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ANALYSIS OF NUMERICAL ASPECTS ON THE SIMULATION OF RANKINE CYCLES

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Abstract. *This article presents a comparative study concerning two methods of solution of the algebraic set of equations generated by the simulation of Rankine cycles. The solutions and the performance obtained by using the Successive Substitution method and the Newton-Raphson method, also called here as Single Block method, are compared when applied to the simulation of five different configurations of Rankine cycles. Initially, a simple Rankine cycle is modeled, followed by more complex configurations, including reheating and regeneration. The simulation methods are applied to obtain the most important results in the analysis of power thermal systems. In most simulations, the final results are presented as a function of (1) required electric power, isentropic efficiency, and inlet turbine pressure; (2) thermal efficiency, steam generator pressure and temperature; (3) pressure and subcooling degree of the saturated liquid inside the condenser; and (4) pumps isentropic efficiency. The same results are obtained for both simulation methods employed. The method of Successive Substitution showed the lowest computational time, particularly for the most complex cycle configurations. An alternative application of the Single Block method was inconvenient for cycles with a more complex configuration due to the high computational time when all thermodynamic properties were included in the system of equations to be solved.*

Keywords: *Rankine cycles, simulation methods, thermal systems.*

1. Introduction

The huge number of studies and simulation softwares concerning steam power plants reflects the importance of this type of thermal systems to the electric energy generation worldwide. As showed by Badr et all (1990), several modifications and improvements regarding basic cycles are investigated with the aim of increasing their thermodynamic performances. Even small changes in efficiency can have a dramatic impact on the economic feasibility and also affects global energy resources due to the extensive generation of electric energy today.

Researches and engineers that build their own simulation codes have to make a choice regarding the numerical method or strategy of solution of the set of equations from the resulting coupled problem (Stoecker, 1989). Different alternatives of numerical methods can be employed to solve the same mathematical problem, but each of them can provide proper solutions for other aspects, like calculation running time, facility of construction of the code, and the use of commercial numerical subroutines.

This article discusses the use of some simulation methods for different alternatives of Rankine cycles, where a specific code is constructed allowing to explore different configurations of solutions.

2. Simulation Methods

The knowledge of different simulation methods provides several alternatives for researchers dedicated to the simulation of systems. Stoecker (1989) classifies the simulation methods of thermal systems as sequential or simultaneous. Considering a system composed by several equipments or components, the sequential method is applied when the output of one component is the input of the next one (Fig. 1) in a straight forward calculation, without further iterations. The solution for each equipment of the system exclusively depends on the results obtained in the calculation of the previous component. The system is coupled, but in a very weakly way, and it can be sequentially solved provided it is properly organized.

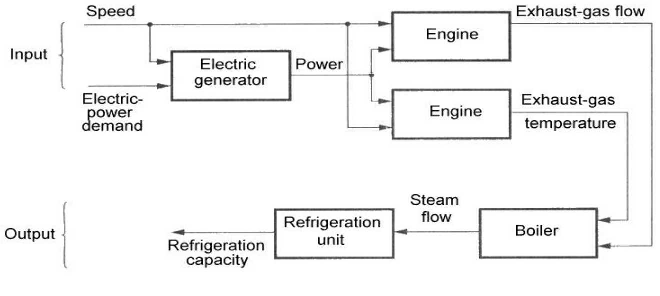


Figure 1 – Information flow diagram of the sequential simulation for a thermal system (Stoecker, 1989)

The method of simultaneous simulation, according to the same author, is applied when a system presents several interconnections and functional relations among its components, as shown in Fig. 2. It involves the simultaneous solution of the set of equations of the problem, followed by the estimation (initial guess) of some variables, if the system is non-linear. This is the most common situation in thermal systems, where many components are coupled together.

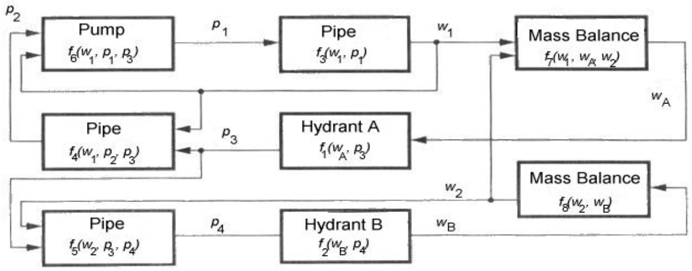


Figure 2 – Information flow diagram of the simultaneous simulation for a thermal system (Stoecker, 1989)

As power plants are usually characterized by several interconnections and information flows among its components, leading to a strongly coupled set of equations, only the methods of simultaneous simulation will be considered in this study. Two of these methods are presented here: the method of Successive Substitution, and the method herein designated as Single Block.

The method of Successive Substitution, which is strongly linked to the diagram of information flow of the system, consists in assuming initial guesses to a set of variables, and starting and iterative process of calculation, that will be repeated until convergence is achieved (Fig. 3). The solution method of this kind of simulation resembles the mathematical model of Gauss-Seidel for the solution of a system of equations.

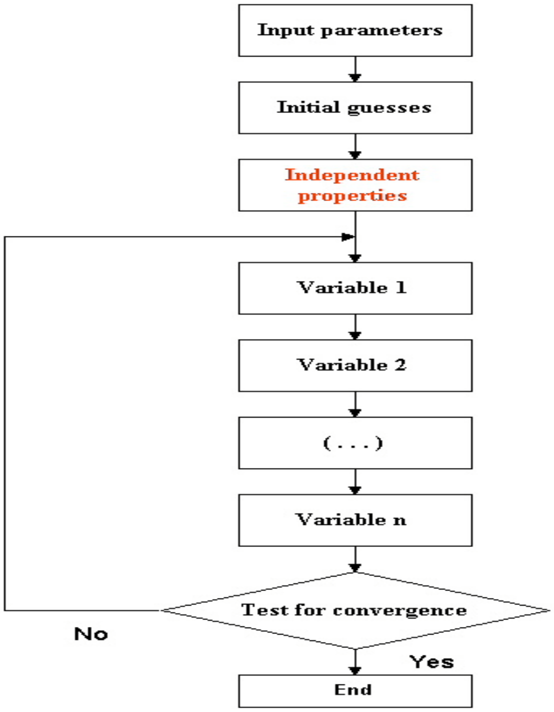
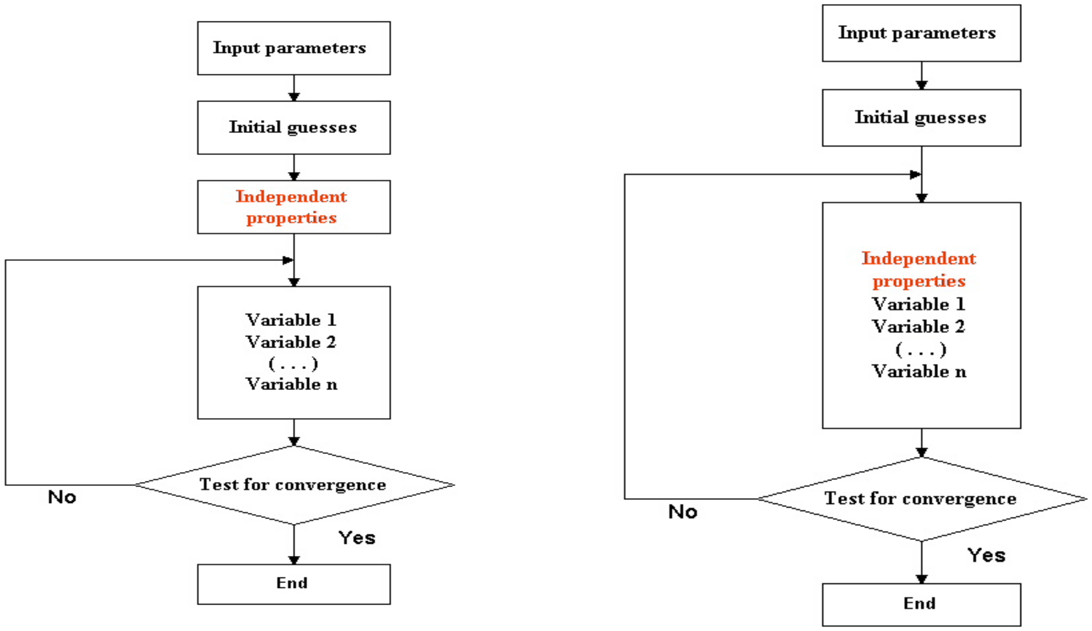


Figure 3 – Algorithm representing the method of Successive Substitution applied to a system

The Single Block method, also called Newton-Raphson method by Stoecker (*op.cit.*), consists in a simulation strategy where the set of coupled equations of the problem is solved in a single block by applying the method of Newton-Raphson or one of its derivatives, such as the method of Powell. As many different alternatives to solve the mathematical non-linear problem can be used, it is assumed in this text that the Newton-Raphson method will identify only the mathematical method and not the simulation strategy.

An algorithm representing the Single Block method is shown in Fig. 4 (a). Another alternative solution involving the same method is presented in Fig. 4 (b), where the determination of independent properties are also included in the set of equations that represents the system. This last option is similar to submit a complete set of equations to a solver, without previous selection or critical analysis of the nature of these equations. This independent properties are thermodynamics states that can be calculated from prescriptions of the cycle that will never change.



(a) (b)

Figure 4 – (a) Single Block algorithm applied to a system; (b) Alternative Single Block algorithm with independent properties.

3. Modeling of the cycles

Five configurations of Rankine cycles are proposed in this study. Most of the cycle components were modeled by applying mass and energy balances, taking into consideration the isentropic efficiencies of the steam turbine and pumps, and the thermal performance of the steam generator. The choice of the set of equations used to determine the thermodynamic properties of water was based on simplicity, low relative deviations in relation to data found in tables, and comprehensive inclusion of the regions of working fluid state (from compressed liquid to superheated steam at 22.1 MPa). The set of equations presented by Irvine and Liley (1984) was adopted as it suitably fit into these considerations. The equations presented by these authors deal with the determination of the thermodynamic properties of water for the saturation region and for the superheated steam region. This reference does not present a specific equation for compressed liquid, and therefore the properties of saturated liquid at the same temperature were adopted for this region. The largest deviations found in the calculation of the properties were 0.25% for the saturation region, and 0.56% for the superheated steam region as compared to the tables presented by Van Wylen and Sonntag (1993).

3.1 Simple Rankine cycle (RS)

Figure 5 presents the cycle model, herein designated as Simple Rankine (RS), and its respective $T \, x \, s$ diagram considered in the simulations.

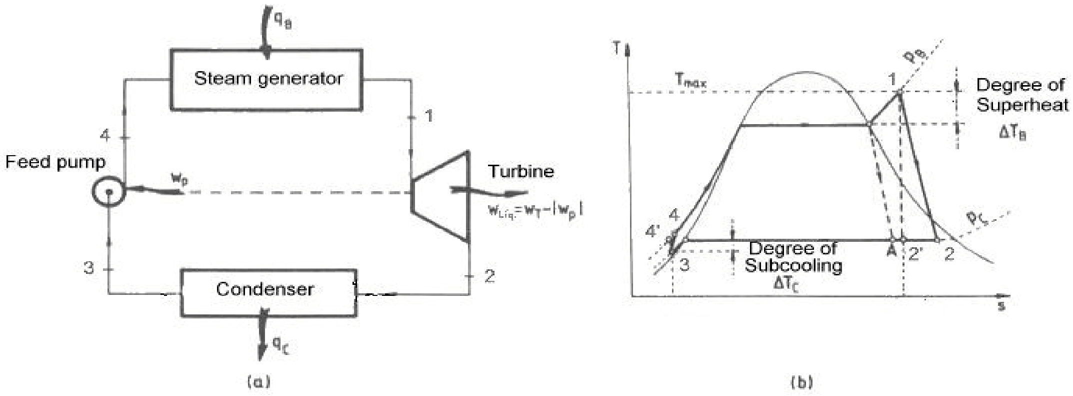
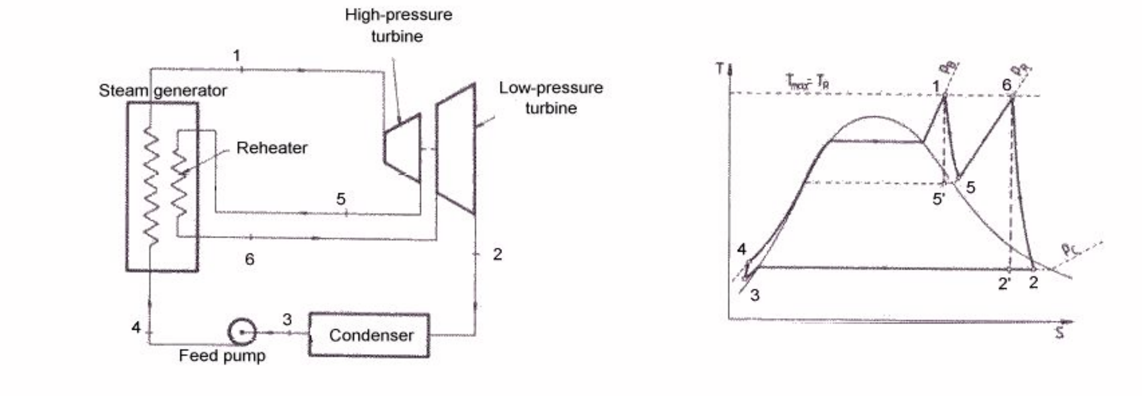


Figure 5 – (a) Simple Rankine cycle RS, and (b) corresponding $T \, x \, s$ diagram

The model presented in Figure 5 presents three main implementations, which differentiate it from an ideal Rankine cycle: (1) superheating of the steam generator, in order to increase mean temperature of the heat added to the system, consequently increasing the thermal efficiency of the cycle; (2) inefficiencies in the turbine and in the water feed pump, in order bring then closer to reality, where a completely isentropic behavior does not occur; and (3) subcooling of the condensed liquid before the water feed pump, in order to obtain a better and smoother performance in this operation.

3.2. Rankine cycle with reheating (RH)

Figure 6 shows the Rankine cycle with reheating (RH), with a two-stage steam turbine.

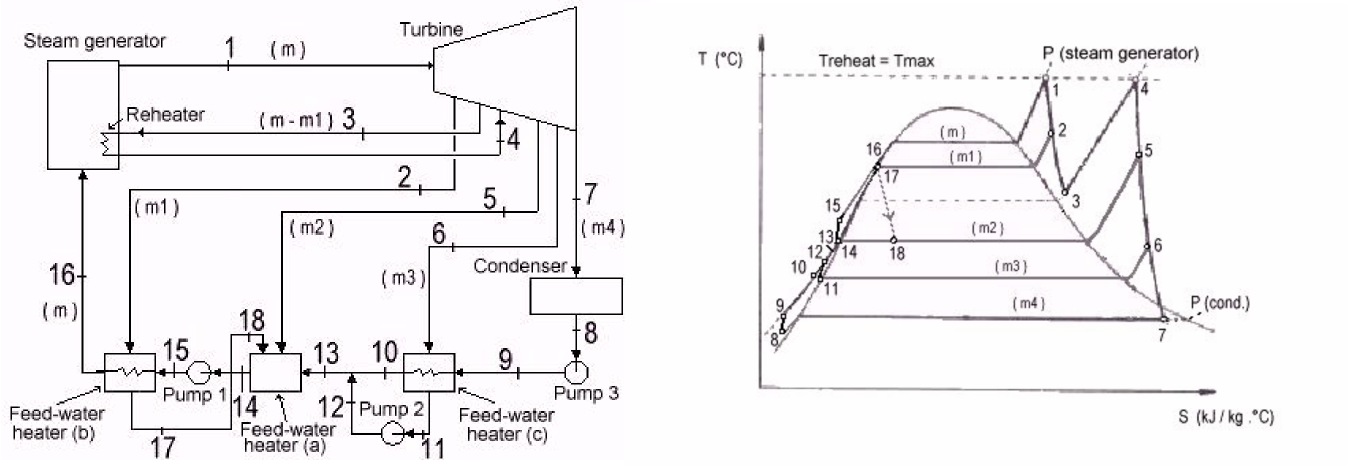


(a) (b)

Figure 6 – (a) Rankine cycle RH with a reheating stage; (b) respective *T x s* diagram

3.3. Rankine cycle with reheating and regeneration (RC)

The Rankine cycle with reheating and regeneration (RC) proposed in this study presents the three most conventional types of regeneration heat exchangers considered in thermoelectric power plant designs, according to Badr et al (1990). These regeneration types involve feeding water heaters commonly known as: (a) open (or direct-contact) type; (b) closed type with cascaded backward drains, and (c) closed type with forward pumped drains, according to Fig. 7.



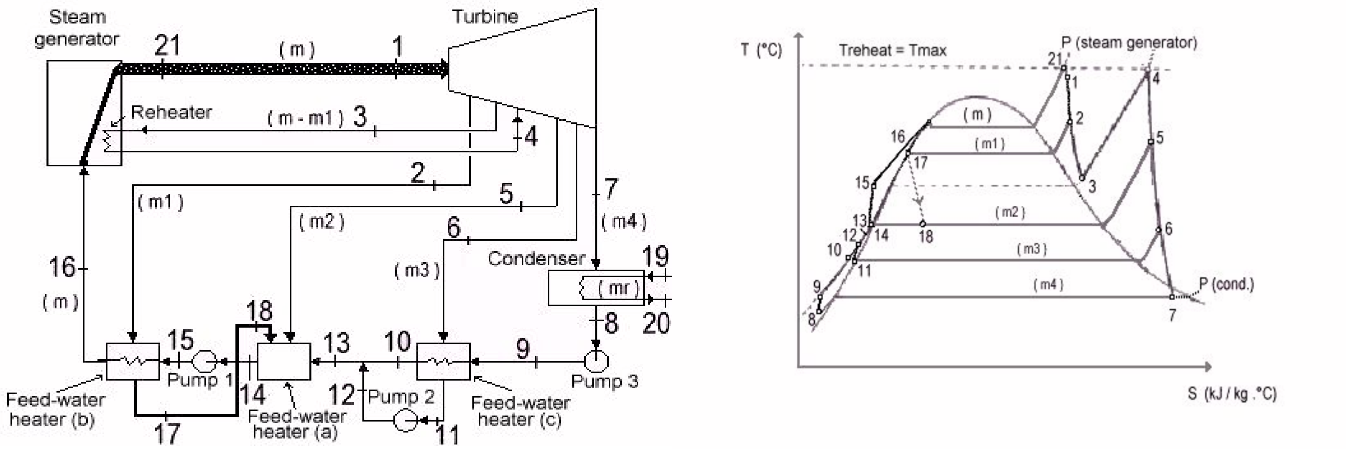
(a) (b)

Figure 7 – (a) Rankine cycle RC with reheating and regeneration developed for simulation; (b) corresponding *T x s* diagram

The feed-water exit temperature from a closed-type heater actually will not reach the inlet temperature of the bled steam. Thus, a *terminal temperature difference* (*TTD*) of around 4 to 6 °C is maintained to represent the real operation condition of the heater. The *TTD* is prescribed as the difference between the saturation temperature of the bled steam and the outlet temperature of the feed water (Badr, 1990). Therefore, feed-water exit temperatures are considered here as a function of the saturation temperature of the bled steam.

3.4. Rankine cycle with reheating and regeneration including piping friction losses (RA)

Based on the previous configuration, three new implementations are considered: friction losses in the piping that connects the steam generator to the turbine, internal pressure losses in the steam generator, and sizing calculation of the condenser. A diagram of the model of this cycle (RA) is shown in Fig. 8.



(a) (b)

Figure 8 – (a) Rankine cycle RA with reheating, regeneration and losses in the steam generator-turbine piping; (b) corresponding *T x s* diagram

3.5. Rankine cycle with reheating and regeneration including piping friction losses and efficiency curves of the turbine and main pump (PAN)

In the previous cycles, the isentropic efficiencies of the turbine and the pumps were previously assumed as inlet fixed parameters. Aiming to obtain more realistic results, the calculation of isentropic efficiencies of the turbine and the feeding pump of the steam generator were included, taking them as a function of the system mass flow, obtained from efficiency curves. The mass flow *m*, in kg/s, of the turbine is given by the equation proposed by Schegliaiev (1978):

$$m = m_n \cdot \sqrt{\frac{p_1^2 - p_2^2}{p_{1n}^2 - p_{2n}^2}} \cdot \sqrt{\frac{T_{1n}}{T_1}} \qquad (1)$$

where m_n is the nominal steam flow, in kg/s; p_1 and p_2 are the working steam pressures at the turbine admission and extraction, in kPa; p_{1n} and p_{2n} are the nominal steam pressures at the same positions, in kPa; T_1 and T_{1n} are the working and nominal steam temperatures at the turbine inlet, in °C.

Equation (1) is exclusively applied for the stage included between point 1 (turbine inlet) and point 2 (first extraction) showed in Fig. 8, that represents the high-pressure turbine. The resulting mass flow determines the value of the turbine isentropic efficiency, which is used as an fixed parameter for the other turbine stages. The isentropic efficiency of the other stages can be determined by the same procedure used before, once the nominal values of mass flow and pressure of turbine extractions are known. As to the pumps, the isentropic efficiency found for pump 1, which feeds the steam generator, was used as fixed parameter for pumps 2 and 3.

4. Simulations and results

The simulations were carried out under the hypothesis of continuous systems in steady state conditions. The five configurations of Rankine cycles where submitted to Successive Substitution and Single Block strategies of solution in order to obtain relevant results for the analysis of power thermal systems, such as cycle thermal efficiency, steam quality at the turbine exit, mass flow, pump power, and heat exchanged in the steam generator and in the condenser.

The simulation codes where developed in FORTRAN, using IMSL libraries. The NEQNF subroutine was employed on the Single Block method. This subroutine solves a system of non-linear equations using a modified Powell hybrid algorithm, and an approximation for the Jacobian matrix by finite differences.

In order to compare the performance of the simulation strategies, a Rankine cycle with reheating and regeneration (RC) was chosen. Table 1 presents the prescriptions considered in this case.

Table 1 – Inlet data for solving the Rankine cycle with reheating and regeneration (RC)

Inlet Parameter	**Symbol**	**Value**
Total power required in the turbine	P_t	63,00 MW
Pressure in the steam generator (turbine inlet)	P_1	11,30 MPa
Temperature in the steam generator (turbine inlet)	T_1	530,00 ºC
Pressure in the first turbine extraction	P_2	2,30 MPa
Reheating pressure	p_3, p_4	2,20 MPa
Reheating temperature	T_4	530,00 ºC
Pressure in the second turbine extraction	P_5	0,900 MPa
Pressure in the third turbine extraction	P_6	0,090 MPa
Condenser pressure (turbine exit)	P_7	0,007 MPa
Turbine isentropic efficiency	η_t	80 %
Pump isentropic efficiency	η_b	70 %
Steam generator isentropic efficiency	η_g	80 %
Subcooling degree of the liquid in the condenser	*GSRco*	5,00 °C
Subcooling degree of the liquid in the feed-water heater R1	*GSRr1*	0,00 °C
Subcooling degree of the liquid in the feed-water heater R2	*GSRr2*	0,00 °C
Subcooling degree of the liquid in the feed-water heater R3	*GSRr3*	0,00 °C
Terminal temperature difference in closed feed-water heaters	*TTD*	4,00 ºC
Quality adopted for the working fluid in point 18	X_{18}	0,20
Water specific volume of saturated liquid	v_{agua}	0,00101 m^3/kg
Convergence criterion	*Crit*	10^{-4}

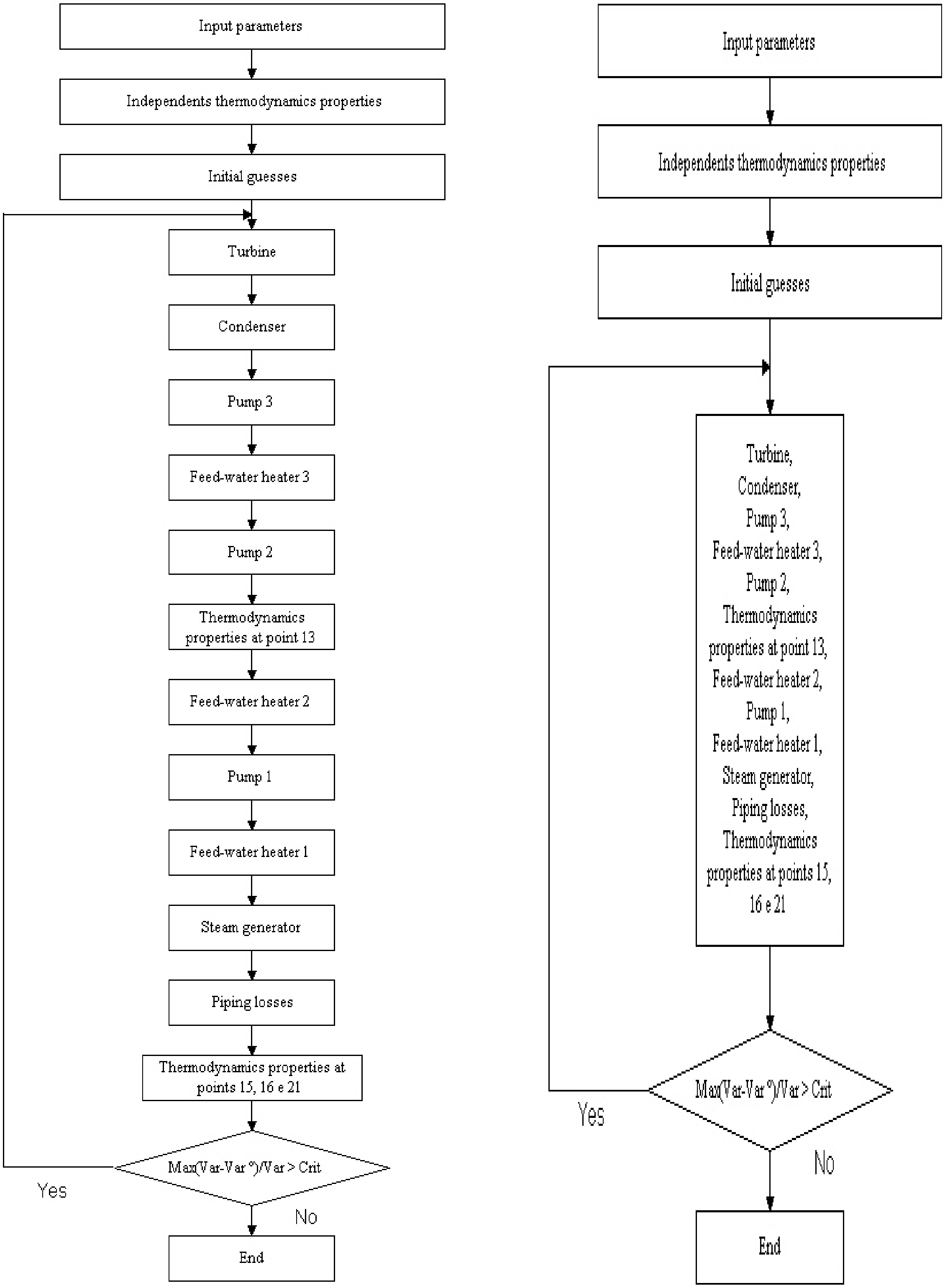
The proposed cycle was simulated using the Successive Substitution method and 2 versions of the Single Block method: alternative 1 solves only the system of coupled equations of the problem and alternative 2 does the same but also considers the set of independent thermodynamic variables. The diagrams corresponding to this two alternatives were displayed in Figure 4. Table 2 presents the most important results concerning this three options of strategies of solution.

Table 2 – Main results of the simulations of the Rankine cycle with reheating and regeneration (RC)

Results	**Symbol**	**Successive Substitution**	**Single Block Alternative 1**	**Single Block Alternative 2**
Heat supplied to the steam generator	q_g	194,88 MW	194,88 MW	194,88 MW
Heat exchanged in the condenser	q_c	94,48 MW	94,48 MW	94,48 MW
Total power required by the pumps	P_b	0,86 MW	0,86 MW	0,86 MW
Mass flow in the system	m	53,30 kg/s	53,30 kg/s	53,30 kg/s
Mass flow in turbine extraction 1	m	4,23 kg/s	4,23 kg/s	4,23 kg/s
Mass flow in turbine extraction 2	m_2	5,34 kg/s	5,34 kg/s	5,34 kg/s
Mass flow in turbine extraction 3	m_3	3,94 kg/s	3,94 kg/s	3,94 kg/s
Mass flow at the turbine exit	m_4	39,79 kg/s	39,79 kg/s	39,79 kg/s
Quality at the turbine exit	x_7	0,98	0,98	0,98
Cycle thermal efficiency	η_y	31,89 %	31,89 %	31,89 %
Relative computational time	***RCT***	**1,00**	**1,32**	**102,66**

Differences are observed only in *relative computational time* (*RCT*), which is defined as the ratio between the computational time of each method and the lowest computational time found among them. The *RCT* of the fastest method (Successive Substitution) is equal to 1. The Single Block Alternative 2 method presented a simpler and more practical configuration, where all the equations that characterize the system are inserted in a single routine, and in any sequence, what makes this alternative close to a common solver of equations. However, it brings the inconvenience of recalculating all independent thermodynamic properties for each iteration, despite being already determined in a previous iteration. This leads to a significant increase in computational time, that in approximately 103 higher than the lowest *RCT* time obtained for this case (Panosso, 2003).

A more complete configuration was simulated, considering a Rankine system RA (Fig. 8) with reheating and regeneration, including the condenser unit sizing, the calculation of the pressure drop along the piping connecting the steam generator to the turbine and a prescription of the pressure drop on the steam generator. After the results of time calculation from the former case, the simulations were ran using either the Successive Substitution method or the Single Block-Alternative 1 method, whose block diagrams are shown in Fig. 10.



(a) (b)

Figure 10 - Algorithm for Rankine cycle RA, employing: (a) Successive Substitution method, or (b) the Single Block-Alternative 1 method. Properties are pointed in Fig (8).

4. Analysis and discussion

The five system configurations were submitted to the three strategies of solution presented in this paper, in order to compare their performance. As all the results were pretty much similar, only the computational outputs were analyzed. Table 3 presents the main results of the simulations, where the steam quality and thermal efficiency are shown just to provide some useful and interesting data of these five cycles.

Table 3 – Results of the employment of the simulation methods

			RCT						
Rankine cycle	**Number of equations**	**Convergence criterion**	**Successive Substitution (SS)**	**Single Block (1)**	**Single Block (2)**	**Sub relaxation (SS)**	**Real time (SS)**	**Steam quality turbine exit**	**Cycle thermal efficiency**
RS	**24**	**10^{-4}**	**1,00**	**1,24**	**25,29**	**-**	**5,01 s**	**0,91**	**26,24 %**
RH	**32**	**10^{-4}**	**1,00**	**1,23**	**29,55**	**-**	**6,70 s**	**0,99**	**27,91 %**
RC	**98**	**10^{-4}**	**1,00**	**1,32**	**102,66**	**0,5**	**7,56 s**	**0,98**	**31,89 %**
RA	**120**	**10^{-4}**	**1,00**	**2,45**	**-**	**0,5**	**116,97 s**	**0,98**	**31,86 %**
PAN	**130**	**10^{-5}**	**1,00**	**2,05**	**-**	**0,1**	**776,97 s**	**0,97**	**31,31 %**

The comparison among the five cycle configurations indicated that the best thermal performances are obtained when the Rankine cycle is implemented with reheating and regeneration (RC, RA and PAN). The implementation of reheating allows better steam qualities, closer to the state of dry steam, which are considered as acceptable in order to avoid the corrosion of the turbine blades (RH, RC, RA and PAN). When the internal losses in the steam generator and losses in the piping connecting the steam generator to the turbine are considered, the pump 1 in the RA cycle needs to deliver water at higher pressures in order to compensate the pressure drop. In the performed simulations, the pressure pump 1 for the cycle with no losses (RC) was 11.30 MPa, whereas when losses were assumed, this value increased to 13.65 MPa (RA). The thermal efficiency was reduced in 0.03%, going from 31.89% in (RC) to 31.86% in (RA), which is closer to reality.

The Successive Substitution method had always the lowest computational time (*RCT*=1.00), particularly for the more complex cycle configurations. Table 3 also shows the dimensional computational time spent by the employment of this method, using a 750-MHz Pentium III computer. As expected, the higher number of equations implies in an increase of computational time, that is significantly noted when piping calculations are considered (RC to RA).

Solving more complex configuration cycles with the Successive Substitution method requires the use of a sub relaxation coefficient, in order to achieve convergence. The criterion of convergence for the simulation of the PAN cycle, that involves the calculation of the isentropic efficiencies of the turbine and pump 1, had to be increased from 10^{-4} to 10^{-5}, and the real computational time was 7 times higher compared with the RA cycle.

The Single Block-Alternative 2 presented an inconvenient computational time as compared to Alternative 1 because the independent thermodynamic properties have to be recalculated at each iteration performed on the set of equations of the problem. Therefore, this method was not taken into consideration for configurations with more than 100 equations (RA and PAN).

5. Conclusion

Simultaneous simulation methods were used to solve the set of equations that represents five different configurations of Rankine cycles. The options presented in this paper were: Successive Substitution method, and the two alternatives of Single Block method.

The Successive Substitution and the Single Block-Alternative 1 methods were characterized by the determination of the thermodynamic properties of water for independent states in an external subroutine to the iterative sequence of calculation, in opposition to the Single Block-Alternative 2 method. Despite this former method provided a simpler and more practical way to assemble and solve the set of equations, it had an inconvenient computational time as compared to the Successive Substitution and Single Block-Alternative 1 methods.

The same results were obtained with the use of both analyzed methods, showing their applicability in Rankine cycle simulations. The Successive Substitution method always presented the lowest computational time, particularly in more complex configurations. This method allows routine calculations of the components of a thermal system to be separately developed, even by different programmers, permitting the inter-exchange of routines in the main program. On the other hand, the Single Block method is very practical as it deals with equations that are characteristic of the whole system. The choice of one or other simulation method will therefore depend on the factors required at the moment of the simulation, such as being practical, fast, inter-exchangeable, etc.

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