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ON THE APPLICATION OF THE DANG VAN CRITERION IN FRETTING FATIGUE. PART II: THE USE OF A CRITICAL VOLUME AVERAGING METHODOLOGY

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***Abstract.** It was shown in Part I of this work that the Dang Van mesoscopic scale criterion underestimates the fretting fatigue strength of components subjected to severe stress gradients. In Part II, the existence of a critical stressed layer, or volume, is used to explain these results qualitatively, and an averaging methodology is developed to allow the extension of the mesoscopic scale approach to cases of rapidly varying contact stress fields. It is shown that the proposed averaging methodology provides realistic estimates of fatigue strength and predicts the observed size effect reported in the experimental work, although the critical dimension does not seems to be a material parameter.*

***Keywords.** fretting fatigue, multiaxial fatigue, mesoscopic scale approach, contact size, averaging methodology, critical volume*

1. Introduction

Due to the superficial damage involved in the fretting phenomenon many empirical laws (Ruiz et al., 1984, Ruiz and Chen, 1986), which correlate the amount of microslip on the contacting surfaces with stresses produced by the tangential and bulk fatigue loads, have been proposed in order to predict the fretting fatigue crack nucleation site. Although comparisons with a significant set of experimental data have shown that such empirical parameters can successfully predict the location of crack nucleation (Neu et al., 1999, Kuno et al., 1989, Ruiz et al., 1984, Ruiz and Chen, 1986), they do not provide any measure of fatigue strength. Based on experimental work (Warlow-Davies, 1941, McDowell, 1953) investigating the role of the superficial fretting wear on life Ciavarella (2003) has suggested that fretting fatigue should be treated as notch analogue problem (Araújo and Nowell, 2002). Hence, if the methodology considered to predict the fretting fatigue strength can take in account the effects of the stress concentration caused by the contact problem, the whole process could be treated in a stress basis (for high cycle fretting fatigue).

In a similar way to cracks at notches, fretting cracks nucleate at points of high local stress where the material is under a complex multiaxial stress state. Moreover the stress concentration is extremely localised, decreasing very rapidly away from the initiation point. However, as shown in Part I of this work, multiaxial initiation parameters based on critical plane approaches do not account for the effects of stress gradients. Rather, they suggest that surface stress-strain behaviour at some particular high stressed point is sufficient to quantify fatigue damage. This may well be the case for fretted components with a relatively slowly varying contact stress field (Szolwinski and Farris, 1998). However, for more rapidly varying stress fields, it appears sensible to argue that a high localised maximum is insufficient and that high values of the initiation parameter must be sustained over a characteristic length or volume for the crack to keep on growing (Fouvry et al., 1999). Therefore, the concept of a stress gradient will now be introduced to the mesoscopic scale approach previously discussed. Similar approaches have been attempted by other researchers, mainly in notch fatigue analysis. The concept of a “critical layer” for the analysis of the elastic shakedown limit at notch roots was introduced by Stieler (1954) as early as 1954. Peterson (1959), Neuber (1958) and Siebel and Stieler (1955) derived a relationship to predict the effect of notch size on life. They established that crack initiation depends on the volume of material stressed and on the stress gradient. More recently Flavenot and Skalli (1989) have defined a critical layer for characterising the microstructural state of materials. They suggested that this critical layer is a constant for a specific material and could be related to microstructural dimensions such as grain boundaries. Miller’s short crack work (1993), showing that the material fatigue limit is related to the strongest microstructural barrier, seems to corroborate this idea.

The current paper aims to develop an averaging methodology, which, combined with a multiaxial fatigue strength criterion, will give a less conservative prediction of fretting fatigue strength. Therefore the concept of a stress gradient will now be introduced to the mesoscopic scale approach discussed in Part I (Araújo et al., 2003). In contrast to Flavenot and Skalli (1989) it will not be assumed initially that the averaging dimension is a material constant, such as the grain size. Rather, the solution will involve the search for a dimension or volume, which will best fit the set of experimental data presented. In order to do this the following averaging methodology is proposed.

2. Averaging Methodology

In Part I the Dang Van mesoscopic scale criterion was applied to the analysis of a set of fretting fatigue experiments carried out with different contact sizes. Here we will attempt to extend this analysis to the case of high stress gradients by averaging the model over a critical volume.

An averaging technique can be developed by arguing that high stresses must be sustained over a critical volume, V_c, in order for a crack to breach the strongest microstructural barrier. This characteristic volume might be thought of as corresponding to the grain size or other microstructural dimension, but the analysis will proceed by assuming that the volume may not be a material constant. The averaging methodology involves a square area element surrounding the initiation site to delineate the volume (the current analysis is 2D plane strain, so although a volume is the appropriate physical parameter to discuss, actual averaging is carried out over an area). A schematic visualization of the technique is presented in Fig. 1 and its implementation may be described as follows:

i) Calculate the cyclic stress field over a grid of x and y points.

ii) Average the deviatoric stress components over each element of volume V_c, and at each load step on the load path.

iii) Calculate the deviatoric residual stress field for the corresponding element of volume applying Dang Van's hypersphere algorithm.

iv) Evaluate the Dang Van parameter ,

$$DV = Max\left[\frac{\tau(t)}{\tau_{fl} - mp_h(t)}\right] \tag{1}$$

for each element of volume and find the global maximum. Here, $\tau(t)$ and $p_h(t)$ are the Tresca (maximum difference between principal stresses at each time (t)) and Hydrostatic stresses of the microscopic deviatoric stress tensor averaged over the characteristic element of volume at each time t of the stress history. τ_{fl} is the fatigue limit under fully reversed torsion and m is a material parameter that depends on τ_{fl} and σ_{fl}, the fatigue limit under pure bending.

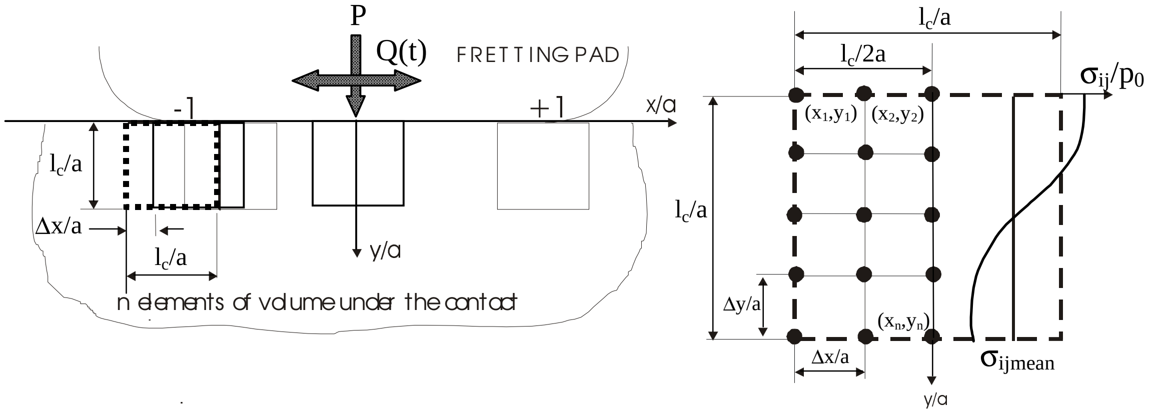

Figure 1– Schematic description of the averaging method.

3. Results

As mentioned in part I of this work, tests produced by Nowell (1988) on an Aluminium alloy have shown a contact size effect where fatigue lives were infinite at smaller contacts and finite for larger ones (Fig. 5). The interesting feature of these data is that within a same series of experiments the stress levels at the surface were identical for tests with different contact sizes. Hence, only the rate of decay of the stress levels changed from test to test. Application of Dang Van's mesoscopic criterion to predict the fatigue strength for such tests have shown that this model is not capable to predict the contact size dependence on fatigue life. We strongly believe that such limitation is related to the punctual nature of the criterion, which assumes that fatigue is controlled by the peak value of the fatigue parameter, irrespective of the affected volume. Hence, the Dang Van criterion cannot account for the presence of sharp stress gradients. Fouvry et al. (1999) have suggested that the parameter needs to be averaged over a characteristic volume in order to properly quantify the fatigue damage for components under severe stress gradients. The averaging method previously described will therefore be applied to reassess the results. To conduct such evaluation the local microscopic stress field was calculated at spatial intervals $\Delta x/a = \Delta y/a = 0.005$ and stresses were averaged over a volume (l_c x l_c x unit thickness in the z-direction) for three different values of l_c. Notice that in the analysis presented in part I a single global maximum of the Dang Van cracking risk parameter $DV_{\max}$ was found for each Al data series. Now, for each combination of l_c and contact size, a, within the same data series a different average global maximum parameter $\overline{DV}_{\max}$ is determined by averaging the stress field over the element of volume. This essentially means that one may use the average parameter in order to try to predict the experimentally observed critical contact size range, a_0, below which infinite fretting fatigue lives were achieved. The largest and smallest contact sizes to show $\overline{DV}_{\max} < 1$ and $\overline{DV}_{\max} \geq 1$, respectively, define the predicted range for each data series. Table 1 records these predictions for the different values of l_c considered. The experimental critical contact size ranges are also recorded for comparisons. It is apparent that good predictions are obtained for most of the data series when l_c varies from 180 to 270 µm. For instance, in *series 1* data the largest contact size to show infinite live was a=0.28 mm, and a=0.38 mm was the smallest contact to show finite live. These results can be exactly predicted by the Dang Van criterion if the stresses are averaged over an element of volume whose side is l_c =270 µm. This is further illustrated in Figs.3 and 4, where the averaged local stress state is plotted together with the cracking risk line. Notice that for a=0.28 mm the local stress state never crosses the cracking risk line (Fig. 3) hence no fretting fatigue initiation is predicted. On the other hand, for a=0.38 mm the local averaged stress state exceeds the fatigue criterion and crack initiation is now expected (Fig.4). However, it is worth noticing that significantly different predictions may be obtained if the stresses are averaged over elements of volume of different dimensions. This may be seen in Fig.5, which depicts the variation of the cracking risk parameter on the surface for l_c varying from 0 to 270µm. The graph was obtained for *Al series 1* data and contact size a=0.28 mm. It is apparent that smaller values of l_c such as l_c =0, which corresponds to no averaging, or l_c =20µm produce conservative results since they predict DV values significantly greater than 1, while the experimental results record no failure. Increasing l_c to 270µm will generate a DV value which is just below unity, but a further increase in l_c would produce a non-conservative lower bound for the critical contact size range a_0, as may also be observed in Table 1.

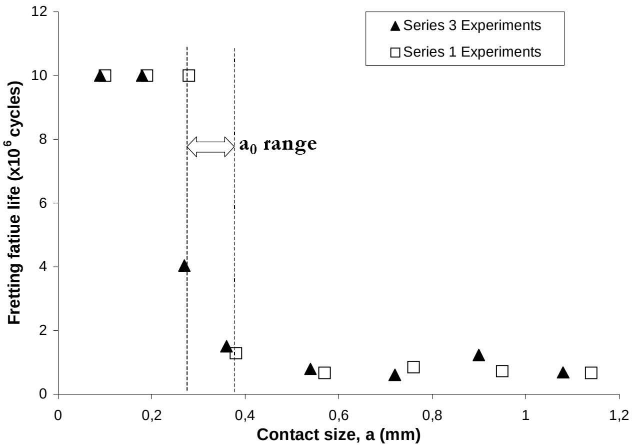

Figure 2– Results of Nowell's (1988) fretting fatigue experiments: variation of life with contact size.

Table 1 – Predicted and experimental critical contact sizes.

Series	a_0(micron)			
	Experiment	l_c=180µm	l_c=210µm	l_c=270µm
1	**280-380**	190-280	190-280	**280-380**
3	**180-270**	**180-270**	180-270	270-360
4	**360-540**	180-360	**360-540**	360-540
5	**570-710**	**570-710**	570-710	710-850

4. Discussion and Conclusions

The analysis of the results previously presented reveals that the Dang Van criterion can not explain the pad size effect in Nowell's experiments with Al4%Cu (Nowell, 1988) if the evaluation of the cracking risk parameter is based on the microscopic local stress state of single superficial severely stressed points. It has been verified that better predictions may be achieved if the local stress state is averaged over an element of volume. This simple procedure developed a mean to introduce the effect of different stress gradients in the Dang Van multiaxial fatigue parameter. In fact, a number of authors (e.g. Miller, 1993, Flavenot and Skalli, 1989, Fouvry et al., 1999) have suggested that high values of stress must be sustained over a characteristic dimension or volume for crack initiation to take place and the averaging methodology successfully incorporates this concept. However, difficulties arose since the results provided by the average Dang Van cracking risk parameter showed that there is no single averaging volume, which was able to correctly predict the critical contact size range for the different data series and load conditions used (see Table 1). For instance, to predict a contact size range of the right order in Al series 1 and 3 data a l_c =270 and 180μm, respectively, are required, while for Al series 4 a l_c =210μm would do and for Al series 5 a value of l_c superior than 180μm would be necessary. Again, these results suggest that either the averaging parameter is not a true material constant, or that the Dang Van multiaxial fatigue parameter does not accurately characterise the initiation process in cases where high stress gradients are present. One should notice however that the value of the fatigue limit in torsion was estimated and this may have brought some inaccuracy to the analysis. On the other hand, it seems clear that the stress gradient plays an important role on the crack nucleation phenomenon and that an average methodology is needed if this effect is to be taken in account. However, it is far from clear how one can estimate an appropriate averaging volume *a priori*, should it not be a material constant.

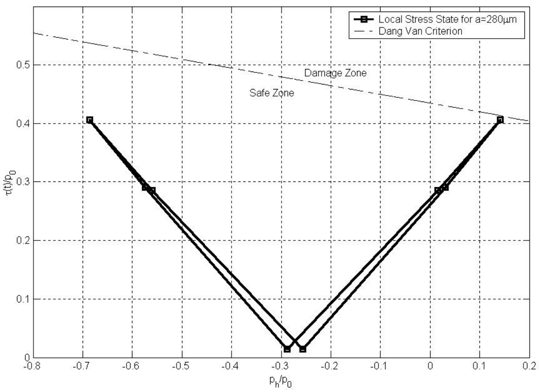

Figure 3 – History of average local stress state and Dang Van fatigue criterion for *Al series 1* data at (x,y)=(-1;0) and a=0.28 mm. Stresses are averaged using l_c=270 μm.

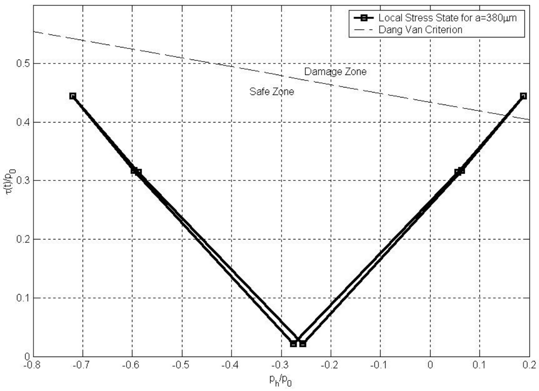

Figure 4 – History of average local stress state and Dang Van fatigue criterion for *Al series 1* data at (x,y)=(-1;0) and a=0.38 mm. Stresses are averaged using l_c=270 μm.

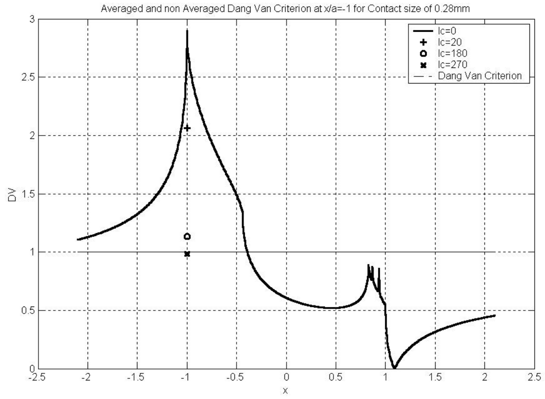

Figure 5– Averaged and non Averaged Dang Van Criterion at x/a=-1 for contact size of 0.28mm.

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