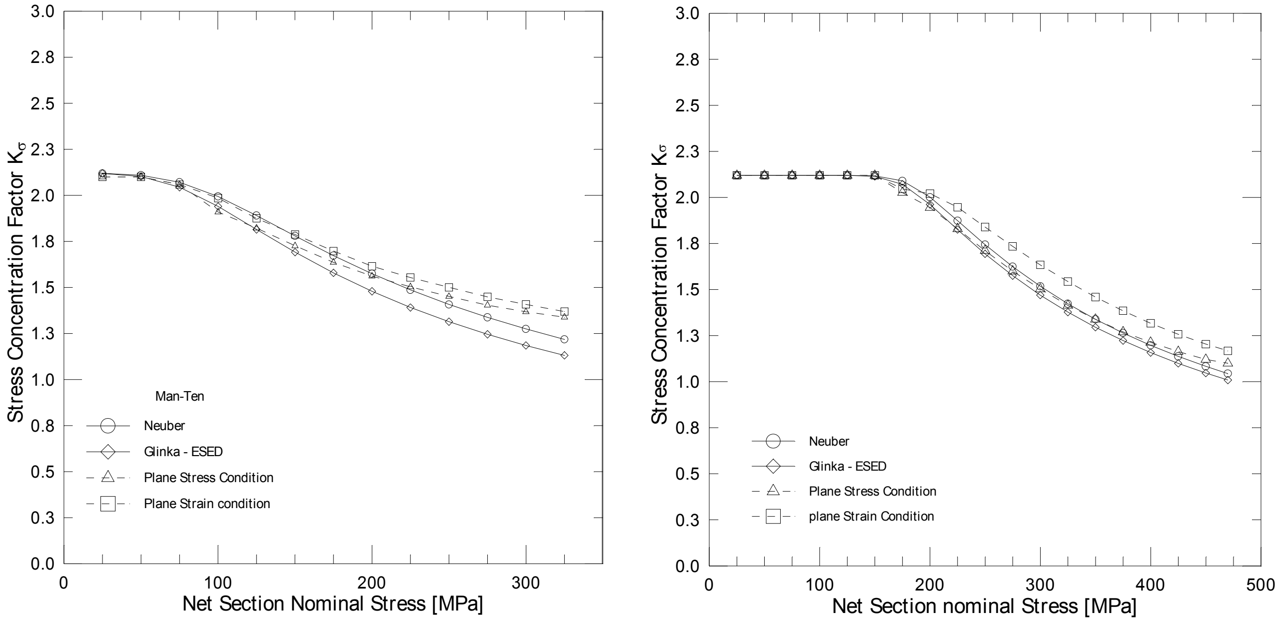

Figure 9. Behaviour of the stress concentration factor K_σ, for different nominal stress levels.

From Fig. (9) it can be seen that the two materials have a similar behaviour. The decrease of K_σ for loadings below yielding, on the opposite of the initial hypothesis of Neuber's model is due to the use of Ramberg-Osgood constitutive equation for the entire load range. This fact is more evident for the Man-Ten steel behaviour in reason of the magnitude of its hardening exponent and strength coefficient. On the other hand, the decrease of the stress concentration factor $K_\sigma$after yielding and the increase of the strain concentration factor, K_ε is confirmed. When the values of K_σ given by the Neuber and Glinka models are compared with the values obtained with the finite element method, underestimations of local stress levels up to 15 % could be observed.

The total strain levels at the notch root, estimated with the different models and by the finite element method using plane elements associated to plane stress state (points near the surface) and plane strain state (points inside the body), are plotted against the amplitude of nominal stress in Fig (10a) and (10b).

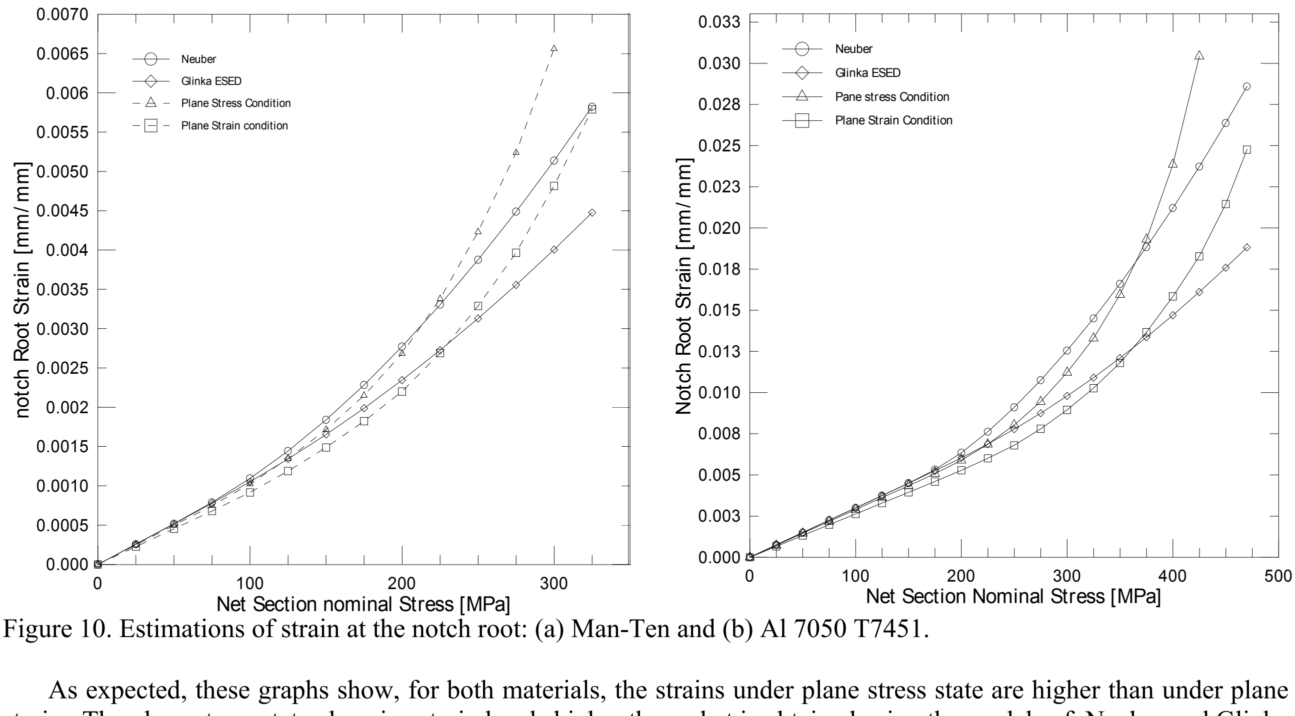

Figure 10. Estimations of strain at the notch root: (a) Man-Ten and (b) Al 7050 T7451.

As expected, these graphs show, for both materials, the strains under plane stress state are higher than under plane strain. The plane stress state also give strain levels higher than what is obtained using the models of Neuber and Glinka for the elastoplastic regime and fit adequately for the nominal stress levels which precede local yielding.

From a simple evaluation of the strain levels predicted by the Neuber and Glinka models it can be observed that these two models predict values with increasing differences to the nominal stress up to 20% in the case of generalized yielding for both materials.

When the results obtained considering the hypothesis of plane strain are analysed, it can be observed the Neuber model overestimates the strain at notch root. This deviation is minimized as the nominal stress on the discontinuity section approaches generalized yielding. The model is affected by the cyclic properties of the materials. It fits better the results for Man-Tem steel, presenting non significant errors while errors of 15% magnitude are found for Al alloy 7050 T7451 when compared with the values estimated under plane strain.

Glinka's model predicts better values for plane strain for nominal stress levels below yielding stress. However, after that point it underestimates the local strain level, giving values around 20% lower for generalized yielding.

5. Conclusions

In the present work, the finite element method and the models proposed by Neuber e de Glinka were used to verify the validity of the local strain approach and evaluate the strain levels and its relation with the nominal stress applied to a geometrical discontinuity section of a plate with a central circular hole. In this sense, the applicability of the models proposed by Neuber and Glinka was assessed comparing the results obtained with both methods and the results from the finite element analysis. The theoretical stress concentration factor for the linear-elastic regime was determined using photoelastic and finite element methods. The stress state at the surface and the strain state inside the component were evaluated using numerical methods and compared with the predictions given by the elastoplastic models proposed by Neuber and Glinka. The results show that Neuber and Glinka underestimates the stress values for plane stress, when compared with values given by finite element method. On the other hand the Neuber rule overestimates the strain levels while Glinka's model underestimates the strain levels under plane strain conditions. Al alloy 7050 T7451 and Man-Ten steel were used to study the effects of the behaviour of the stress/strain curve on the stress distribution, showing a general behaviour similar for both materials with small differences for nominal stress levels near the generalized yielding of the discontinuity section.

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