DEVELOPMENT OF SOFTWARE DEDICATED TO RELIABILITY-BASED DESIGN OF MECHANICAL PARTS SUBJECT TO FATIGUE FAILURE

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Abstract. *The use of probabilistic techniques in engineering design, widely used in the naval, nuclear and aerospace industry, has become more necessary in other engineering areas, such as automobile, electric and civil engineering, aiming the optimization of a specific design and using as constraints the product reliability and the costs associated with design and manufacturing. The main difficulties of applying such techniques as part of a design criteria are the unfamiliarity of efficient probabilistic methods for the designer, the complexity of the calculations, and the acquisition of laboratory or field data to describe the uncertainties of the random variables considered as design parameters. Another important point is the training of the designer to understand this new reliability-based design approach. In order to minimize these difficulties, this study proposes the development of software dedicated to reliability-based design of mechanical parts subject to fatigue failure. This failure mechanism is chosen due to the great dispersion associated with the materials fatigue resistance, increasing the uncertainty of the part operational life, making the fatigue failure the most common failure of mechanical parts or structures subject to dynamic loading. The computer code uses a limit state function based on the Palmgren-Miner's cumulative damage rule, using as random variables, described by probability density functions, the critical damage, the fatigue resistance and the dynamic stress acting on the part. The code executes reliability analysis, checking the failure probability for a specific operational life, or, given a target failure probability, the code suggests the mean dynamic stress, the operational life or the part material, in order to fulfill the reliability requirement.*

Keywords. *reliability, fatigue, design methodology.*

1. Introduction

Fatigue is the most important failure mode to be considered in mechanical and structural design subject to dynamic loading. Approximately, 80% of mechanical and structural failures are caused by the fatigue mechanism, as describes Sundararajan (1995). The fatigue failure process involves the crack nucleation and its consequent propagation until the rupture of mechanical or structural part when submitted to stress action, whose magnitude is variable with the time. The consequence of fatigue failure usually is catastrophic, occurring suddenly and without a previous signal easily detectable, and may cause loss of human life.

Usually, in the design stages of components submitted to cyclic loads, the designer define its operational life, which corresponds to the period of use of the component under specified conditions of external loads, and the dimensions of the component must be calculated to avoid the occurrence of fatigue failure during the operational life. The variables that influence the fatigue failure mechanism are considered deterministic, so the uncertainty associates with them and with the calculation procedure of the fatigue life are considered with the use of a safety factor, aiming to reduce the possibility of the failure, even if the variable, in the real situation of use, assumes different magnitude of the used as design load.

Due to great amount of uncertainty associates to the fatigue phenomenon, a probabilistic approach must be used to get a economically and technically feasible design. However, for the application of probabilistic methods to the project, some difficulties are found, as the designer's lack of knowledge of efficient probabilistic methods of calculation, complexity of the calculations, data acquisition from laboratories or field tests, to feed the failure probability evaluation algorithm, and paradigmatic change of design methodology.

This study aims the development of a computational system to execute probabilistic analysis of the fatigue process in component subject to high-cycle fatigue failure. Such computational system has as main characteristic the capacity of accomplishment the statistical calculations, including some decision-making capacity from its knowledge base and rules. The answers to the users are the failure probability of the component under a loading condition, and the permissible stress or the permissible number of cycles, for which the component could be submitted in order to present a target failure probability, specified for the user of the system. Moreover, the system allows the exchange of the part material analyzed to obtain the failure probability appropriated to the project.

2. Fatigue Phenomenon

Fatigue failure of a component or mechanical structure results from repeated dynamic loads in the form of tension or deformation, that presents the maximum magnitude lower than that necessary to cause rupture under action of static loading, (Dieter, 1981).

2.1. High-Cycle Fatigue

High-cycle fatigue is considered as the process in which the failure occurs for a number of alternating load cycles higher than 10^4 cycles (Fuchs; Stephens, 1980). The base for the application of the method of high-cycle fatigue is the diagram stress *versus* the number of cycles for the failure (*S-N*), determined through the rotating-bending fatigue test (Wöhler test). This method has been traditionally used in projects of mechanical components as shaft, gears, wheels and others mechanical components. The high-cycle fatigue considers that the number of cycles for crack nucleation is much higher than the number of cycles for crack propagation until the final rupture. The hypothesis behind this method is that once the crack is detected, the component residual strength is very small.

2.2. Method of the Limits States Applied to the Fatigue

The conventional fatigue test submits a specimen to a stress cycle with constant amplitude until it breaks. The test can be made for some values of stress in order to determine the *S-N* curve, but always the sine cyclical loading is kept constant. However, in many practical applications, the alternating stress amplitude that acts on the component does not remain constant. In this case it is possible to identify stress cycles, and for each cycle with stress fluctuation higher than the fatigue limit of the material, a damage is introduced in the component.

To each stress fluctuation an amplitude of cyclic stress can be associated, of magnitude equal to the half of the stress fluctuation. The total life of a component subject to this type of loading can be estimated through the percentage of the life consumed for each cycle of applied stress. The formulation of Palmgren-Miner generally is used to represent the accumulated damage of a component subjected to fatigue. Assuming that the total deterioration of the resistance will occur when the damage D assumes value greater than 1, the limit state formulation for fatigue failure will be:

$$D = \sum_{i=1}^{n_S} \frac{n_i}{N_i} = 1 \tag{1}$$

where n_i.is the number of loading cycles with stress amplitude S_{ai};, and N_i is the number of permissible cycles extracted of the *S-N* curve, for the stress S_{ai}. Figure (1) shows an example of the accumulated damage theory using *S-N* curved.

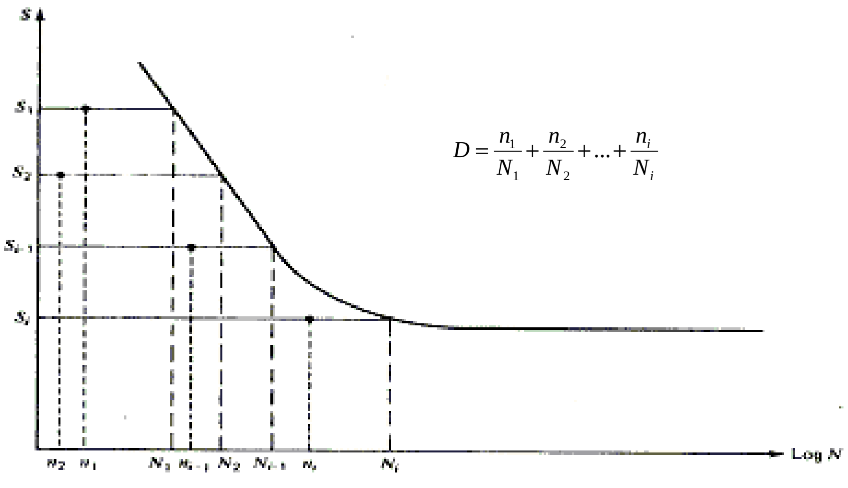


Figure 1. Use of the *S-N* curve for damage determination by Palmgren-Miner’s criterion (Collins, 1993).

The mathematical relation between stress and number of cycles for the *S-N* curve is written as (Sundararajan, 1995):

$$NS^b = K_S \tag{2}$$

where K_S is a constant that defines the position where the straight line cross with the axle *logN*; b is the angular coefficient of the straight line.

For one loading cycle the damage can be expressed with the following formulation:

$$d = \frac{1}{N(S_a)} \tag{3}$$

where d is the damage accumulated in one cycle with cyclic stress with S_a amplitude, and $N(S_a)$ is the number of cycles that the material would support under action of the cyclical stress S_a.

Considering the *S-N* curve in the formulation of damage d the relation can be written below:

$$d = \frac{1}{K_S} S_a{}^b \tag{4}$$

where b is the angular coefficient of the straight line, or either, the inclination of the *S-N* curve in the logarithmic domain.

Considering that the stress fluctuation acting on the component varies with time, the cumulative damage in the structure after n cycles of tension with amplitude S_{ai} will be:

$$\Delta d = \sum_{i=0}^{n-1} d_i \tag{5}$$

where $d = \frac{1}{K_S} S_{ai}{}^b$.

Applying the linear failure criterion of the cumulative damage, the structure will fail when:

$$1 - \frac{1}{K_S} \sum_i S_{ai}{}^b \leq 0 \tag{6}$$

Finally, the limit state equation that governs the fatigue phenomenon of a component, considering the high-cycle fatigue process, can be written in following way (Assakkaf; Ayyub, 1999):

$$Z_S = g_S(x) = \frac{\Delta K_S}{f^b S_{eq}{}^b} - N_{eq} \tag{7}$$

where S_{eq} is the equivalent stress that represents the sum of stress, D is the critical damage associated with Palmgren-Miner's rule, and f is a factor associated with existing uncertainties in the equivalent stress that acts in the structure, which are related to the uncertainties in the external loading and in the stress evaluation method. The fatigue failure occurs when $Z_S \leq 0$.

3. Reliability Fundamentals

The limit state function presented in Eq. (7) can be used for probabilistic analysis if the variables are considered random. The main goal of the probabilistic approach is to calculate the fatigue failure probability, represented by the probability of $Z_S \leq 0$.

If the joint probability density function for the basic random variables x_i's is known, represented by $f_x(x_1, x_2, \ldots, x_n)$, the failure probability can be given by the integral:

$$P_F = \int_\Omega f_x(x_1, x_2, ..., x_n) dx_1 dx_2 ... dx_n \tag{8}$$

where Ω represents the failure domain, where Z_S £ 0.

In general, the joint probability density function is unknown, and the integral is a formidable task. For practical purposes, alternate methods of evaluating the probability of failure are necessary. Considering known the probability density function of each random variable, the limit state function can be expanded using a Taylor series about the mean values of the random variables, and them truncated at the linear terms. A measure of reliability can be estimated by introducing the reliability index b that is based on the relation between the mean and the standard deviation of the truncated function, as presented by the references (Ang et al., 2001), (Castillo et al., 1999), (Madsen; Krenk; Lind, 1986), (Melchers, 1987), (Shen, 1999), (Souza, 1994) and (Sundararajan, 1995). This method is named conditional probabilistic method, or Level II method, and is the most used in the determination of the reliability of structural or mechanical problems. This optional must be placed before the reference list.

3.1. Advanced First Order Second Moment Method

The greatest imprecision in the method of the average values is consequence of the limit sate equation linearization, once the linearization point corresponds to the vector formed by the mean value of the basic variables, which is placed in the space region for $Z_S > 0$. The greater the distance of this point to the surface $Z_S = 0$, the greater will be the error in the calculation of the reliability index or in the failure probability. To solve this problem the Advanced First Order Second Moment Method (AFOSM) was developed, in which the point chosen for the linearization of the limit state function is located on the surface where $Z_S = 0$. Defining the limit state function in the following form:

$$Z_S \cong g(\underline{x}^*) + \sum_{i=1}^{n} (x_i - x_i^*) g_i'(\underline{x}^*) \tag{9}$$

where $\underline{x}^*$ is the vector representing the coordinate of the linearization point., $g(\underline{x}^*)$ is the limit state function calculated at $\underline{x}^*$, and $g_i'(\underline{x}^*)$ is the partial derivative of $g(\cdot)$ regarding the random variable x_i calculated at $\underline{x}^*$.

The mean value of function Z_S is express as:

$$m_z \cong g(\underline{x}^*) + \sum_{i=1}^{n} (m_i - x_i^*) g_i'(\underline{x}^*) \tag{10}$$

and its standard deviation is written as:

$$s_z \cong \left[\sum_{i=1}^{n} (g_i'(\underline{x}^*) s_i)^2 \right]^{1/2} \tag{11}$$

In this method, the standard deviation of the linearized limit state function is expressed from a linear combination of standard deviation of the involved basics variables in the problem in the following way (Dai; Wang, 1992) and (Souza, 1994):

$$s_z = \left[\sum_{i=1}^{n} a_i g_i'(\underline{x}^*) s_i \right] \tag{12}$$

where the value of the constant of proportionality a, referring to each basic variable, is determined for the relation:

$$a_i = \frac{g_i'(\underline{x}^*) s_i}{\left[\sum_{j=1}^{n} (g_j'(\underline{x}^*) s_j)^2 \right]^{1/2}} \tag{13}$$

Once the linearization point is located on the surface defined for $Z_S = 0$, and combining Eq. (10) and (11) the reliability index is defined as:

$$b = \frac{\sum_{i=1}^{n} (m_i - x_i^*) g_i'(\underline{x}^*)}{\sum_{i=1}^{n} a_i g_i'(\underline{x}^*) s_i} \tag{14}$$

and the failure probability is defined as $p_f = F(-b)$, where $F(\cdot)$ is the cumulative distribution function of the standard normal variate. In this method the definition of the linearization point of the limit state function is fundamental, which can to be determined through the solution of Eq. (15), is defined from a regrouping of the terms of Eq. (14).

$$x_i = m_i - a_i\, b\, s_i \tag{15}$$

The set of the points $(x_1^*, x_2^* \ldots x_n^*)$ defines the linearization point of the limit state function. For the solution of Eq. (15) is necessary to adopt an initial value for the reliability index b, and, later, to use an algorithm to define the point of linearization $\underline{x}^*$ and the value of b for which the limit state function equals zero, that is, the point that defines the failure. The references (Dai; Wang, 1992), (Souza, 1994) and (Sundararajan, 1995) suggest a procedure for the determination of the linearization point, used in the development of the computational system.

Beyond an optimized linearization of the limit state function, this method is valid for non-correlated normal distributions. In case the basics variables are correlated or have another distribution type, the model above presented will have to undergo corrections aiming the improvement of the estimate of the reliability index. In case of some variables present non normal distribution and knowing their probability density function, the references (Dai; Wang, 1992), (Souza, 1994) and (Sundararajan, 1995) suggest the application of the Rackwitz method. This method introduces a correction in the calculation of the mean value and the standard deviation of the basics variables, looking for getting at value x_i*, an equivalence among the values of the probability density functions and accumulated distribution that really describe the behavior of the variable and the values of these same functions calculated for the normal distribution. The correction is made through the following functions:

$$m_i^n = x_i^* - F^{--1}[Fx_i\,(x_i^*)]\,s_i^n \tag{16}$$

$$s_i^n = \frac{f\{F^{-1}[Fx_i\,(x_i^*)]\}}{fx_i\,(x_i^*)} \tag{17}$$

where m_i^n is the corrected mean value of basic variable x_i; s_i^n is the standard deviation corrected of basic variable x_i; $fx_i(x_i^*)$ is the probability density function magnitude of basic variable x_i, calculated at point $x_i = x_i^*$; $Fx_i(x_i^*)$ is the cumulative distribution function magnitude of basic variable x_i calculated at point $x_i = x_i^*$; $f^n(a)$ is the magnitude of the probability density function of standard normal variate, calculated at point a; $F^{--1}(b)$ is the coordinate that correspond to magnitude equal to b for cumulative distribution function of standard normal variate.

These values must be used in the calculation of the proportionality constants a of the basics variable, Eq. (13), as well as in the definition of the linearization point, Eq. (15), used in the procedure of calculation of the reliability index, in accordance with the advanced first order second moment method.

4. Software Characteristics

To execute the fatigue probabilistic analysis based on the limit state functions presented on Eq. (7) a specific software was developed (Freitas, 2002). The main goal of the software is to calculate the fatigue failure probability considering as random variables the constant K_S, the equivalent stress S_{eq}, the critical damage of the Palmgren-Miner's rule D and the coefficient f. The algorithm for reliability analysis is the AFOSM and in Fig. (2) a flowchart of the software is presented.

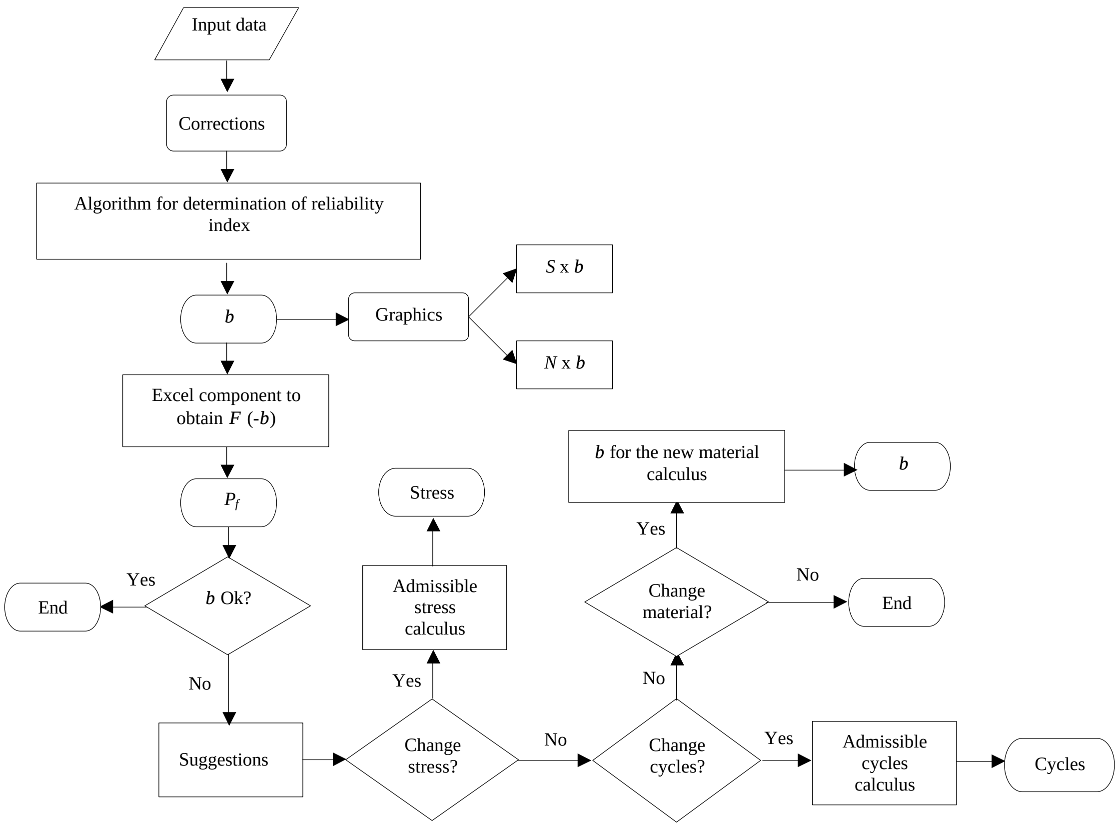


Figure 2. Flowchart of the software.

The input variables of this software are the critical damage D, the constant K_S, the equivalent stress S_{eq}, the coefficient b, the factor f, and the number of cycles N. Each one of the variables is defined with it mean and standard deviation, except the number of cycles and coefficient b, considered deterministic. This program offers for the designer the options of change the stress, cycles or material to obtain the target reliability index.

The reliability analysis considering a great variety of probability density functions is a great advantage in this program when compared to the method recently proposed by Shigley (2001), where the fatigue reliability is evaluated in a close form, but the method is valid only for lognormally distributed variables. In many fatigue reliability analysis, the stress is distributed according to a Weibull or Rayleigh probability functions that can be used in the software. The code also allows the definition of mean value of random variables in order to achieve a target reliability helping the designer for some decision-making about the sizing of the mechanical part.

5. Case Study

This example uses the shaft indicated in Fig. (3), which is submitted to a lateral loading, represented by concentrated loads of magnitude 8900 N and 13350 N. This shaft is supported in two bearings, mounted in the extremities of the shaft, being its center lines indicated in the positions *A* and *B* of Fig. (3). The shaft rotates with constant angular speed and is manufactured with steel whose yielding strength is 613.21 MPa.

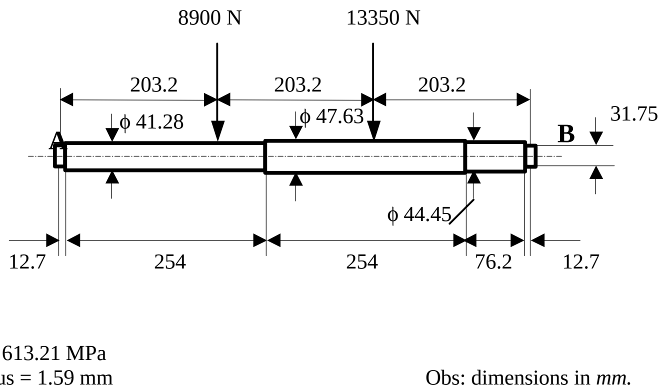


Figure 3. Sketch of the shaft.

5.1. Analysis of the Case Example

Initially the traditional deterministic analysis of the shaft is executed, which aims at the determination of the number of loading cycles that the part can be submitted until the occurrence of fatigue failure. The graphic of the bending moment is presented in Fig. (4).

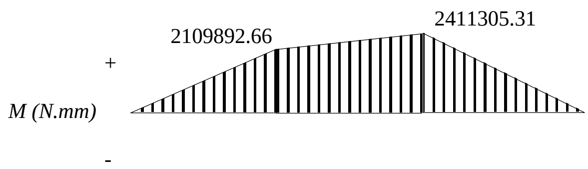


Figure 4. Bending moment on the shaft.

The bending moments in the transitions are:

a) Transitions *f* 41.28 – *f* 47.63 mm → *M* = 2204084.11 N.mm.
b) Transitions *f* 47.63 – *f* 44.45 mm → *M* = 1054944.93 N.mm.

Therefore, the critical section to be analyzed in the shaft is the transition *f* 41.28 – *f* 47.63, which the bending moment magnitude is of 2204084.11 N.mm. The nominal alternating stress S_{nom} that acting in the critical section of the shaft, calculated through the equation below, will be 319.16 MPa.

$$S_{nom} = \frac{M}{W} \tag{18}$$

where *W* is the transversal resistance modulus of section, in the region where the shaft has the smallest diameter. The stress concentration fatigue factor K_f, used in the correction of the cyclical stress in the transition, is determined through the following relation (Shigley, 1984):

$$Kf = 1+q(Kt - 1) \tag{19}$$

The stress concentration factor K_t and the notch sensitivity coefficient q used in Eq. (19) are determined through charts present by Shigley (1984), then Kt = 2.0, q = 0.76 and the stress concentration fatigue factor is Kf = 1.76. The cyclic stress S_a acting in the transition of the shaft is determined with the Eq. (20), where the nominal stress is multiplied by the stress concentration factor:

$$S_a = Kf.S_{nom} \tag{20}$$

S_a = 561.72 MPa

To determine the fatigue limit of the material, the relation proposal by Shigley (1984) is used, that affirms the proportionality between the fatigue limit and the ultimate strength. The fatigue limit considered in the study is:

$$S_e = k_a.k_b.k_c.k_d.k_e.k_R\ (0.504.S_u) \tag{21}$$

where the load factor k_c, temperature factor k_d, others factor k_e and reliability factor k_R are equal to 1.0.

$$k_a = 1.58.(S_u)^{-0.085}\ \therefore\ k_a = 0.916 \text{ (surface roughness factor)} \tag{22}$$

$$k_b = \left(\frac{d}{7.62}\right)^{-0.1133}\ \therefore\ k_b = 0.826 \text{ (size factor)} \tag{23}$$

S_e = 233.84 MPa

The coefficient of 50% reliability was used, and this related with the scatter of the fatigue strength of the material. As S_a assumes magnitude greater than S_e, the shaft will have finite life, being necessary to determine which will be this life. The equation that represents the *S-N* curve is written in the following way (Shigley, 1984):

$$S_{a,N} = C.N^{b'} \tag{24}$$

$$C = \frac{(0.9.S_u)^2}{S_e} \tag{25}$$

$$b' = -\frac{1}{3}\log\left(\frac{0.9.S_u}{S_e}\right) \tag{26}$$

where b' is slop of fatigue curve (in relation to log$N$ axle).

In case that the new *S-N* curve is very influenced by the fatigue limit, which must incorporate the correction due the stress concentration, since, physically, the *S-N* curve for a body with stress concentration is different of the *S-N* for the situation without stress concentration. So:

$$S_e = \frac{Se}{Kf} \tag{27}$$

Hence S_e = 132.86 MPa, S_a = 319.16 MPa and the fatigue curve equation will be $S_{nom} = 2292{,}50.(N)^{-0.206}$. Therefore with S_a = 319.16 MPa = $S_{nom} = 2292.50.(N)^{-0.206}$, we have:

N = 14348.3 cycles

5.2. Probabilistic Analysis of the Case Example

The failure probability of the component will be calculated with the data presented in Tab. (1). For the determination of standard deviations of the variables D, S, and f involved in the analysis is used a coefficient of variation of small magnitude, with value 0.01 to guarantee low scatter of the data, validating the deterministic hypotheses for the probabilistic calculation. The mean of variable K_S is determined by Eq. (28), and its standard deviation is determined through a coefficient of variation with value equal to 0.08, as recommends Shigley (1984). The variable b and N assume constant values.

$$K_S = \left(\frac{1}{C}\right)^b \tag{28}$$

where $b = \left|\frac{1}{b'}\right|$ is the angular coefficient of the fatigue curve.

Table 1. Input data to the software.

	Mean	*Standard Deviation*	*Probability Distributions*
D	1	0.01	lognormal
K_S	$2.052.10^{16}$	$1.642.10^{15}$	lognormal
b	4.854	---	---
S_{eq}	319.16	3.19	lognormal
f	1	0.01	normal
N	14348	---	---

Using the software developed for probabilistic fatigue analysis a reliability index equal to –0.1 was calculated, according to the data presented in Fig. (5), that represents the software main screen. This reliability index corresponds to a 53.98% probability of failure, which is coherent with the deterministic calculation, since the choice of the coefficient of reliability, used for the choice of the *S-N* curve, was made assuming a 50% reliability, or a 50% possibility of the fatigue strength to be lower than the calculated value. In this situation, being the others variables deterministic, the failure probability of the shaft is directly related with the reliability applied to the *S-N* curve.

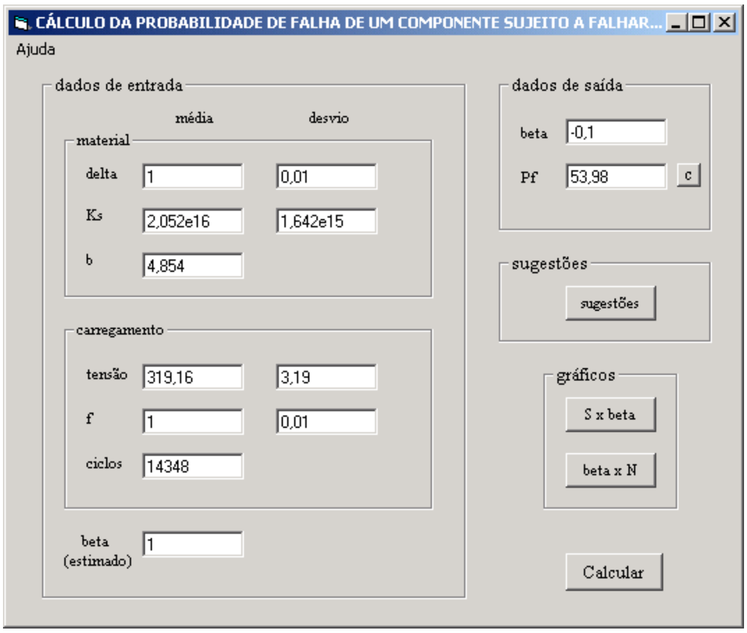


Figure 5. Software main screen.

Assuming that the value of the target reliability index in this project is 3.0, that is, a failure probability of 0.13%, the program suggests three change possibilities: the reduction of the cyclic stress magnitude; the reduction of the shaft life; the change in the shaft material.

To get the value of *b* equal to 3.0, the maximum cyclic stress must be of 297.47 MPa. However, to reach this stress level, lower than the original value, the operating loading has to be reduced, which is not interested for the project, or the diameter of the shaft has to be increased. Calculating the shaft diameter necessary to reach the stress of 297.47 MPa according to deterministic approach, the following results is obtained:

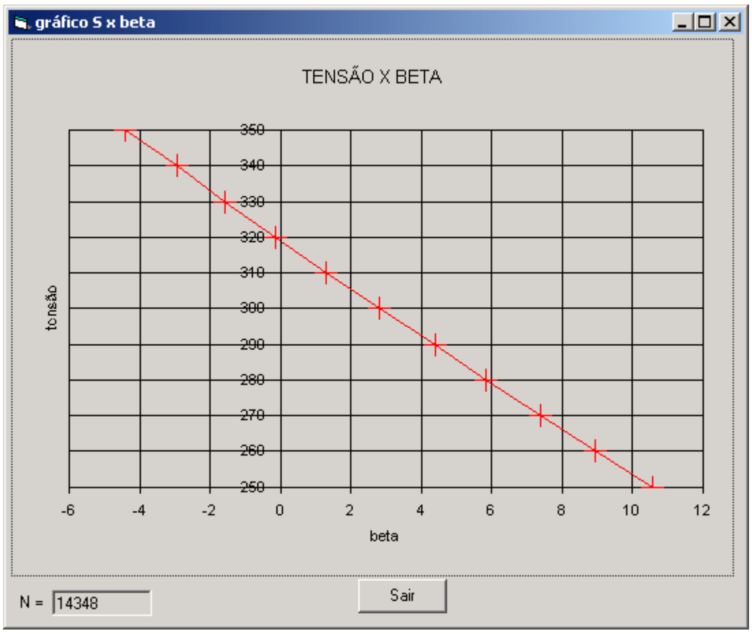
d = 42.26 mm

With the aid of the developed software, the maximum number of cycles to which the shaft could be submitted, remaining the original stress of 319.16 MPa, for a failure probability equal to 0.13 %, will be 10402 cycles. Table (2) presents the analysis made with two types of materials for attainment the target reliability index, with the cycle stress equal to 319.16 MPa.

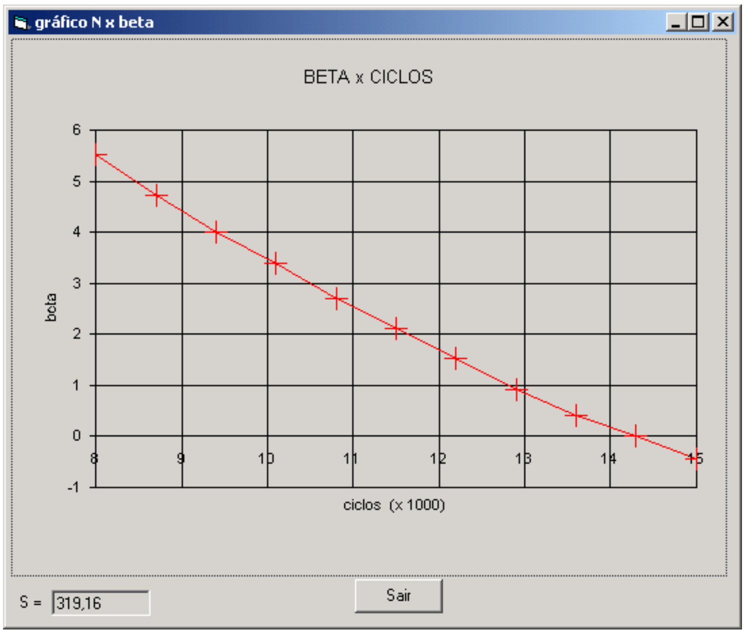
Table 2. Reliability index obtained for two materials.

Material	*b*
AISI 4340	11.42
AL 2024-T4	-0.45

Its clearly noticed that with the coefficients of variation used in this analysis the increase of the mechanics strength of the material implies in an improvement of the reliability index. In case the designer is interested in carrying through a graphical analysis in the band of interest for stress between 250 and 350 MPa and for the number of cycles between 8000 the 15000, a graphical analysis is present, as shown in Fig. (6).



(a)



(b)

Figure 6. (a) Graphical analysis to stress *versus* reliability index and (b) reliability index *versus* cycles.

The graphic in Fig. (6a) show the importance for the designer to make use of a mechanism of probabilistic evaluation for the execution of a project, seen the sensitivity of the value of the reliability index in relation to the variation of the stress level. A small increment in the value of the stress decreases significant the reliability index, raising the failure probability of projected shaft.

Again, in Fig. (6b), it is possible to the sensitivity of variation of the reliability index, in this case, in function of the number of cycles. This resource allows the designer to determine and to visualize in way more insurance and exact the shaft life for a target failure probability.

6. Conclusions

The system developed in this work showed to be of fundamental importance to assist users who are involved in projects in which the failure for fatigue are studied and where also takes in account the scatter of the involved variables. This because, without the aid of an automatized system, becomes practically impossible the designer to make use of the techniques of the advanced conditional probabilistic method, presented in this work.

With the aid of programs capable to execute such task the paradigm of not develop mechanical projects considering the statistical scatters of variables is broken, what allows the designer, to have greater domain and perception on the consequences that some change in one of the variables will cause in the behavior of the structure. This means the possibility of carrying through a well sizing, optimized and safe project.

This program, besides carrying through the calculation of the reliability index and the failure probability, also offers to the designer the specialized suggestions to get greater reliability index.

Through the case example analyzed in item 5 of this work, it can be verified that this system, besides being in accordance with resulted given by literature, supplies to the designer a vision that is not obtained when he executes only the traditional deterministic analysis, propitiating a greater precision in the evaluation the effects of some changes made in the design.

In accordance with the workmanships cited in the bibliographical reference the used theoretical methods in this work currently are being used in fatigue reliability analysis. Moreover, a scarcity of works published regarding expertise systems that deal with the phenomenon of the high-cycle fatigue. Therefore, research regarding the implementation techniques and development of specialist systems, as well as the use of more efficient methods in the determination of the fatigue life of a mechanical component must be done in the future.

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